Comparing Small Population Genetic Algorithms over Changing Landscapes

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Abstract: This paper examines the performance and adaptability of a number of small population Genetic Algorithms (GAs) over a selection of dynamic landscapes. Much of the research in this area tends to focus on GA with relatively large populations for problem optimisation. However there is research, which suggests that GAs with smaller populations can also be effective over changing landscapes. This research compares the performance and adaptability of a number of these small population GA over changing landscapes. With small population GAs, convergence can occur quickly, which in turn affects the adaptability of a GA over dynamic landscapes. In this paper five GA variants using small population sizes are run over well-known unimodal and multimodal problems, which were tailored to produce dynamic landscapes. Adaptability within the population is considered a desirable feature for a GA to optimise a changing landscape and different methods are used to maintain a level of diversity within a population to avoid the problem of premature convergence, thereby allowing the GA population adapt to the dynamic nature of the search space. Initial results indicate that small population GAs can perform well in searching changing landscapes, with GAs which possess the ability to maintain diversity within the population, outperforming those that do not.

1 INTRODUCTION

Genetic Algorithms (GAs) (Holland, 1992) (Goldberg, 1989) are search algorithms based on Darwinian principals of survival of the fittest. GAs are population based optimisers and as population evolve from generation to generation they tend to become homogeneous as the population converges. GAs are widely used for various optimisation problems, however optimisation over changing landscapes, can prove more difficult compared to that of static landscapes as the populations needs to be able to adapt and escape local optima as the landscape changes. Therefore, if populations tend to converge on a particular solution, a GA needs to be able to evolve its population to respond to changes over a changing landscape. Because of this, the ability for the GA population to adapt to changes in the search space is critical in optimising changing landscapes. Adaptability (Mori and Kita, 2000)(Krishnakumar, 1990)(Mumford, 2004) is considered to play an important role in how GA searches through a changing landscapes. Associated with adaptability is the maintenance of diversity within the population.

With this in mind, a decision needs to be taken whether it is better to use a standard GA and restart the population when the landscape alters or to use a GA which maintains an element of diversity within the population. Research such as (Cobb and Grefenstette, 1993; Krishnakumar, 1990; Grefenstette et al., 1992; Goldberg and Smith, 1987) looked at stationary and non-stationary landscapes and found that GAs were able to optimise functions over both landscapes, with the maintenance of population diversity being a critical feature. Population size is also an important consideration in the optimisation of a changing landscape. Researchers such as (Ahn and Ramakrishna, 2002) (Grefenstette, 1986) (Whitely, 1989) (Krishnakumar, 1990) found small populations to be just as effective in optimizing certain landscapes, thereby avoiding the additional overhead associated with a larger population.

This paper examines and compares the performance and adaptability over changing landscapes, of a number of GA implementations using small populations. The motivation is to compare the performance and adaptability of a number of small population GAs over changing landscapes in order to examine the impact of diversity maintenance and determine whether restarting a standard GA or the inclusion of a diversity maintenance technique is most ben-

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eficial over the given landscapes. The contribution is to develop an insight into the ability of various small population GAs to adapt and optimise changing landscapes by comparing their performance over specifically designed landscapes. Initial results indicate that the use of particular diversity maintenance techniques in small population GAs, proves beneficial over the chosen landscapes. The paper is laid out as follows: Section 2 examines the background literature. Section 3 outlines the GAs and the test suite used in the experiments. Section 4 describes the findings, Section 5 concludes and Section 6 outlines future work.

2 BACKGROUND

In relation to population diversity and convergence, De Jong (DeJong, 1975) set a threshold value of 95% similarity between loci alleles to indicate convergence. By examining fluctuations in online performance we can get a indication of the level of convergence within the population. Work carried out by Mahfoud (Mahfoud, 1995) defined convergence as occurring when the average population fitness value for the previous 4 generations is greater than the average fitness value for the present generation. Allen et al. (Allen and Karjalainen, 1999) used a simple approach to measure convergence and defined that if no progress has been made after 25 generations then the GA has converged. The approach used is this research is Allen's method because of its simplicity; furthermore this method give the GA a better opportunity of avoiding getting caught in local optima in the landscapes used in this research.

