Assigning Business Processes: A Game-Theoretic Approach

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Abstract: Business processes are essential for the successful growth of an organisation. Business models aim to organise such processes and invoke the necessary processes for particular tasks. Such a mechanism is responsible for the appropriate response and time management of the business strategy of a corporation. To this end, we formulate a scenario where business processes need to be invoked according to the expenditures of their operation, in order to complete a transaction in a market model. We model such a mechanism using game theory and we produce a characteristic function that we maximise, in order to reduce the cost of the business processes. We employ the well-known assignment game to form business process coalitions and minimise the business operation cost in the market.

1 INTRODUCTION

A current trend in the business oriented research is the emergence of business intelligence, since it reflects on real problems that businesses deal and their respective solutions (Chen et al., 2012). In particular, bid data has shown specific trends that businesses follow in their market domains (Minelli et al., 2012). Bid take place in a real-time fashion; hence, real-time strategic interactions constitute a major issue in business intelligence. Such decisions may introduce enormous consequences to the development of a business; thus, uncertainties must be taken into serious considerations by managers. A risk-minimum estimate needs to be identified, in order to proceed with business process deployment. This belongs to a class of problems named strategic business planning (Sonteya and Seymour, 2012).

Business analysts face unique issues when attempting to address a specific strategic problem. Decision time is usually limited. The potential of a wrong decision increases the cost and overall consequences of a business plan. In most of the cases, real-time strategic planning is associated with cost. Hence, a business analyst needs to have expert knowledge, in order to act in a timely fashion and address the business problem by keeping the cost at reasonable levels. Business planning can be thought of a complex system (Snowden and Boone, 2007); hence, often, debates in meeting rooms are at hand, in order to come up with the ideal strategic plan. Business processes can assist to such a problem by trying to automate responses to real world business problems (Scheer et al., 2004).

Game theory utilises models of conflict and cooperation (Von Neumann et al., 2007), between business processes in this paper. Aumann (Aumann and Dreze, 1974) provides the difference between cooperative and non-cooperative games. We deal with the class of cooperative games in this paper. Agreements between players can take place before the start of a cooperative game. The cooperation in such games is given by a set of players a set of strategies, and a set of payoffs that represent the outcome of the strategies played, in the form of a utility. A coalition is characterised by the achievement of the coordination of the members' strategies (Saad et al., 2009), (Curiel, 1988). As we read in (Weber, 1994), should we consider the business process assignment as a market game, a business process not participating in the coalition does not affect the trading within the coalition. Thus, the corresponding strategy profiles and their respective utilities define the characteristic function of our game formulation. Furthermore, when considering a Cournot game in which business processes will select quantities, the outcome of the coalition depends on the behaviour assumption of the business processes outside the coalition.

We aim to construct a cooperative model and define a characteristic function in order to maximise the efficiency of a business process network. There has been other research works that dealt with business issues in a game theoretic and cooperative manner (Binmore and Vulkan, 1999), (Yahyaoui, 2012), (Katsanakis and Kossyva, 2012), (Li et al., 2002), (Yasir et al., 2010). We encapsulate the characteristic function to indicate the formed coalitions between business processes. Games that involve forming of coalitions are distinguished between the ones which include transferable utilities and the ones that with non-transferable utilities. In the former, utility is divided within a coalition and in the latter, it is difficult to show what the utility can define when a coalition is formed.

In this paper, we address the assignment of business processes to operational business processes that need to be executed with time constraints. We produce a cooperative coalition formation game-theoretic model and we solve it to provide the optimal business process assignment. Specifically we show the following contributions:

- We build a game theoretic model of the business process assignment problem
- We construct a characteristic function based on the time/cost for an execution of a business process
- We show that it may be more efficient for a business process to form a coalition with another process to fulfill a business process if the time spent is less than the direct business process assignment
- We show the distribution of the time of completion with our model

This paper is structured as follows: Section 2 provides the method of our game-theoretic formulation, section 3 gives results on a specific scenario and section 4 provides the conclusions of our approach.

