A New Technique for Phase Shift Measurements based on Amplitude Estimations

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Abstract: The paper presents a new original technique for the accurate real time measuring of the phase shift between two quasi-harmonic optical signals based upon the estimation of amplitudes of the both initial quasi-harmonic signals and the third signal that is formed by summation the first two ones. The required phase difference is then calculated as an angle of a triangle formed by the reconstructed undistorted signals' amplitudes values. An important peculiarity of the proposed technique consists in the fact that the phase data are obtained as a result of the amplitude measurements only what significantly decreases the demands to the measuring equipment. For the amplitude values estimation the methods of the Rician data analysis are proposed to be applied. The paper provides both the mathematical substantiation of the technique and its computer simulation results. The elaborated method is meaningful for various applied tasks to be solved in numerous ranging and communication systems.

1 INTRODUCTION

The accurate measuring of two signals' phase difference is one of the most important problems in various fields of science and technology, such as radio-physics, optics, radiolocation, radio-navigation, etc. Such measurements are in the use at distance measurements, in ranging systems, at determining the object's geometrical parameters, at non-destructive control and in many other applied tasks (Kinkulkin, Rubtsov, Fabrik, 1979; Chmykh, 1993; Smirnov, Kucherov, 2004).

The problem of measuring the phase difference has been investigated for a long time and many various methods for its solving have been elaborated. These methods include the phase compensation technique, the transformation of the time interval into the voltage (Chmykh, 1993), the digital technique of accounting the number of pulses (Webster, 2004; Mahmud, 1989), the phase measuring method accompanied by the frequency transform (Chmykh, 1993; Webster, 2004), the correlation methods (Chmykh, 1993; Webster, 2004; Liang, Duan, Yeh, Luo, 2012), the Fourier transformation technique with the further extraction of the phase component (Webster, 2004; Mahmud, 1989, Mahmud,1990), the least square adjustment method with the data fitting for a sinus-shaped signal (Sedlacek, Krumpholc, 2005).

A number of existing phase measuring methods a-priori use a harmonic signal model (Kinkulkin, Rubtsov, Fabrik, 1979), i.e. imply the constant amplitude's value, what does not correspond to the real circumstances. In practice we normally have the so-called quasi-harmonic signal that is characterized by the random variations of the signal's amplitude due to the Gaussian noise. Such amplitude's variation is a serious obstacle for the accurate phase measuring (Chmykh, 1993; Ignat'ev, Nikitin, Yushanov, 2010). A number of various parametric techniques have been proposed for the signal's phase measurements (Ignat'ev, Nikitin, Yushanov, 2013; Ramos, Serra, 2008; Hing, Cheung So, Zhenhua, Zhou, 2013), which imply the calculation of rather a big number of the signals' parameters and normally demand a significant volume of computational resources.

The original method of the signals' phase difference measuring elaborated in the present paper differs in principle from the methods of the prior art as it is based entirely upon measuring and processing the amplitude values only.

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2 THE PROBLEM SETTING AND BASIC DEFINITIONS

In order to consider the phase difference between two quasi-harmonic signals let us clarify the concepts to be used. In practice an inevitable noise influence results in the random variations of the signal's amplitude. Therefore the quasi-harmonic, or quasi-sinusoidal signal is to be considered instead of a sine-shaped signal. In each moment of time t a signal to be analyzed can be presented as follows:

$$x(t) = R(t)\cdot\sin\left(\omega t + \varphi(t)\right) \tag{1}$$

where ω is the common frequency, $R(t)$ is the signal's amplitude, or envelope that randomly varies due to the Gaussian noise influence, and $\varphi(t)$ is the phase shift that also changes randomly in time under the noise influence. Normally the signal contains also the slowly changing additive "white" Gaussian noise. It can be filtered and its presence is not critical for measuring the phase $\varphi(t)$. To ensure the convenient graphical representation we'll consider the signal (1) in a complex plane (as a complex value) denoting it as $S(t)$:

$$\begin{aligned} S(t) &= R(t)\cdot\exp\left[i\left(\omega t+\varphi(t)\right)\right] \\ &= s(t)\cdot\exp(i\omega t) \end{aligned} \tag{2}$$