GA theory would suggest that larger GA populations should maintain an element of diversity within the population for a longer period when compared to small population GAs (Leung et al., 1997) (Goldberg and Deb, 1991). If diversity can be maintained, the additional genetic material or building blocks should assist in the adaptability of the GA. However, this comes at a cost of time and complexity as the larger population takes longer to search through (Ahn and Ramakrishna, 2002) there exists a trade off between the advantages of having a large population in terms of searching a search space and the associated additional overhead. Research by Greffenstette (Grefenstette, 1986) and also by Whitley (Whitely, 1989), found that smaller population sizes with slightly higher crossover and mutation rates to be just as affective when searching a landscape. The mutation operator plays a role in introducing diversity by allowing a population which has converged on a particular solution, to open up the landscape to further exploration. However, there is a limit to it's ability to introduce diversity, before it becomes a random walk.

3 EXPERIMENTAL SET UP

In this paper we compare the performance and adaptability of GAs using small populations over changing landscapes. We contrast the results of a simple GA (SGA) (Holland, 1992) to four GA variations: GA with Elitism (GAE), Immigration GA (GAI), Micro GA (MGA) and Diploid GA (DGA). The GA with elitism (GAE) (Goldberg, 1989) retains the fittest chromosomes from generation to generation. As there is no guarantee of a chromosome surviving selection, with a standard GA, elitism guarantees that the fittest individual will be maintained in the population. The immigration GA (GAI), implemented as outlined by (Yang, 2008), results in the best solution being maintained in the population, while the worst individual is replaced by a random immigrant. The Micro GA (MGA) (Krishnakumar and Bailey, 1990) retains the best individual from generation to generation and once convergence is detected, the remainder of the population is randomly initialised. Finally, the Diploid GA (DGA) used in this research is as outlined in (Goldberg and Smith, 1987).

3.1 Test Suite

De Jong(DeJong, 1975) used the Sphere function and Shekel's function in his paper and these functions were used to test a number of parameters at different values. Population size, crossover and mutation rates were examined at different levels and the results produced from these experiments lay the foundation parameters which are still in use today on these functions. The Sphere function is used to test the general efficiency of a GA and Shekel's function is used to test the GAs ability to avoid getting caught in the 25 local optima or foxholes that are present in that landscape. DeJong found that the GA was able to optimise both landscapes and the that the values of mutation population and crossover values play an important in how efficiently a GA optimises a landscape. Other researchers such as Digalakis et al. (Digalakis and Margaritis, 2002) carried out these experiments and were able to optimise the Sphere and Shekel's function by using DeJong parameter values.

3.1.1 Sphere Function

According to DeJong (DeJong, 1975) the sphere function is a unimodal low-dimensional quadratic function with a minimum of zero, in other words it contains no local optima. The Sphere Model (DeJong, 1975) is relatively easy to optimise as it is continuous, convex and unimodal. This function is normally used to measure the efficiency of a particular algorithm. The Sphere function is represented as follows:

$$f(x) = \sum_{i=1}^{d} x_i^2$$

where: $\min(fx) = fx(0,\ldots,0) = 0$ and d is the number of dimensions, $-5.12 \le x_i \le 5.12$ and for all $i = 1,\ldots,d$. The changing environment experiments allow the GAs to search the landscape defined by the Sphere model, with the function being inverted after 100 generations. By changing the fitness function in this manner, what was the global optimum becomes the furthest point from the new global optimum.

3.1.2 Shekel's Function

Shekel's function is a continuous non-convex, non quadratic two-dimensional multimodal function with 25 local minima based at the points. This 2-dimensional function contains 25 different *foxholes*, each varying in depth, surrounded by relatively flat surfaces. Shekel's Foxhole function has the following definition:

$$f(x) = \left[\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right]^{-1}$$

where:

$$-65.536 \le x_i \le 65.536$$

$$a_{ij} = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & \ldots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & \ldots & 32 & 32 & 32 \end{bmatrix}$$

Shekel's Foxhole is a minimising problem, where $f(x) = f(0,\ldots,0) = f(-32,-32) \approx 1$. At generation 500 the objective function is inverted, meaning that the old global optimum is now the furthest point from the new global optimum.

