On the Efficient Graph Representation of Collinear Relation in the Shape Grammars

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Abstract: Shape grammars are a powerful, generative approach to description, interpretation and evaluation of many designs. However, their practical implementation has problems related to computational and spatial efficiency. Because of that, successful examples are restricted to small graphs or those with reduced numbers of rules and shapes. While executing a project inspired by Antonio Gaudi's designs, we found those limitations critical and initiated a series of research tasks to improve of the efficiency of their implementation. The most important task consists in developing an efficient graph representation of the collinear segments. The proposed solution, based on a classical application of shape grammars – Stiny's Chinese lattice design – has been compared with two most popular existing representations.

1 INTRODUCTION

Shape Grammars have been introduced in 1971 by Stiny (Stiny and Gips, 1971) and further developed by himself. Stiny defined them as a "set of rules of transformation applied recursively to an initial form, generating new forms" (Stiny, 1980). Since then, they have been used as a powerful generative approach to description, interpretation and evaluation of many designs. For over 45 years, the notation of the formalism has been significantly changed and developed. Despite that, shape grammars were used constantly as a rule-based system for describing and generating designs (Knight, 1999), (Stiny, 2006).

Shape Grammars are used for creating and understanding designs directly, by performing computations with shapes. They have been applied to tackle a variety of design tasks from analysis to synthesis. Basing on a classification by Terry Knight and George Stiny (Knight, 2015), which distinguishes seven areas of application of shape grammars, the following representatives can be pointed out in each area.

Painting is an area where shape grammars are used to analyse transformations of the style of a painting. Terry Knight (Knight, 1989) has developed her research on transformations of De Stijl Art: The Paintings of Georges Vantongerloo and Fritz Glarner. She used the formalism to describe stylistic changes in design.

Shape grammars can be also used for product design. Jay P. McCormack, Jonathan Cagan and Craig M. Vogel (Jay P McCormack, 2004) used them to explore brand identity and showed that they might have an important role in studio work, engineering and marketing.

Terry Knight and George Stiny also mentioned the area of craft. In his work, Rizal Muslimin (Muslimin, 2010) restructured weaving performance in architecture. He has analysed the tacit knowledge of traditional weavers through perceptual study and converted it into explicit rules in computational design.

Mechanical design is another area where shape grammar-based methods have been developed. Agarwal, Cagan and Stiny (Agarwal et al., 2000) have shown that they are capable of generating coupled forms, by means of so-called function shape grammars. They have achieved that by satisfying the minimal required functionality and then modifying the device to obtain the desired specifications.

Stiny and Mitchell (Stiny and Mitchell, 1980) also used shape grammars for landscape design. They used parametric shape grammars to design Mughal gardens.

Jos Pinto Duarte (Duarte, 2005) researched shape grammars in the area of architecture. He has described an interactive computer system for design of customised mass housing. The role of shape grammars in such a system is to systematise the design

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rules.

One of the most developed area of shape grammar application is urban design. Jos Nuno Beiro (Beirao, 2012) defined all the aspects of using shape grammars in this field and successfully applied his theses in practice. Although he mentioned that rules had be hidden from the designer, he gave the designer some freedom by using discernible names for the moves and their parameters.

Analysis of the mentioned examples leads us to the conclusion that Shape Grammars can be successfully applied in many fields. However, the graph structures, which formally represent problems, are based on small graphs or have limited numbers of rules.

During our work on the project inspired by Antonio Gaudi's designs we have noticed a serious problem with graph processing efficiency. We work on different types of structures, which are described by large numbers of nodes. The project requires representation of thousands or even millions of edges, nodes and shapes which are not represented by lines. Even the most advanced among the exiting methods are incapable to process such vast collections data. Real-life application of shape grammars demands their efficient implementation for shapes consisting of thousands of elements.

Translation of shapes into graph grammars requires many problems to be solved, including:

- Complexity reduction of designation of the left-hand side of transformation rules: In a general case, it is an NP-complete problem, just like subgraph isomorphism. But with some restrictions on the application rule form and the generated graph structure, it can be solved in a polynomial time (cf. (Flasinski, 1990)).
- Introduction of a mechanism of parallel application of the transformation rules with implicit synchronization; cf. (Kotulski and Sedziwy, 2010).

