A Multi-objective Mathematical Model for Problems Optimization in Multi-modal Transportation Network

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Abstract: In order to reach a sustainable planning in a rather complicated transport system, it is of high interest to use methods included in Operations Research areas. This study has been conducted to solve the transportation network planning problems, in accordance with the optimization problem and multi-objective transport network in multi-modal transportation. Firstly, we improve the implementation of the existing literature model proposed in (Cai, Zhang, and Shao, 2010; Zhang and Peng, 2009) because after the conducted experimentation, we show that there are two previously proposed constraints that make the solution unrealizable for the transportation problem solving. Secondly, we develop the proposed multi-objective programming model with linear constraints. Computational experiments are conducted to test the effectiveness of the proposed model. The mathematical formulation is developed to contribute to success solving the optimization problem, taking into account important aspects of the real system which were not included in previous proposals in the literature, and review. Thus, it gives ample new research directions for future studies.

1 INTRODUCTION

The multimodal transportation offers a full range of transportation modes and routing options, allowing them to coordinate supply, production, storage, finance, and distribution functions to achieve the most efficient relationships. The goal is to move from the starting city to the destination city through other intermediate cities, of which there are several routes between two cities. In the multi-objective optimization problem, the decision maker is charged by an efficiency choice of existing routes in order to select the best itinerary according to a compromise solution between a set of objectives such as the minimization of the transport cost and the duration of transport, the maximization of service quality, etc.

The multimodal transportation network studies were carried out by several problems such as planning networks, shortest path, maritime or airline with urban centers, freight transport, transmission line, loading-unloading terminals, schedules, etc. The focus of most widely research in the literature has been based on planning network.

There are various measures to evaluate a multi-modal path, for example, the travel cost, in-vehicle time, waiting time, length, travel time, transfer time, the number of transfers and so on. The optimization and the operation research play an important role to solve this problem. The main objective of this problem is to determine the shortest and efficient way of satisfying a set of objectives, and a set of operational constraints according to customer demands.

In general, the objective of a multimodal network planning problem is to optimize reliable transport chains for passenger or freight. The mathematical formulation of the transit network design is usually intractable by exact approaches. In (Wan and Lo, 2003) a MILP formulation that minimizes the operating cost to a bus capacity constraint is proposed. A characteristic of their formulation is that it allows generating implicitly the structure of the routes. However, this requires that a maximum number of routes in the solution should be specified. The objective minimizing the operating costs, according to constraints within the system considering the capacity and bounded exchange action frequency.

The multimodal shortest path problem (M-SPP) is concerned with finding a path from a specific origin

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to a particular destination in a given multimodal network while minimizing total costs associated. The complexity of finding multi-modal route is obviously much higher than single modal one. The multi-criteria multi-modal shortest path problem (MM-SPP) with transfer delaying and arriving time-window belongs to the set of problems, which are known as NP-hard. In (Liu, Mu, and Yang, 2014; Liu, Yang, Mu, Li, and Wu, 2013) the exact algorithms for solving the MM-SPP to minimize the total travel time have been suggested, in which the delaying time in the transfer parking and the arriving at a time window of destination are considered, as well as the total travel cost.

Each case treated is defined by the specific parameters at the problem type. Since, each variant possesses its own characteristics, it requires a different decision depending on the considered context. These decisions are based on the special characteristics of the transportation mode, and on specific constraints of the treated problem. These constraints are specified for each customer, vehicle, mode, road or means of transport, as well as the type of the problem.

Being based on the existing works of the literature research, we have adopted a mathematical model, while relying on the existent works with the objectives, and constraints set to take into account the recommendations made by experts according to the hypotheses of our treated problem. This proposed formulation will be cited and validated by tests, which will be detailed in the rest of this paper.

The structure of our paper is organized as follows: In section 2, we'll present the construction model with the proposed formulation in order to solve the multi-objective and multimodal transportation problem. Section 3 discusses our contribution and motivation. Section 4 provides the proposed model by some numerical experiments. Thereafter, in section 5 we will discuss a critical comment on our work by a synthesis of the obtained results. Finally, section 6 concludes our work with a summary and proposes some future research directions.