3.2 Parameter Settings

Parameter setting in GA is a key component in getting the algorithm to operate in an efficient and effective manner. Researchers have come up with slightly different parameters over time but are problem depended. The values used in this research are taken from Grefenstette (Grefenstette, 1986) and found that smaller population sizes with slightly higher crossover and mutation rates to be just as positive to converge on a solution and with quicker results time due to less processing time.

Table 1: Parameter Settings.

Parameter settings	Changing Sphere	Changing Shekels
population size	30	30
crossover	0.7	0.7
mutation	0.01	0.01
generations	200	1000
runs	20	20

4 FINDINGS

This section outlines the results for each of the GAs over the changing Sphere function and changing Shekel's function. With graphs plotting the Online (average) and Offline (best) results. As these are minimising problems the results tend to zero on the graph as the GA evolves. The dynamic Sphere function experiments were run over 200 generations, with the objective function altering at generation 100. The dynamic Shekel's function experiments ran for 1000 generations, with the fitness function changing at generation 500. A two sided paired Wilcoxon test was carried out on the offline results for each of the experiments to access whether the population means differ for the variant GA when compared to the SGA. Results were considered to be statistically significant with a $Pvalue < 0.005$.

4.1 SGA (SGA) Changing Sphere Function Results

The SGA results indicate that the algorithm had little difficulty in solving the problem, both before and after the landscape change. Figure 1 illustrates both the online and offline performance. As shown in Table 2, the SGA located the global optimum value of 0, both before and after the change. The results also indicate that the population converges quite quickly, shown by the closeness of the online and offline results. However, as the population had converged on chromosomes representing the original global optimum, once the objective function altered, this turns what were good chromosomes with high fitness values, into very poor solutions. This change in the landscape is designed to examine the GAs ability to adapt and locate the new global optimum. Optimisation takes longer at this point, as the population is made up of solutions that are much further away from the solution than the initialised population at the start.

The SGA adapting to the change in the objective function and searching for the new global optimum is shown by the slight increase in distance between the online and offline results, as the population begins to move away from the area of the search space where is had converged. Following the change in the fitness function, the SGA eventually locates the global optimum (see Table 2) and begins to converge as the distance between the offline and online results decreases. Overall the SGA, using a small population, had little difficulty in optimising the landscape both before and after the fitness solution representations were changed, this is largely due to the nature of the Sphere function.

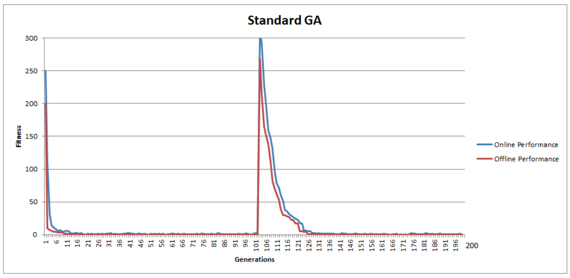

Figure 1: SGA Changing Sphere Function.

4.2 SGA Restart (SGAR) Changing Sphere Function Results

Figure 2 outlines the online and offline performance of restarting or reinitialising the population of the SGA when the landscape alters. The results indicate that in the initial phase of the search, the performance of the SGAR is similar to that of the SGA. However, once the landscape changes, the early online and offline results of the SGAR are better than those of the SGA. This appears to indicate that the SGAR, through random initialisation, appears find it easier to locate the new global optimum. One reason for this is that in the random initialisation of the population, conducted after the change in the objective function, leaves the population, on average, closer to the new global optimum, rather than converged around the old global optimum as is the case for the SGA. Table 2 indicates that in terms of locating the global optimum the performance of the SGAR is comparable with that of the SGA, but marginally better after the landscape change. Comparing the offline results of the SGAR to

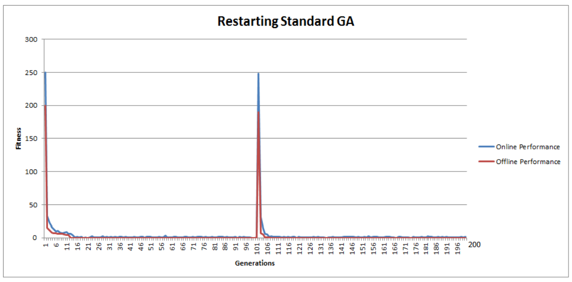

Figure 2: SGA Changing Sphere Function.

those of the SGA, the results were shown to be statistically highly significant ($p - value < 2.2e^{16}$).