However, the most basic and yet still open problem is proper representation of the shape elements.

The shape graph presented in Figure 1 consists of two overlapping squares (basic elements). In this example, the line segment (a,b) has been split into three collinear segments (the other line segments also have been split). The representation of relations among them and segment (a,b) has a strong influence on the implementation of the shape grammar.

This problem has been outlined by Grasl and Economou (Grasl and Economou, 2011), the authors of one of the best shape graph interpreters called GRAPE. The efficiency of the line segment representation, with each line consisting of several segments, will be considered here. The representation proposed

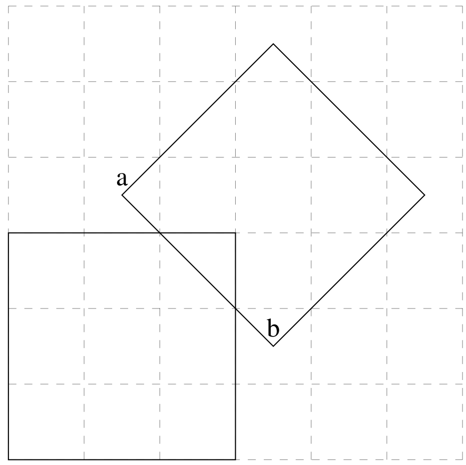

Figure 1: Collinearity problem in Shape Grammars.

in the following part of the paper has been compared with two others – Grasl (Grasl and Economou, 2011) and Kelles et al. (Keles et al., 2010) – based on the classical Stiny shape grammar application for Chinese ice-ray lattice design (Stiny, 1977).

2 COLLINEAR SEGMENTS IMPLEMENTATION

One of the most important problems associated with graph representation of shapes is the representation of collinear shapes in a way that allows their use it for the analogical relations for Bezir curve elements. The maximum line segments are a set of lines created by combining all collinear line segments that touch or overlap. The most straightforward approach is to map points to the nodes and segments to the edges. In such a representation, if a line is divided into several segments by a crossing line, it is difficult to designate the other collinear segments. Note that two or more connected collinear segments can appear in the transformation rule as one segment.

To overcome this problem, Keles et al. (Keles et al., 2010) join all node pairs in such lines. Unfortunately, this approach results in creation of too many edges. For n points along the line, $n \cdot (n-1)/2$ edges have to be created. Thus, even for a small grid of lines (over a dozen intersecting lines), like the one in Stiny's ice-ray (Stiny, 1977), a large number of edges is generated: 1653 edges for Chengtu Szeehwan, 1825 AD and 1392 edges for Hanchow, Szechwan 1875 AD. The mentioned examples of Chinese lattice design creates a good background for evaluation of the efficiency of the considered methods.

Grasl and Economou analyse 7 other representa-

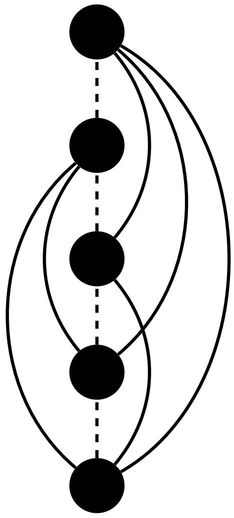

Figure 2: Keles's long line representation.

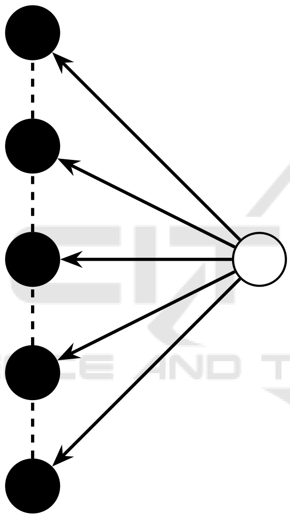

Figure 3: GRAPE long line representation.

tions and finally introduce the representation in which both points and segment lines are represented as nodes (points as a black nodes and lines as a white nodes). The collinear 5-segment line in both approaches is presented in Figure 3 and Figure 2. This representation is efficient for collinear lines, but we have to analyse it in the entire context of the generated shapes. Thus, an intermediate solution has to be considered and two such approaches are presented here.