2 COOPERATIVE BUSINESS PROCESS ASSIGNMENT

We consider a simple market model where business processes may be assigned to intermediate (relay) business processes within a time frame, in order to save cost. Initially, we produce a cooperative model for the relay business process selection problem and we attempt to distribute the saved completion time between different business processes. This will be accomplished once we manage the completion time appropriately. To this end, we propose a sellers - buyers approach based on (Shapley and Shubik, 1971) to solve the completion time distribution issue. We define the coalition formation game as an ordered pair $< N, \phi >$, where $N = 1, 2, 3...N$ is the set of business processes and ϕ is the value of the characteristic function, which is given over 2^n possible coalitions of N. Also, note that $\phi(\emptyset) = 0$. If we have a set of all business processes in a coalition, then we can claim that we have formed a grand coalition. On the other hand, if our set does not include the entire set of the business processes, the resulting subset is called coalition. The price that a coalition C is worth, is obtained by the value of the characteristic function $\phi(C)$. This value constitutes the maximum common payoff of the business processes in C upon cooperation. We denote the T value that a source business process j requires to complete by allocating it to another business process further up the process stack k by T_{jk}. Also, denote the time demand of completion time required for a buyer business process be allocated to a seller business process by T_{ji}. Furthermore, denote the completion time value of the i^{th} seller to her own offer to the completion time saving of the cooperative coalition by c_i and the the value of the j^{th} buyer to the cooperation of the i_{th} seller by T_{ij}. We denote the value required by the relay business process to reach the target next business process as T_{frwd}. At this point, we assume that $c_i = T_{frwd}$, since a seller business process has T_{frwd} amount of completion time, favouring its corresponding source business process. Note that the sellers payoff will not be maximised unless its time of completion is compensated. On the contrary, the cooperation between a buyer and a seller and the completion time that will be saved constitutes the requirement of each buyer to form a coalition with a seller business process. Therefore we calculate T_{ij} as

$$T_{ij} = T_{jk} - T_{ji} \tag{1}$$

if $T_{ji} > c_i$, then a value exists that both the sellers and buyers select. When we refer to time of business process completion saving, we mean the selection of the best completion time required to finish the process and move to the next business process of the business plan network.

2.1 The Characteristic Function

We define the characteristic function $\phi(C)$ as the maximum completion time saving that the business processes accomplish by cooperating between them, in order to form a coalition. Let B and S denote the buyers and the sellers respectively. If there is only one business process present in the coalition, the business process has no cooperation process; hence, its preference is the direct transmission to the business process with the less completion time T. Thus, we have

$$\phi(C) = 0 \text{ if } |C| = 0 \text{ or } 1 \tag{2}$$

since, there is no improvement we have

$$\phi(C) = 0 \text{ if } (C \cap B = \emptyset) \text{ or } (C \cap S = \emptyset) \tag{3}$$

In order to further explain equation (3), for a connection efficiency improvement to take place, a coalition that consists of buyers and sellers business processes must be established. Hence, we will separate business processes into buyer and seller pairs respectively. Before we move into business process assignment, we provide the simplest form of coalition, which is given below:

$$\sigma_{ij} = \max[0, T_{ij} - c_i] \text{ if } i \in B \text{ and } j \in N \tag{4}$$

Equation (4) states that cooperation between a buyer and a seller business process will be instantiated only if the direct connection requires a larger completion time than the cooperative connection. In the case that the direct connection is better than the cooperative connection, $\sigma_{ij} = 0$. Our aim is to calculate the function ϕ for reasonably large coalitions; hence, we are trying to identify the best buyers assignments to the respective sellers business processes, which maximize the time efficiency and minimise the cost. This is represented as

$$\phi(C) = \max[\sigma_{i_1,j_1} + \sigma_{i_2,j_2} + ... + \sigma_{i_n,j_n}] \tag{5}$$

where $n = \min[|C \cap B|, |C \cap S|]$. We maximize (5) through all the arrangements of the players i in $C \cap B$ and j in $C \cap S$. As we can see, we can formulate the assignment game as a linear programming (LP) problem. Let mn be a set of binary decision variables that satisfy

$$x_{ij} = \begin{cases} 1, & \text{if } i \text{ relay process assigned to process j} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $i = 1,2,3...m$ and $j = 1,2,3...n$. Each binary variable indicates whether a business process i acting as a relay process will be allocated to a business process j that is wishes to execute.