4.3 GA with Elitism (GAE) Changing Sphere Function Results

Figure 3 contains the offline and online results for the GAE. Overall, the GAE's online and offline performances appear marginally better than that of the SGA, due to the introduction of elitism (see Table 2). The offline results of the SGA compared to GAE were shown, using to be statistically highly significant with a $p - value$ of $2.817e^{-08}$.

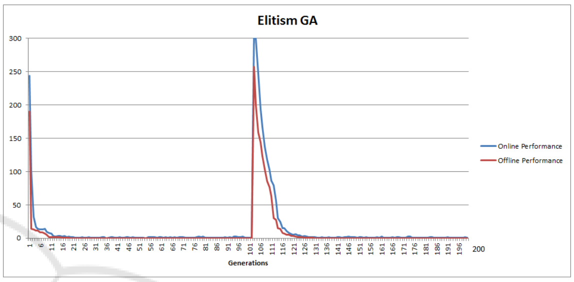

Figure 3: GA with Elitism (GAE) Dynamic Sphere Function.

4.4 Immigration GA (GAI) Changing Sphere Function Results

The GAI results (see Figure 4), the global optimum value of 0 was located on average, at generation 4. Once the landscape altered, the offline global optimum of 0 was located at generation 138. The GAI used in this paper, removes the worst solution in each generation and replaces it with the best solution from the previous generation. This allows the GAI to insert and remove solutions based on the fitness function for the particular landscape. The results, illustrated in Table 2, indicate that over the changing Sphere function, the Immigration GA outperforms the SGA, SGAR and GAE, before and after the landscape change. Although the offline results compare favourably, it is interesting to note that the online results shown in Figure 4, after the landscape change, illustrate the GAI's ability to adapt through the introduction of diversity into the population. This is shown in the graph by the increase in distance between the online and offline lines. The offline results of the SGA and GAI for the Dynamic Sphere experiment were statistically highly significant ($p - value < 2.2e^{16}$).

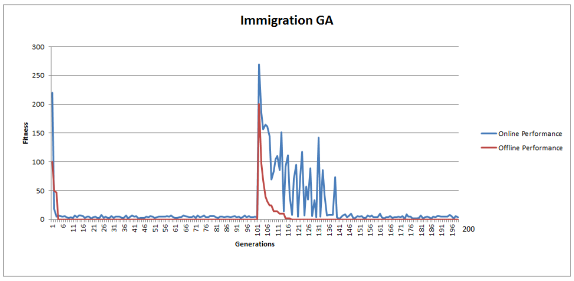

Figure 4: Immigration GA (GAI) Changing Sphere Function.

4.5 Micro GA (MGA) Changing Sphere Function Results

The results of the MGA are shown in Figure 5. The results of the MGA, in locating the global optimum, indicate an improvement over the SGA, SGAR and GAE algorithms, both before and after the objective function changes (see Table 2). The restarting mechanism and elitism of the MGA appears to allowed the algorithm adapt much quicker to the altered landscape, thus optimization the changing Sphere problem in the least amount of generations. The offline MGA result, were statistically highly significant when compared to the offline SGA results, with a $p-value = 0.0009585$. The other interesting feature of the MGA relates to the online performance, both before and after the landscape alters, the MGA's online performance suggests a significant degree of diversity is maintained in the population. This differs dramatically from the SGA's online performance, which suggests a large degree of convergence. This suggests that the ability to maintain diversity within the population, as implemented by the MGA, assists in adapting to the landscape change.