3 CHINESE LATTICE DESIGN

Stine (Stiny, 1977) analyses several examples of Chinese lattice design to show the expressiveness and capabilities of shape grammars. In this paper we will

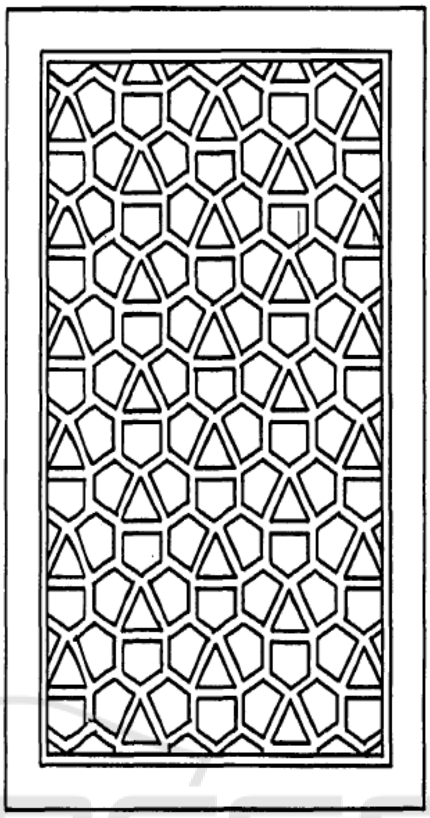

Figure 4: Chinese Lattice design - Chengtu, Szechwan 1800 AD.

consider the Chengtu Szeehwan, from 1800 AD (see Figure 4) and from 1825 AD (see Figure 5), as well as Hanchow, Szechwan from 1875 AD (see Figure 6).

Table 1 shows number of lines consisting from n collinear nodes in the designs from 1800, 1825 and 1875 years.

Table 1: Number of collinear nodes in the design.

NCN	1800 AD	1825 AD	1875 AD
2	262	42	0
3	0	84	24
4	0	0	4
5	0	0	6
7	0	0	4
9	0	0	3
11	0	0	2
13	0	8	5
15	0	7	0
17	0	0	4

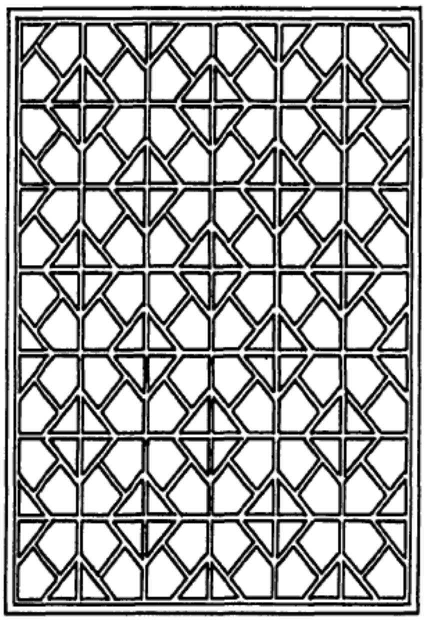

Figure 5: Chinese Lattice design - Chengtu, Szechwan 1825 AD.

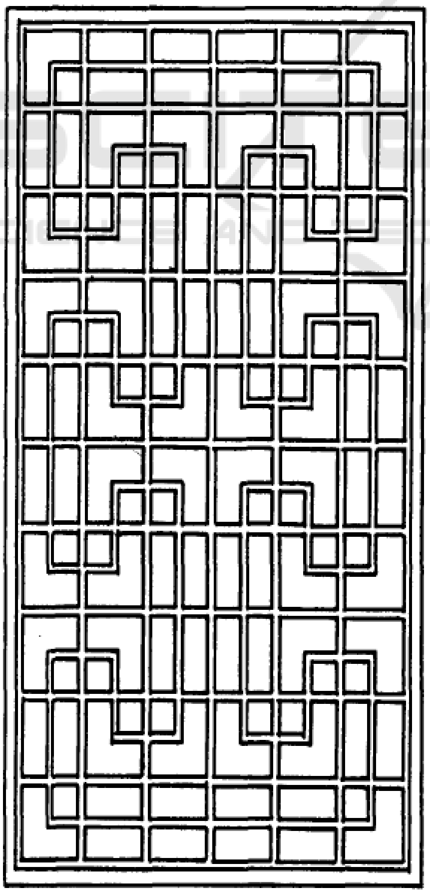

Figure 6: Chinese Lattice design - Hanchow, Szechwan 1875 AD.