We denote as ξ the total time saving of the cooperative coalition formation and we formulate the business process relay selection problem as an LP.

$$\text{Maximize } \xi = \sum_{i \in B} \sum_{j \in S} \sigma_{ij} x_{i,j} \tag{7}$$

s.t

$$\sum_{i \in B} x_{ij} \leq 1 \text{ for } i = 1,2,...m$$
$$\sum_{i \in S} x_{ij} \leq 1 \text{ for } i = 1,2,...n$$

The first constraint states that each relay business process may be assigned to at most one business process. The second constraint specifies that every business process has to be connected to at least one relay business process. Solving this LP problem will give us the maximum completion time saved when a coalition is formed of B relay business processes and S source business processes. Hence, we have

$$\xi_{max} = \phi(B \cup S) \tag{8}$$

Thereafter, we transform the LP problem to its equivalent matrix formulation.

$$\begin{aligned} &\max \mathbf{c^T x} \\ &\text{s.t. } \mathbf{A} \cdot \mathbf{x} \leq \mathbf{b} \\ &\mathbf{x} \geq 0 \end{aligned} \tag{9}$$

Note that the constraint $\mathbf{x} \leq 1$ has been folded into the constraint $\mathbf{A} \cdot \mathbf{x} \leq \mathbf{b}$

2.2 Business Process Selection Core

We proceed to the core of the coalition, which should not be empty or consisting of one business process only. According to Shapley and Shubik (Shapley and Shubik, 1971), the core of the relay selection game is the set of solutions of the dual LP problem of the assignment problem. In this paper, we introduce the Lagrangian dual. We take the nonnegative Lagrangian multipliers $(\mathbf{y}, \lambda)$ to the constraints $\mathbf{Ax} \leq \mathbf{b}$ and $\mathbf{x} \geq 0$ as follows

$$\mathcal{L}(\mathbf{x}, \mathbf{y}, \lambda) = \mathbf{c^T x} + \mathbf{y^T}(\mathbf{b} - \mathbf{Ax}) + \lambda^T \mathbf{x} \tag{10}$$

which serves as an upper bound of the characteristic function (9), whenever $\mathbf{x}$ is feasible or not. Therefore, $\max_x \mathcal{L}(\mathbf{x}, \mathbf{y}, \lambda)$ bounds the optimum of (9). In order to obtain the upper bound, we have to solve the following

$$\begin{aligned} &\min_{\mathbf{y},\lambda} \max_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{y}, \lambda) = \\ &\min_{\mathbf{y},\lambda} \max_{\mathbf{x}} \mathbf{c^T x} + \mathbf{y^T}(\mathbf{b} - \mathbf{Ax}) + \lambda^T \mathbf{x} \end{aligned} \tag{11}$$

We have the third equality since $\mathbf{c} - \mathbf{A^T y} + \lambda \neq 0$ and we may select an appropriate $\mathbf{x}$ such that $\mathcal{L}(\mathbf{x}, \mathbf{y}, \lambda)$ goes to infinity. Therefore, we have a finite bound when $\mathbf{c} - \mathbf{A^T y} + \lambda = 0$. By taking the strong duality of the linear program, the optimum of (9) coincides with (Boyd and Vandenberghe, 2004). We can make the formulation more simple by assuming that $\mathbf{b} > 0$, since T is always positive. We denote P as a convex set on $\mathbf{x}$ and $f_i(\mathbf{x}) = (i = 1, ...m)$ as a set of convex functions. Moreover, we define the general min-max problem as

$$\min_{x \in P} \max_{i \in [k]} f_k(\mathbf{x}) \tag{12}$$

where $[k] = \{1, ...m\}$ is a set of indexes. For detailed report on the solving method of this problem, we refer the reader to (Spyrou and Mitrakos, 2017).