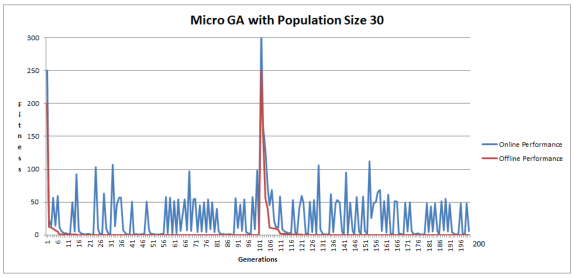

Figure 5: Micro GA (MGA) Changing Sphere Function.

4.6 Diploid GA (DGA) Changing Sphere Function Results

The DGA results, seen in Figure 6, shows the offline value of 0 was found at generation 5. At generation 100 the landscape alters, with the offline value of 0 located at generation 141. The results indicate that over the dynamic Sphere landscape the DGA outperformed the SGA and the offline results, when compared were statistically significant with a $p-value = 0.03033$. When looking at the results of the DGA, it appears that the maintenance of diversity associated with the diploid structure, assists in adapting to a changing landscape. There is however, an additional overhead associated with the diploid structure compared to haploid GA. Further, in terms of the fitness of each of the diploid solutions, there is a large degree of convergence as indicated by the online results.

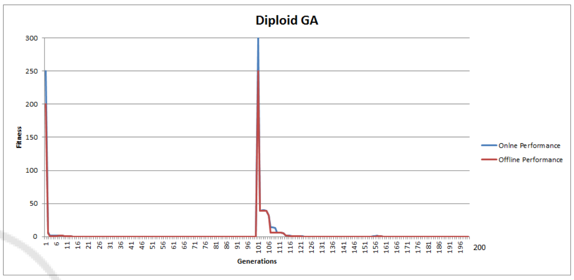

Figure 6: Diploid GA (DGA) Changing Sphere Function.

Table 2 outlines the results of the changing Sphere Function experiments, identifying whether the global optimum was located both before and after the landscape changed. The results indicate that the most adaptable small population GAs were the GAI, MGA and DGA. However, in terms of adaptability over the dynamic Sphere function landscape, all of the GAs succeeded in locating the global optimum. One reason for this is the relative ease of the Sphere function, however using the Sphere function indicates the efficiencies of each GA variant.

Table 2: Changing Sphere Optimisation

	Changing Sphere Function Experiments			
	Before Landscape Change		After Landscape Change	
	Gen.	Optimum	Gen.	Optimum
SGA	85	Yes	170	Yes
SGAR	90	Yes	165	Yes
GAE	58	Yes	161	Yes
GAI	4	Yes	138	Yes
MGA	34	Yes	136	Yes
DGA	5	Yes	141	Yes

4.7 SGA Changing Shekel's Function Results

The SGA results (Figure 7) evolved for a predetermined number of generations and the problem was optimised in generation 470. The results indicate that Shekel's function proved more difficult for the SGA compared to the Sphere function. At generation 500 the objective function inverted causing the landscape to change. At this point the population has evolved around the old global optimum which is much further away from the solution than the initialized population, thereby making the adaptation more difficult. The SGA however, managed to locate the new global optimum at generation 990. The SGA located the global optimum both before and after the landscape change (see Table 3). The online results are interesting as, they illustrate the increase in difficulty of Shekel's function in comparison to the Sphere function, with slightly less convergence within the population.

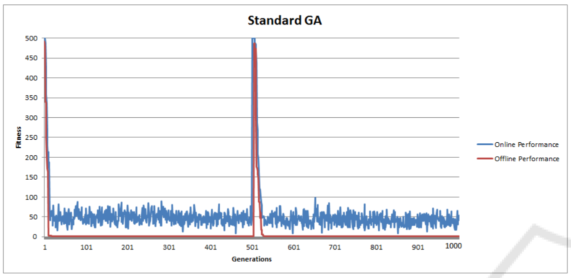

Figure 7: SGA Dynamic Shekel's Function.

4.8 SGA Restart (SGAR) Dynamic Shekel's Function Results

Figure 8 illustrates the results of restarting the SGA when the landscape alters. The global optimum was located before and after the change in the objective function, as shown in Table 3. The performance was in line with that of the SGA, with a minor improvement. The offline results, compared to those of the SGA were statistically highly significant.