2 A MULTI-OBJECTIVE MATHEMATICAL MODEL

The main objective of the multimodal network problem is to determine a shortest and an optimal path between a start point and an end point to according to several criteria relating to the transportation mode or the itinerary, etc., to satisfy a set of objectives that are distinct to the treated case problem. In fact, a multimodal problem requires the consideration of multiple objectives and linked constraint of a sequence of frequently used modes. For an optimal choice of a transport mode or an itinerary by a transport mode selected, the various criteria must be taken into consideration, although these criteria are conflicting.

The multi-objective optimization can be defined as the problem that is finding a vector of decision variables which satisfies all constraints and optimizes a vector of objective functions. These functions from a mathematical description of performance criteria are usually in conflict with each other. In this paper, we have treated a multi-objective optimization problem. We consider the problem studied is to find viable multimodal and multi-objective transport processes, in order to minimize the total transportation cost and the total time of the itinerary, while respecting the arriving of goods at a customer in the corresponding time window. First, we will present and discuss the model of (Cai et al., 2010).

2.1 Model Assumptions and Code Description

2.1.1 The Assumptions

Let us define the following assumptions as:

- Only one mode of transport and a path can be chosen between two nodes to carry the goods.
- Transport costs are directly proportional to the realization, namely, the choice of the quantity and the unit transportation cost.
- The limited capacity constraint of each mode is respected.
- If a vehicle arrives at the node before the start date of his time window, he waits.
- Transshipment of goods can only happen once more at each node.

2.1.2 The Sets and Settings

N : The set of all nodes;

K : The set of transport mode;

Q : The total quantity of goods;

P : The maximum transfers duration;

p_i : The delay period at node i if delay occurred;

f_i : The overhead expenses per hour if delay occurred at node i;

$\boldsymbol{C_{i,j}^{k}}$: The transport cost of a unit quantity from node *i* to node *j*, by using k^{th} transport mode;

$\boldsymbol{c_{i}^{k,l}}$: The fee for transport mode changed from k to l at the node *i*;

$\boldsymbol{t_{i,j}^{k}}$: The transport time from node *i* to node *j*, with k^{th} transport mode selected;

$\boldsymbol{TW_{ij}}$: The largest time windows of cargos arriving from node *i* to node *j*;

$\boldsymbol{tw_{ij}}$: The shortest time windows of cargos arriving from node *i* to node *j*;

$\mathbf{a_{i}^{k,l}}$: The transfer time from transport mode *k* to the transport mode *l* at the node *i*;

$\boldsymbol{F^{k}}$: The vehicle capacity from the k^{th} transportation mode.

$\boldsymbol{S^{k}}$: The number of vehicles used by the k^{th} transportation mode in order to transport the whole quantity of the freights. With, $S^{k} = Rounds\left(\frac{Q}{F^{k}}\right)$ upward, that returning the smallest integral value that is not less than $\left(\frac{Q}{F^{k}}\right)$.

2.1.3 The Decision Variables

The decision is related to the optimization process which focuses on itinerary scheduling and the decision of selecting each transportation mode of corresponding transportation means. Thus, we need to define the decision variables that explain the variables associated with each considered parameter of our treated problem. These decision variables are used to express the constraints and optimization criteria.

$$\boldsymbol{x_{i,j}^{k}} = \begin{cases} 1 \text{ if the } k^{th} \text{ transport mode is selected from } i \text{ to } j, \\ 0 \text{ otherwise} \end{cases}$$

$$\boldsymbol{y_{i}^{k,l}} = \begin{cases} 1 \text{ transport mode changed from } k \text{ to } l \text{ at } i, \\ when\ k \neq l, \\ 0 \text{ otherwise} \end{cases}$$

$$\boldsymbol{u_{i}} = \begin{cases} 1 \text{ if there is a delay at the node i} \\ 0 \text{ otherwise} \end{cases}$$

2.1.4 The Formulation

The mathematical formulation is a determinant step in the resolution step and the optimization step of any problem. Indeed, it allows us to define and characterize the sets, the parameters, the decision variables, the optimization criteria and constraints that will satisfy the specific decisions. The paper presents an effective solution for determining the shortest and efficient way of satisfying a certain set of demands under several criteria and also a large set of operational constraints. Our proposed formulation is presented as follows:

Minimize

$$\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} C_{i,j}^{k}.S^{k}.x_{i,j}^{k} + \sum_{i\in N}\sum_{k\in K}\sum_{l\in K} c_{i}^{k,l}.y_{i}^{k,l} + \sum_{i\in N} u_{i}.f_{i}.p_{i} \quad (1)$$

$$\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} t_{i,j}^{k}.x_{i,j}^{k} + \sum_{i\in N}\sum_{k\in K}\sum_{l\in K} a_{i}^{k,l}.y_{i}^{k,l} + \sum_{i\in N} u_{i}.p_{i} \quad (2)$$

$$Max\left[\left(\left(\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} t_{i,j}^{k}.x_{i,j}^{k} + \sum_{i\in N}\sum_{k\in K}\sum_{l\in K} a_{i}^{k,l}.y_{i}^{k,l} + \sum_{i\in N} u_{i}.p_{i}\right) - \sum_{i\in N}\sum_{j\in N} TW_{ij}\right),0\right] +$$
$$Max\left[\left(\sum_{i\in N}\sum_{j\in N} tw_{ij} - \left(\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} t_{i,j}^{k}.x_{i,j}^{k} + \sum_{i\in N}\sum_{k\in K}\sum_{l\in K} a_{i}^{k,l}.y_{i}^{k,l} + \sum_{i\in N} u_{i}.p_{i}\right)\right),0\right] \quad (3)$$

Subject to

$$\sum_{k\in K} x_{i,j}^{k} = 1 \forall i, j \in N \quad (4)$$

$$\sum_{k\in K} x_{i,j}^{k} - x_{j,i}^{k} = 0 \forall i, j \in N \quad (5)$$

$$x_{j,i}^{k} + x_{i,j}^{l} \geq 2.y_{i}^{k,l} \forall i, j \in N \quad (6)$$

$$\sum_{k\in K}\sum_{l\in K} y_{i}^{k,l} = 1 \forall i \in N \quad (7)$$

$$\sum_{i\in N}\sum_{k\in K}\sum_{l\in K} y_{i}^{k,l}.a_{i}^{k,l} \leq P \forall i \in N \quad (8)$$

$$x_{i,j}^{k}, y_{i}^{k,l} \in \{1,0\} \forall i, j \in Net \forall k, l \in K \quad (9)$$

The equations that describe this mathematical formulation can be summarized as follows. Equation (1) represents the first objective that seeks to minimize the total cost of the multimodal network, including the cost of the itinerary, transshipment cost and overhead cost on delay. Equation (2) defines the second objective, which seeks to minimize the total duration of multimodal transportation, including the period of the itinerary, changing period and delay duration. Equation (3) expresses the third objective that guaranteed the arriving at the destination in the time window. Constraint (4) is specific to the selection of transportation mode, that only one mode of transport and one itinerary can be selected between two nodes. If it is zero, it means that the *i* node is not included in the transport. Equation (5) demonstrates that in the itinerary the destination node is the start node for the next itinerary. Constraint (6) shows that the selection of the route should be ensured by a continuous itinerary. Equations (7) and (8) are relative to the transshipment constraints. Constraint (7) indicates that one change of transport mode can happen once at each node. Constraint (8) represents the maximum time to be respected by the total transshipment time. The decision-making variables taking the integer binary value are described by the equation (9).

3 CONTRIBUTION AND MOTIVATION

In this paper, we have proposed a multi-objective mathematical program inspired from (Cai et al., 2010) and (Zhang and Peng, 2009). The authors have addressed the multi-modal transport problem with full loads at time limit. The problem is defined by the search of a multimodal path in order to reach the destination through several cities, knowing that there are several transport modes possible between any two cities. Each itinerary is characterized by a transport duration, a cost, and a transportation capacity between two cities. In fact, the authors proposed a combination model for multi-modal transport of full loads with time window constraint. The considered assumptions are: firstly only one mode of transport can be selected between two cities and secondly, the transport cost is linear with distance. The same model defined in (Zhang and Peng, 2009)is presented in (Cai et al., 2010). The distinction parameters between our mathematical formulation and the one presented by (Cai et al., 2010; Zhang and Peng, 2009) are presented as follows:

$l_{i,j}^k$: The transport distance from i to j with the k^{th} transport mode selected, for j=i+1.

T: The time limit from start point to the end.

The distinction constraints that indicates:

- The cargos to be arriving in limited time.