4 SHAPE GRAPH REPRESENTATION

During definition of a shape grammar, we have to define what kinds of the basic elements can be used as building blocks for shapes generated by the grammar. In the Virtual Gaudi project we assume that we will describe the shape as Bezir curves (linear, quadratic, cubic or higher order) connected by common nodes. Such a curve will be denoted by $S^{A_n}(x,y)$, where x and y are the nodes representing end points, and A_n is a set of other $(n-2)$ attributes defining the Bezier curve of the order n. For two curves, we denote $S^{A_n}(x,y) \subset S^{A_n}(w,z)$ if all points of the first curve belong to the second one. For a given n and a set of attributes A_n, we will say that two shapes, are co-n-bezier shapes (denoted $\leftrightarrow$) as if the following conditions are met: for any nodes x,y,u,w,v,z:

- $S^{A_n}(x,y) \leftrightarrow S^{A_n}(u,z)$ if $S^{A_n}(x,y) \subset S^{A_n}(u,z)$
- $\leftrightarrow$ relation is closed under symmetry i.e. $S^{A_n}(x,y) \leftrightarrow S^{A_n}(u,z) \Rightarrow S^{A_n}(u,z) \leftrightarrow S^{A_n}(x,y)$
- $\leftrightarrow$ relation is closed under transitivity i.e. $S^{A_n}(x,y) \leftrightarrow S^{A_n}(u,w)$ and $S^{A_n}(u,w) \leftrightarrow S^{A_n}(v,z)$ $\Rightarrow S^{A_n}(x,y) \leftrightarrow S^{A_n}(v,z)$

The $S^{A_n}(x,y)$ that does not contain any curves $\leftrightarrow$ related with them will be called minimal and its end nodes will be connected with direct edges.
The shapes grammars use hybrid notation:

- for the presentation layer, the mentioned basic shapes are used to express the designed item,
- for efficient generation of this item, a more complex graph-based representation (based on graph transformation rules) is used.

Here, we compare the efficiency of several graph representations according to the number of source items (nodes and edges). There is a necessity to remember the information about the generated shapes; the number of nodes and edges in the graph are the basic parameters while considering the computational complexity of graph-based algorithms.

Let us note that in $S^{A_n}(x,y)$ representation of the Bezier curves we separate the notation of end nodes and A attributes. Thus, in the graph representation we will represent only these end nodes, while the A attributes will be remembered either as attributes of the edge representing shapes (in case of Keles' approach) or as attributes of the node representing shapes (according to Grasl). The advantage of such a representation is that it is the same for all orders of Bezier curves. For the simplicity of presentation, we will illustrate the mentioned representation based on the co-1-bezier shape called line segments (shortened to $S(x,y)$) and collinear shape relation.

As already mentioned, two opposite graph representations of the collinearity problem have been considered in literature. The first one has been proposed by Keles (Keles et al., 2010). He mentioned that shape nodes are represented as graph nodes and shapes are represented as edges (A attributes are also remembered in edges). The representation of collinear segments defiend using 5 nodes is illustrated in Figure 2. We will have 10 collinear segments and the same number of edges. A more compact representation suggested by Grasl and Economu (Grasl and Economou, 2011) introduces white nodes for representation of the shape edges. The representation of collinear segments defined by 5 nodes is illustrated in Figure 3. The A attributes will be remembered in white nodes. Two segments are collinear if their end nodes are connected with the same white node.

The advantage of the Grasl representation is not obvious in the case of more complex shapes, such as the one presented in Figure 7.