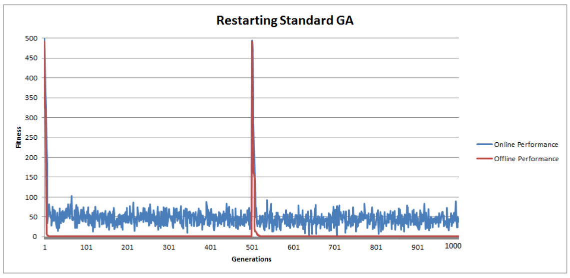

Figure 8: SGA Dynamic Sphere Function.

4.9 GA with Elitism (GAE) Dynamic Shekel's Function Results

The GAE results (see Figure 9) shows the global optimum being discovered before and after the landscape changed. The performance was largely similar to that of the SGA and SGAR, this is shown in Table 3. The offline performance results of the GAE and SGA were shown to be statistically highly significant ($p - value < 2.2e^{-16}$).

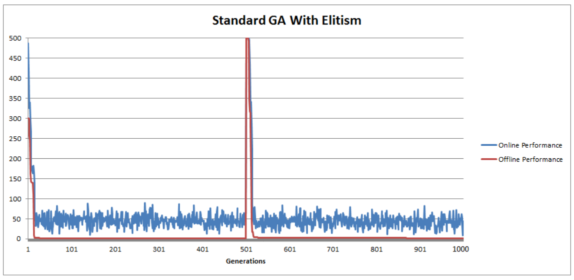

Figure 9: GA with Elitism - Dynamic Shekel's Function.

4.10 Immigration GA (GAI) Dynamic Shekel's Function Results

The online and offline results of the GAI experiments are illustrated in Figure 10. The results indicate that there is a higher degree of diversity maintained within the population in comparison to the SGA, SGAR and GAE (illustrated by the online performance). The offline performance shows the global optimum being located in both phases of the search, see Table 3. The performance of the GAI showed an improvement over that of the SGA before the objective function changes. After the change, however the results indicate that the GAI adapted quicker in locating the new global optimum. This is due to the ability of the GAI to maintain diversity into the population. The offline performances of both algorithms was statistically highly significant with a probability value $< 2.2e^{16}$.

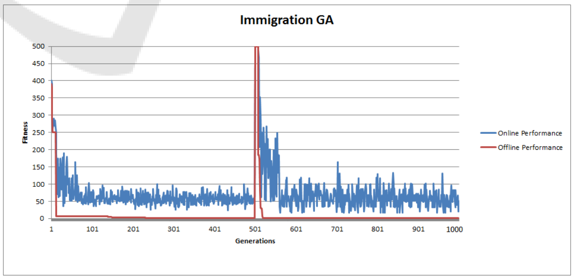

Figure 10: Immigration GA (GAI) Dynamic Shekel's Function.

4.11 Micro GA (MGA) Dynamic Shekel's Function Results

Figure 11 shows the offline and online results for MGA. The offline performance illustrates the success in locating the global both before and after the landscape changes. The online performances shows the large amount of diversity being introduced into the population as the average fitness values vary erratically. However, Table 3 indicates that the MGA exhibited the ability to succeed in the search, locating the global optimum before and after the alteration, in the fewest number of generations, compared to the other GA variations. The ability to continuously introduce new individuals into the population, appears to beneficial over the given landscape. Statistically, the offline performance of the MGA compared to that of the SGA was shown to be highly significant with a $p-value < 2.2e^{16}$.

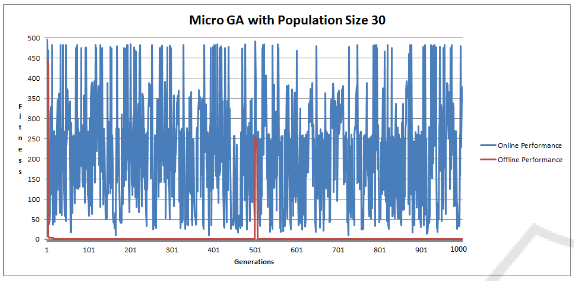

Figure 11: Micro GA Dynamic Shekel’s Function.