  $\sum_{i\in N}\sum_{k\in K} t_{i,j}^k\, x_{i,j}^k + \sum_{i\in N}\sum_{k\in K}\sum_{l\in K} a_i^{kl} y_i^{kl} \leq T\ ,\forall\, j = i+1$

- That a number of cargos cannot exceed the capacity of conveyance.

  $Q \leq F_{i,j}^k, \forall\, i \in N, k \in K, j = i+1$

- The time-window between two nodes.

  $$tw_{i,j} \leq t_{i,j}^k\, x_{i,j}^k + a_i^{kl} y_i^{kl} + u_i p_i \leq TW_{i,j}$$

Although, we describe the main improvements made to the literature model. The first objective is to minimize the total cost of transport. This cost is measured by the sum of three terms. At the transportation cost term, we replaced the distance parameter by a measured relative to the goods transported (the number of used units of transport), according to experts of the domain. In our case, we will ignore the setting of the distance since there is only one path between two nodes made by the same transport mode k. We also note that it is useless to consider the distance parameter when calculating the transportation cost. On the other hand, we show the importance of considering the goods quantity and the number of transport units used in measuring the transportation cost.

The third objective is provided by the transformation of the time window constraint $tw_{ij} \leq \left(t_{i,j}^k . x_{i,j}^k + a_i^{k,l} . y_i^{k,l} + u_i . p_i \right) \leq TW_{ij}$, to an objective, which assured to the goods arrive at a well-determined interval of time. The main reason for this transformation is on the one hand, in order to give more chance to find a compromise solution, which can be eliminated when it is a constraint. In fact, the given solution of a problem must satisfy all the constraints while minimizing (or maximizing) one or more objectives. On the other hand, when it is an objective we can easily play on their weight or priority relative to the other objective. Moreover, that we can find a solution that is preferred according to a customer that will be eliminated by this constraint. Indeed, the requirements of customers are different. In some cases, the customer prefers that the arrival of their goods, regardless of the time of this merchandise’s arrival.

Pointing out the limitation of the previous formulation is indicated as follows. Taking into account the constraints (4) and (5) on the model of (Cai et al., 2010), the model resolution remains non-feasible. With regard to the definition of constraint (4) that express that cargos will be arriving in limited time, we consider that it is useless to introduce the first part of the expression proposed by the authors. So, we limit the constraint to its second part.

The constraint (5) makes it a non-feasible problem, because if we consider the example of a product fertilizer with a quantity Q = 122 tons, we can't transport all this quantity in a single vehicle. One railroad car can carry 61 tons of fertilizers. Therefore, two railroad cars are needed if railway transport is selected. Each car can carry 35 tons per vehicle, if the road transport is chosen, the total of 3.5 cars, therefore, four cars are needed to transport all the quantity. Each boat can carry 10 tons, if water transport is selected, a total of 12.2 vessels, is taken as 13 boats. So, the equation (5) with these data are expressed as the following: $Q \le 61$ by railroad mode, $Q \le 35$ by road transport mode and $Q \le 10$ by maritime transport mode, which is an impossible inequality. Consequently, this constraint is missing a whole other decision variable. A new parameter must be added a such as n^k that is the number of vehicle used by the transport mode k. Therefore, this constraint becomes, $Q \le n^k F^k_{i,i+1} \forall i \in N,\ k \in K$. We consider the example test that is provided by one compound fertilizer company located in Linyi City.

The element of the first objective $\sum_{i\in N}\sum_{k\in K} u_i q_i p_i$ depends only on to the parameter i, so it must be replaced by $\sum_{i\in N} u_i q_i p_i$. In regards to the element of the second objective $\sum_{i\in N}\sum_{k\in K} u_i p_i$ depends only on to the parameter i, so it must be replaced by $\sum_{i\in N} u_i p_i$. Therefore, the delay in a city is compared to the desired arrival time.

When the index of the parameters are considered according to i, j for $j = i+1$, then the passage is forced by all the nodes according to an increasing order. Therefore, the parameters should be defined by the index i, j in order to guarantee that the choice of the nodes and the order are provided by the model.

Based on the discussion of features, the organization of multi-modal transportation modes, and time-windows introduced, we defined a more efficient model for multi-modal transport of full loads with time-windows.