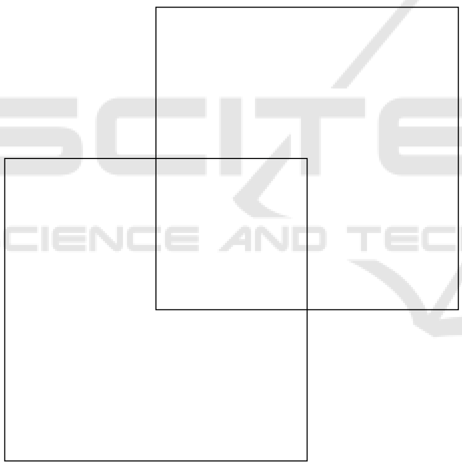

Figure 7: Complex shape.

Keles's representation of the shapes presented in Figure 7 is presented in Figure 8.

Grasl's representation of the same shape presented in Figure 9 is not that simple. The shape is now represented using 18 nodes and 28 edges, while in Keles's approach, the same shape is represented using 10 nodes and 16 edges.

4.1 Virtual Gaudi Project

The Virtual Gaudi Project, developed in our department, is inspired by the art of Antonio Gaudi. The general idea is to formally describe the existing works of Antonio Gaudi, analyse them using various AI systems and finally start to generate a new design in the

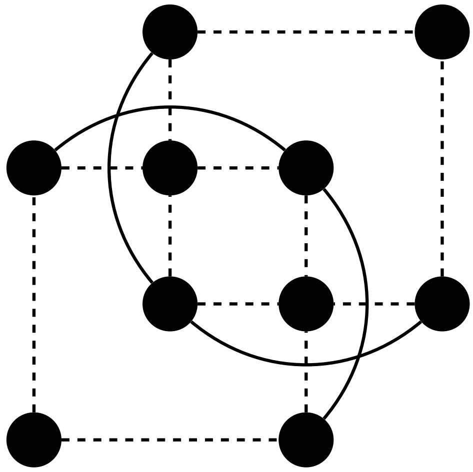

Figure 8: Keles's two-square representation.

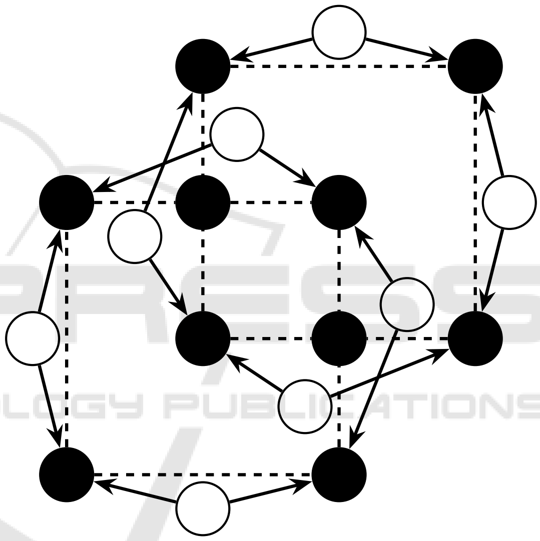

Figure 9: Grasl's two square representation.

"Gaudi style". It is obvious that composed artworks consist of millions of details. Because of that, efficient graph representation is a principal requirement for this idea. In contrast to human perception, which prefers one consistent representation of the problem, the graph grammar formal notation has no problem in describing the same concept in two or more ways, depending on some parameters e.g. the size of the represented elements. This fact is the basis for the proposed solution.

We Will Merge Both Presented Representations. For the two (alternatively three) collinear segments, we will use direct inline edges (as in Keles's solution) and if there are more segments, Grasl's representation will be used.

We analyse these four approaches on the Chinese lattice designs.

In Chengtu, Szechwan 1800 AD design we have 167 (black) nodes and 262 direct (dashed) edges. The (white) nodes, additional edges and the overall number of the given method of representation are presented in Table 2.

Table 2: Comparision of representation for 1800 AD.

represen-tation	Extra nodes	Extra edges	Together	
Keles'2	0	0	429	100,00%
Grasl's	262	524	1215	238,22%
VG-2	0	0	429	100,00%
VG-3	0	0	429	100,00%

In Chengtu, Szechwan 1825 AD design we have 237 (black) nodes and 262 direct (dashed) edges. The (white) nodes, additional edges and the overall number of the given method of representation are presented in Table 3.