4.12 DGA Dynamic Shekel’s Function Results

For the DGA, the offline global optimum was found at generation 450. At generation 500, when the landscape was altered, the new global optimum was located during generation 900 (see Table 3). When examining Figure 12, the offline performance is similar to that of the SGA, SGAR and GAE. The Diploid GA maintained a high level of diversity within the population due to the double chromosome structure. This can be seen through the fluctuating offline performance associated with the DGA, particularly after the landscape changes as it attempts to adapt to the changing objective function. However, in terms of adapting to the more difficult Shekel’s function, the DGA did not perform as well as the other diversity maintenance GA variations, the MGA and GAI. One reason for this may be due to the diploid structure, which through its dominance scheme, struggles with the landscape in question. Again, when comparing the results of the offline performance of the SGA to those of the DGA, the results were found to be statistically highly significant with a $p-value < 2.2e^{16}$.

A summary of the Dynamic Shekel’s function experiments is shown in Table 3. The results indicate that although all of the GAs managed to locate the global optimum, before and after the landscape altered, the GAI and MGA were the most successful.

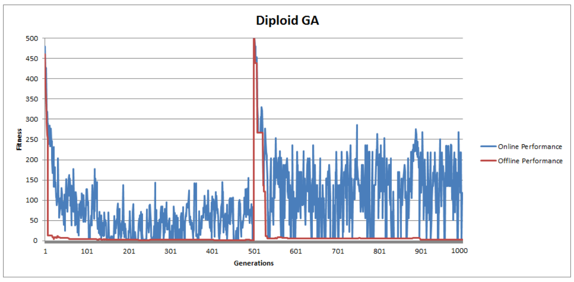

Figure 12: Diploid GA - Dynamic Shekel’s Function.

Table 3: Dynamic Shekle’s Optimisation.

	Dynamic Shekel’s Function			
	Before Landscape Change		After Landscape Change	
	Gen.	Optimum	Gen.	Optimum
SGA	470	Yes	990	Yes
SGAR	460	Yes	950	Yes
GAE	450	Yes	938	Yes
GAI	437	Yes	622	Yes
MGA	23	Yes	614	Yes
DGA	450	Yes	900	Yes

5 CONCLUSION

The results presented indicate that the small population GAs variations were capable of optimising both the dynamic Sphere landscape and the dynamic Shekel’s landscape. The research suggests, as expected, that GAs which maintained diversity into the population, were quicker to adapt when a changing environment was presented. However, what was interesting is that each of the GAs had a relatively small population to work with, in comparison to previous research. The findings of the DGA experiments tended to concur with previous research where Goldberg (Goldberg and Smith, 1987) suggested not much improvement over the SGA in terms of number of generations required to optimize the landscapes. The greater level of diversity in the population produced disappointing results in the changing landscape given the additional overhead required.

The dynamic Sphere results indicate that the GAs associated with diversity maintenance,that is the GAI, MGA and DGA all possess an ability to adapt to a changing landscape and outperform the SGA, SGAR and GAE. The changing Shekel’s function results show some evidence that the GAE had more difficulty in getting out ofthe local optima when compare to the GAI and MGA. The algorithm would continuously keep adding the best chromosome to the population and so may end up getting caught in some of the foxholes of the changing Shekel’s landscape while the algorithm optimized the solution it required more generations to work through the landscape.

Overall the MGA exhibited the greatest ability to adapt over the more difficult dynamic Shekel's landscape, in comparison to the other GAs. The MGA also proved a useful GA variation over the dynamic Sphere landscape, producing, along with the GAI, the lowest number of generations required in order to adapt to the change in objective function. The results seem to imply that over changing landscapes, the combination of small populations and diversity maintenance can prove successful. This may suggest that diversity maintenance techniques coupled with small populations, can reduce the computational overhead and assist GAs over changing landscapes, particularly as the level of difficulty increases.

6 FUTURE WORK

Future work includes further examination into the coupling of small population GA and diversity maintenance techniques over more difficult and varied dynamic landscapes to see if the maintenance of diversity within a small population GA continues to offer an advantage.

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