4 EXPERIMENTATION RESULTS

In this section, we present the obtained results that show the capability of the proposed model for solving a complex problem with multiple objectives (linear and non-linear) simultaneously that proves their efficiency in decision-making. This section is devoted to presenting the computational experiments carried out for assessing the performance of our mathematical model. In fact, we implemented an integer program for solving the multi-objective and multimodal transportation networks planning models, by using Concert Technology of CPLEX 12.4 Optimizers, with Microsoft Visual Studio 2010. Computational experiments are conducted to test the effectiveness of the proposed model.

For a visual representation of the experimental results, the reader is referred to Figure 1 as illustrated below. The optimal solutions are represented by the curve of the set of Pareto solution. The feasible optimistic of the first objective vary between $Z_{\max}$ = 2475.200 Yuan and $Z_{\min}$ = 804.96 Yuan. The optimistic solutions of the second objective vary between $Z_{\max}$ = 81H and $Z_{\min}$ = 14 H. The optimistic solutions of the third objective vary between $Z_{\max}$ = 81H and $Z_{\min}$ = 34 H. These solutions are obtained by a set of tests sample generation.

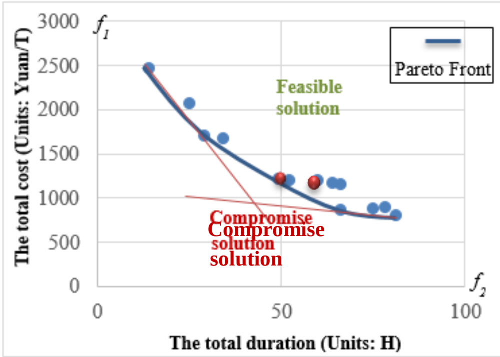

Figure 1: Illustration of the Pareto optimality of a two objectives minimization problem.

We observed that the third objective and the second objective were synchronized objectives. But, the two objectives are showing that they are clearly contradictory with the first objective.

The overall measurement results are summarized in Table 1 and 2 that are presented in the Appendix section. The best solution that minimizes the first objective gives a bad value for the second objective, and vice versa. In this case, we must seek the solution that satisfies the best compromise between these objectives. This solution is presented in Figure 1 with a red point, which is the closest value of the origin. The Multiple Pareto optimal solutions based on five distinct solutions found by the experimentations tests, are represented by a blue curve in Figure 1. Hence, a solution is called a Pareto optimal, if no feasible vector exists which can decrease some criterion without causing a simultaneous increase in at least one criterion. In figure 1, a continuous line is used to mark this boundary for a bi-objective minimization problem in which, there is no single perfect solution that minimizes both f_1 and f_2. The aim is of minimizing the compromise between the total cost and the total duration for an itinerary of the multimodal transportation network.

The set of all Pareto optimal solutions, called non-dominated set or Pareto front, is located on the boundary of the objective vector space (feasible solution space) showing the tradeoff information between the conflicting objectives. Instead, there are compromises between optimal solutions such as the solution presented by red. We can say that the solutions presented in the Pareto front are more optimistic than the dominated solutions.

The various solutions which belong to the front of Pareto are optimistic solutions. However, the decision-maker is provided by the search of the best compromise solution between the goals, which is included on the Pareto Front. The CPLEX tool uses the dynamic mixed-integer programming (MIP) as a search method, the balance optimality, and feasibility MIP emphasis. Indeed, the relaxation solution obtained by a Branch and Cut algorithm through a deterministic parallel mode, uses up to 4 threads.

The definition of Pareto optimality is similar to that of efficiency, and a Pareto optimal point in the criterion space is often considered the same as a non-dominated point. Therefore, a solution is considered as Pareto optimal, in a multi-objective minimization problem, if there exists no other feasible solution which would decrease some criteria without causing a simultaneous increase in at least one criterion. This set also called Pareto front helps the decision maker to identify the best compromise solution by an elimination of inferior ones. Then, the retained solution as elite is the one which has the best compromise between all the objectives.

5 DISCUSSION AND COMMENTS

According to the tests achieved for our work, giving the priority to the first objective by minimizing the total cost is much better than giving the executing priority of the total duration which perfectly complies with the third objective that respects the arriving at a time window. Therefore, we note that the first objective significantly improves the results as shown in the Appendix.