We have to mention that the dual problem consists of $m+n$ variables

$$\mathbf{y} = [q_1, ...q_m, r_1, ..., r_n]$$

Moreover, we can see that the constraints of the problem are

$$q_i + r_j \geq \sigma_{ij} \; \forall i \in B \text{ and } \forall j \in S \tag{13}$$

Essentially, the solution of min-max problem is equivalent to the solution of $\phi(B \cup S)$. The remark above dictates the incentive of relay and source business processes to cooperate. Specifically, q_i and r_j comprise the T values that a relay business process i and a source business process j receive, in order to perform a cooperative transmission. Furthermore, the vector $\mathbf{y} = [q_1, ...q_m, r_1, ..., r_n]$ provides the distribution of the T enhancement and the equivalence of the dual problem with the solution of $\phi(B \cup S)$ constitutes an imputation of the coalition formation relay selection game. Additionally, from (5) and (13)

$$\sum_{i \in C \cap B} q_i + \sum_{C \in S} r_i \geq \phi(C), \forall C \subset S \tag{14}$$

Thus, we defined the core of the relay selection game using (12) and (14),since we encapsulate the imputation efficiency and the fact that an improvement move on the coalition cannot be made.

2.2.1 Completion Time Distribution

The enhancement of completion time T is shared between the business processes. On the other hand, the T completion time enhancement cannot be transferred to the relay business processes, unless the source business processes receive the value first; however, every source business process my be a relay business process; thus, transferring the completion time T enhancement. Thereafter, we construct the T function, distinguished between the cooperative and selfish business processes and prevents any undesired behavior. The T function of the relay business process i at a cooperative scenario is given by

$$T_i[n] = T_i[n-1] + c_i + q_i \tag{15}$$

Note that c_i is the T completion time received by the relay business process i and q_i is the amount of T available for the relay business process to proceed with a cooperation. On the other hand, the T compensation of the relay business process needs to be provided by the source business process. Thus, the T function of source business process j is given by

$$T_j[n] = T_j[n-1] - c_j - q_j \tag{16}$$

Note that, even after transferring $c_i + q_i$ amount of completion time T to the relay business process i, the source business process j will have ϕ_j of T enhancement.

3 RESULTS

We consider a simple market and we take two scenarios on board; the first scenario is a single source process - relay business process and the second is a multiple source - relay business processes scenario.

3.1 Single Source and Relay Business Process

As we can see in figure 1, the source business process executes by reaching the end-business process directly, since its incentive is not to cooperate with the relay business process. This is the case due to the fact that the completion time of going directly to the end-business process is less than the completion time after forming the coalition with the relay business process.

On the other hand, in figure 2, we see that the source process forms a coalition with the relay business process, since it is in its benefit to cooperate, since it saves completion time. This is only the simplest scenario. We investigate thoroughly a more complicated scenario with a network of business processes. In both figures the solid lines represent the preferences of the source business process, while the dashed lines the discarded choices.

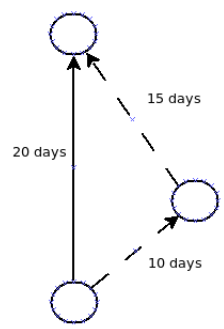

Figure 1: Source Business Process Direct communication with End-Business Process.

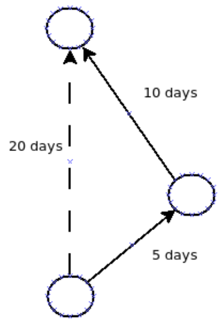

Figure 2: Source Business Process Direct forming Coalition with Relay Business Process.