For measuring the signals' phases we'll analyze the "slow" signal's component $s(t) = R(t)\cdot\exp\left[i\varphi(t)\right]$. Let us denote the initial, undistorted complex signal as vector $\vec{A}(A,\varphi_0)$. It is characterized by a determined amplitude value A and a phase φ_0. The signal's propagation through any medium is inevitably accompanied by its noising, namely – the initial signal's real $A\cos\varphi_0$ and imaginary $A\sin\varphi_0$ parts are independently varied by a lot of random noise components. Let us denote by $\vec{r}(r,\psi)$ a noise component that is superimposed on the initial signal $\vec{A}$. The components r_x, r_y of the noise vector $\vec{r}$ are independent and obey the normal distribution: $\overline{r_x} = \overline{r_y} = 0$, $\overline{r_x^2} = \overline{r_y^2} = \sigma^2$, where σ^2 is a noise dispersion value. Obviously the amplitude r and the phase ψ are distributed as follows: amplitude r obeys the Rayleigh distribution, while the noise components' phase ψ is distributed uniformly in interval (0, 2π).

We'll denote by vector $\vec{R}(R,\varphi)$ the resulting signal that is formed by summing the initial signal $\vec{A}$ and noise $\vec{r}$: $\vec{R} = \vec{A} + \vec{r}$. The real and imaginary parts of $\vec{R}$ can be written as follows:

$$\begin{aligned} R\cos\varphi &= A\cos\varphi_0 + r\cos\psi; \\ R\sin\varphi &= A\sin\varphi_0 + r\sin\psi \end{aligned} \tag{3}$$

The statistical distribution of amplitude R and phase φ of resulting signal $\vec{R}$ is determined by their joint distribution function (Rytov, 1976) that can be calculated from (3).

$$\begin{aligned} &W(R,\varphi)\,dR d\varphi = \frac{1}{2\pi\sigma^2}\cdot \\ &\exp\left\{-\frac{1}{2\sigma^2}\left[A^2 + R^2 - 2AR\cos(a-\varphi)\right]\right\} R dR d\varphi \end{aligned} \tag{4}$$

As one can see from (4), the distributions of the resulting signal's amplitude R and its phase φ are not independent, and phase φ as distinct from phase ψ is not a uniformly distributed value.

Having integrated (4) by φ between the limits from 0 to 2π one can obtain an expressions for the distribution function for amplitude R of resulting signal $\vec{R} = \vec{A} + \vec{r}$:

$$\begin{aligned} &W_R(R)\,dR = dR\int_0^{2\pi} W(R,\varphi)\,d\varphi = \\ &\frac{RdR}{\sigma^2}\cdot I_0\left(\frac{RA}{\sigma^2}\right) e^{-\left(R^2+A^2\right)/2\sigma^2} \end{aligned} \tag{5}$$

At obtaining (5) an integral representation for the modified Bessel function has been used (Abramowitz and Stegun, 1964): $I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos t}dt$. From (5) it follows that amplitude R obeys to the Rice distribution with parameters A, σ^2 (σ^2 is the Gaussian noise dispersion value). So, the influence of noise can be mathematically described as "blurring" the initial signal's vector $\vec{A}$ of amplitude A so that its amplitude becomes a random value $R = |\vec{R}|$ that obeys the Rice distribution.

3 ESSENCE OF THE PROPOSED TECHNIQUE

The mathematical problem to be solved consists in measuring the phase shift between two quasi-harmonic signals that are propagating in different channels. The task consists in measuring these signals' phase difference as an indicator of the object or the process to be studied. We can present these signals as the following vectors: $\vec{R}_1(R_1, \varphi_1), \vec{R}_2(R_2, \varphi_2)$ as illustrated in Fig.1.