For real case problems, the mathematical formulation is not always reliable for a user, since we cannot consider fixed rules for all possible alternatives and treated cases associated with each customer. According to the experts of a transit company, the choice of transportation mode is depends on the several factors, such as customer requirements, the nature, and characteristics of the goods, that can be a major condition for the selection of the mode's problem. There are several features of the goods, such as the expensive, bulky or perishable goods, also the goods category or type such as dangerous, fragile, light, stackable or non-stackable goods, etc.

On the basis of the promising findings presented in this paper, further research will be needed to solve more optimization constraints. Artificial intelligence approaches to the issue are still required. (Bouamama, 2010) proposed a multi-agent approach based on a dynamic, distributed Practical Swarm Optimization algorithm, which is proven to be useful for hard optimization problems. (Mathlouthi and Bouamama, 2015) proposed two new approaches, a centralized and distributed honey-bee optimization, enhanced by a new parameter called local optimum detector. These two approaches are applied to solve the maximal constraint satisfaction problems.

6 CONCLUSIONS AND FUTURE WORKS

This paper presents a new mathematical model in order to solve multi-modal transport problems that satisfy multiple objectives according to several criteria. The proposed multi-objective model is defined by three objectives, the minimization of the total cost, and the total duration of an itinerary while respecting the arriving of goods to a customer at the time window.

In fact, the following conclusions can be drawn as: Firstly, an improvement over the literature model was achieved in order to find feasible solutions. Secondly, a validation of our proposed model by implementation. Thirdly, we summarized our work by computation's test and experimentation results in order to prove the efficiency of our model.

This paper presents a decision method based on a mathematical model which plays a significant role in resolving the transportation problem. Although there has been a fruitful development of models and solution techniques to solve this problem by a relevant decision in transport networks, many future pieces of research prospects are still missing, such as the following:

- There are still opportunities for integrating problems that can be solved separately, by using a multi-criteria analysis approach.
- The development of robust, or dynamic approaches used to solve a planning problem.
- The consideration of several types of products, around on the corresponding product’s cluster with the same characteristics, since the choice of the transport mode depends on the product volume and the value associated with each product.
- The activation or cancellation of a goal according to customer wishes by adding weights to each objective, according to the client's need.

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APPENDIX

Table 1: The experimentations tests in order of priority: objective 2 then objective 1.

		Objective	IInf	Best integer	Cuts/Best Bound	ItCnt	Gap	Solutions
S1	***Z2****			25		1	--	2
		integral	0	**14**	14	1	0.00%	
	Z1 if Z2*	**2475.2**						
S2	***Z1 if Z2<=29***			2003.40		2	--	2
		integral	0	1701.57	1701.57	2	0.00%	
S4	***Z1 if Z2<=64***			2725.69		8	---	3
		904.3059	6	2725.69	904.3059	8	66.82%	
				1244.74	904.3059	8	27.35%	
		1158.2820	2	1244.74	Cuts:7	11	6.95%	
				1169.49	1158.2820	11	0.96%	
		Cutoff		1169.49		11	0.00%	
S5	***Z1 if Z2<=75***			1455.72		8	--	4
		827.544	2	1455.72	827.544	8	43.15%	
				1168.19	827.544	8	29.16%	
		878.02	1	1168.19	Cuts:4	9	24.84%	
				884.62	878.02	9	0.75%	
				883.62	878.02	9	0.63%	
		Cutoff		883.62	883.62	9	0.00%	

Table 2. The experimentations tests in order of priority: objective 1 then objective 2.

		Objective	IInf	Best integer	Cuts/Best Bound	ItCnt	Gap	Solutions
S1	***Z1****			1153.89		2	--	2
		integral	0	**804.96**	804.96	2	0.00%	
	Z2 if Z1*	**81**						
S2	***Z2? if***			75.0		5	--	3
	Z1<=1170	51.6294	6	75.0	51.6294	5	31.16%	
				66.0	51.6294	5	21.77%	
				64.0	51.6294	5	19.33%	
		cutoff		64.0		5	0.00%	
S3	***Z2? if***			75.0		7	---	3
	Z1<=1500	37.4761	3	75.0	37.4761	7	50.03%	
				49.0	37.4761	7	23.52%	
				46.0	37.4761	7	18.53%	
		39.2491	9	46.0	Cuts:5	13	14.68%	
		Cutoff		46.0	46.0	13	0.00%	