3.2 Multiple Source and Relay Business Processes

The objective is to maximise the time completion of the business processes that will be formed after the cooperation and the coalitions formed. In our scenario, business process 1 is the business process that the business plan leads to, business processes $2-4$ are the relay business processes and business processes $6-8$ are the source business processes. The source processes do not form coalitions with all the relay business processes. In particular, source business processes 6 and 8 form 3 coalitions respectively and source business process 7 establishes 4 coalitions respectively. These coalitions include a connection to the end business process, in order to show the difference in the T value and the necessity of cooperation between the business processes. The respective completion time values reside in table 1. Finally the completion time values between the relay business processes $2,3,4$ and the end-business process are $1,1.5,3$ respectively.

Table 1: Network connections and T values.

Source Process	Seller Connections	T
6	1,2,3	23,20,16
7	1,2,3,4	16,12.5,11,12
8	1,3,4	21,15.10,14

Thereafter, we provide the reader with the valuations of the business processes (sellers and buyers). Notably, the sellers' valuations are calculated by simply obtaining the completion time values required to transmit operate a business process to business process 1. On the other hand, the valuations of the buyers are estimated by the result of equation (1), which gives us the difference of completion time between the one-hop connection of each buyer with the end business process and the completion time of each buyer with a seller. Note that when a connection does not exist between a seller and a buyer, we set the result of (1) as 0. We provide the valuations in the table 2, which is counted in days of completion time.

Table 2: Business Processes and Completion Time Values.

Source	Seller Val	Buyer Val		
(i)	(c_i)	T_{i6}	T_{i7}	T_{i8}
2	1.5	3	3.5	0
3	2	7	5	6.5
4	2	0	4	7

The result of the assignment of the most appropriate seller to a buyer is the enhancement of completion time value per pair. The outcome of the game is a profit matrix that shows the resulting completion time enhancement from coalitions between source and relay business processes. We provide this information in table 3. Furthermore we highlight the optimal assignment between relay and source business processes in bold numbers, which indicate the maximisation of the completion time enhancement for each pair that form a coalition.

We are able to identify the core solution of the problem by solving the dual LP problem described in the previous section. However, we are not in the position to claim that the core solution is unique. One of the core solutions obtained, which is close to the Shapley value (Shapley, 1988)

$$y = [2.1708\ 0.95\ 2.55\ 0.55\ 3.30\ 2.44] \qquad (17)$$

Furthermore, we can derive from Table 1 that the completion time required for a direct communication of the buyer business process with the destination - business process 1 - is 45. On the other hand, the successful cooperation between the sellers and the buyers is 17.5, which gives us an completion time enhancement of 68.1%. In order to accomplish that we need to connect business process 6 with business process 2, business process 8 with business process 3 and finally, business process 7 with business process 4. We

Table 3: Completion Time Enhancement.

		Buyers		
		6	7	8
Sellers	2	1.5	**2**	0
	3	**5**	3	4.5
	4	0	2	**5**

can see the final configuration of the business process selection process in figure 3.

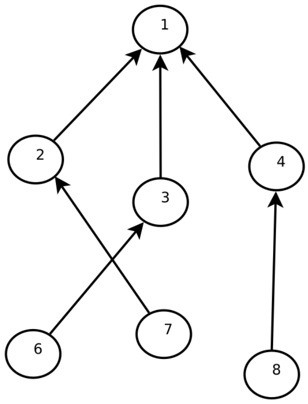

Figure 3: Final Transmission Configuration.

4 CONCLUSIONS

In this paper we attempted to approach the collaboration between business processes in order to accomplish tasks of a business plan. To that end, business processes either connect directly with the business process they require to finish the task or they form coalitions by finding a relay business process to connect to, depending on the business process completion time.

Subsequently, the business processes establish a cooperative network in a game theoretic manner. Our model is based on combinatorial optimisation, which target the maximisation of the completion time enhancement when a relay and a source business process cooperate. We derived the characteristic function used in our game, the coalition core and the credit that each business process has for playing the relay selection game. We evaluated a simple and a more complicated scenario, which indicated the fact that using cooperative business process cooperation the process network exhibits a better completion time. This is due to the fact that each source business process gets assigned to the relay business process that has the best completion time enhancement.

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