Table 3: Comparision of representation for 1825 AD.

represen-tation	Extra nodes	Extra edges	Together	
Keles'2	0	1249	1890	100,00%
Grasl's	141	545	1327	70,21%
VG-2	15	293	949	50,21%
VG-3	15	293	949	50,21%

In Hanchow, Szechwan 1875 AD design we have 165 (black) nodes and 276 direct (dashed) edges. The (white) nodes, additional edges and the overall number of the given method of representation are presented in Table 4.

Table 4: Comparision of representation for 1875 AD.

represen-tation	Extra nodes	Extra edges	Together	
Keles'2	0	1116	1557	100,00%
Grasl's	52	328	821	52,73%
VG-2	28	280	749	48,11%
VG-3	24	276	741	47,59%

5 APPLICATION OF VG-X NOTATION TO SHAPE TRANSFORMATION RULES

The analysed examples show that the introduced VG-2 and VG-3 representations have the same or better expressiveness in comparison with Keles's approach; their advantage grows when longer collinear segments appear in the shape. They are always better than Grasl's approach. The VG-3 representation seems to be slightly better, but the implementation of splitting one of the collinear segments becomes more complex. Thus, we will sketch the solution for VG-2.

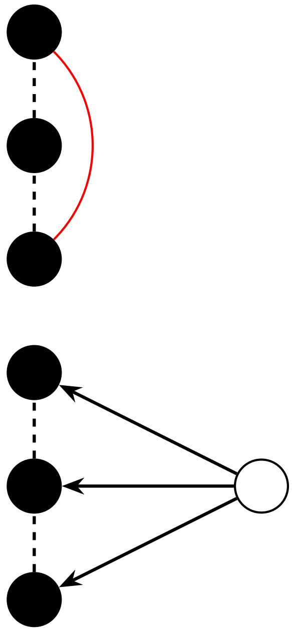

Figure 10: Online conversion of Keles's to Grasl's representations - inline edge.

As mentioned above, there is no problem for graph transformation rules to use the both representations of collinearity in the same system. The left side of the transformation rule lhs will use the Grasl's collinearity representation. While searching the subgraph of the entire graph G that is isomorphic to lhs, we will convert:

- Keles's in-line edge (black line) and the (only) node connected with the in-line endnodes with direct edges (dashed one) — into four nodes in Grasl's representation (see Figure 10),
- direct edges that are not considered in the previous in-line relation — into three nodes in Grasl's representation (see Figure 11).

The opposite transformation will be performed after the application of the transformation rule – all the white nodes that participate in this rule and do not point to at least three black nodes are converted to the Grasl's notation.

In the real life the final project consist of many different styles and shapes. we can present such a situation by summing up Chengtu, Szechwan 1800 AD, 1825 AD and Hanchow, Szechwan 1875 AD. That proves that introduced VG-2 and VG-3 representations are the most efficient ones (see table 5).

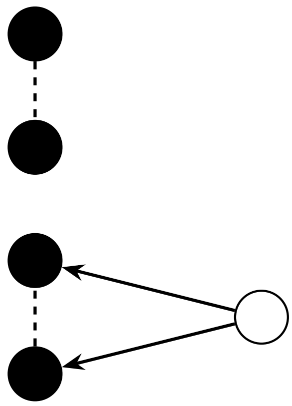

Figure 11: Online conversion of Keles's to Grasl's representations - direct edge.

Table 5: Comparision of representation for the sum.

represen-tation	Extra nodes	Extra edges	Together	
Keles'2	0	2365	3876	100,00%
Grasl's	455	1397	3363	86,73%
VG-2	43	573	2127	54,88%
VG-3	39	569	2119	54,67%

6 CONCLUSIONS

The more compact graph representation of the shapes generated by shape grammars is very important, as the efficiency of graph transformation algorithms depends on the graph size (number of nodes and edges). In this context, efficient representation of the collinear segments is proposed. The definition of transformation rule application, extended by the conversion mechanisms, allows us to combine two different representations of the collinear segments. It should also be noted that the mentioned approach can be extended to representation of co-n-bezier shapes (defined in section 4), which is important in real-world application of shape grammars. For example, in Antoni Gaudi's Sagrada Familia, line nearly do not appear.

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