The values of quasi-harmonic signals' amplitudes R_1 and R_2 obey the Rice distribution with parameters (A_1, σ^2), and (A_2, σ^2), where A_1 and A_2 are the initial, undistorted signals' amplitudes, σ^2 is the Gaussian noise dispersion. It is natural to suppose that such a dispersion value is the same for the both channels by which the two signals are propagating, although the mathematical analysis provided below can be easily generalized for a case of different dispersion values. In the further calculations we'll use *a-priori* knowledge that the phase difference $\Delta\varphi = \varphi_2 - \varphi_1$ between the considered signals is unambiguously determined by the physical properties of the object or the process being studied.

The noised signals to be measured can be put down as follows: $\vec{R}_1 = \vec{A}_1 + \vec{r}_1$, $\vec{R}_2 = \vec{A}_2 + \vec{r}_2$, where vectors $\vec{A}_1$ and $\vec{A}_2$ denote the two initial, undistorted signals, $\vec{r}_1, \vec{r}_2$ - the noise vectors, each of them being characteristic for a corresponding channel of the signal propagation. The phase difference $\Delta\varphi$ between the two signals is equal to an angle between the corresponding vectors.

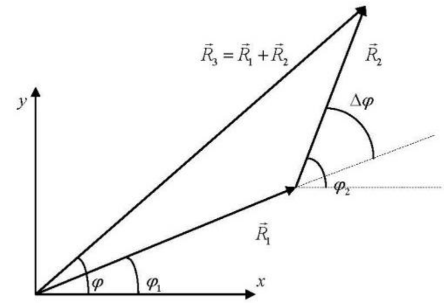

Figure 1: Illustration of the signals to be analysed.

Let us introduce the third vector that is equal to the sum of the two signals being analyzed. We denote it as vector $\vec{R}_3 = \vec{A}_3 + \vec{r}_3$, where $\vec{A}_3 = \vec{A}_1 + \vec{A}_2$ - the sum of the first two undistorted signals. Vectors $\vec{R}_1$, $\vec{R}_2$ and $\vec{R}_3$ form a triangle, and the phase difference between the two signals can be determined from this triangle on the basis of the triangle sides' values, i.e. the signals amplitudes' values.

Obviously, the sought for phase difference between $\vec{A}_1$ and $\vec{A}_2$ could be most precisely calculated if we would be able to "freeze" the triangle at the undistorted, noise-free state. However, the inevitable noise distorts each vector independently and the amplitudes measured in each moment of time would provide a false, distorted value for the sought for phase shift, whereas the required phase shift may be correctly found only from the triangle formed by the initial, undistorted amplitudes: A_1, A_2, A_3. As it has been shown above the signals' amplitudes obey the Rice distribution with the Rician parameters (A_i, σ^2), $i = 1, 2$. As for the third signal $\vec{R}_3 = \vec{A}_3 + \vec{r}_3$, its amplitude can be shown to obey the Rice distribution as well. The parameters of this distribution are: $(A_3, 2\sigma^2)$, where $A_3 = |\vec{A}_3|$. As the amplitudes measured in samples provide the distorted data for the lengths of the triangle sides, they need to be processed in such a way that would allow getting the undistorted values A_1, A_2, A_3 . This means that we have to determine the corresponding Rician parameters' values.

The so-called two-parameter methods elaborated in (Yakovleva, Kulberg, 2013; Yakovleva, Kulberg, 2014; Yakovleva, 2014; Yakovleva, 2015) allow an accurate estimating of both the signal $\left(A_i, i=1,2,3\right)$ and noise (σ^2) parameters based upon the sampled measurements. In other words, by means of calculating the initial, undistorted values of the three signals' amplitudes we would "freeze" the picture as a noise-free one and thus calculate the needed phase difference value just on the basis of geometrical considerations by the formula:

$$\Delta\varphi = \arccos\left(\frac{A_3^2 - A_1^2 - A_2^2}{2A_1A_2}\right) \qquad (6)$$

Below some results of the numerical simulation of the proposed technique are presented. Table 1 demonstrates the dependence of the absolute error modulus $err = \left|\Delta\varphi_{calc} - \Delta\varphi\right|$ at calculating the sought for signal's phase shift upon a number of parameters such as the sample length, the signal-to-noise ratio, etc. The denotations are as follows: $\Delta\varphi_{calc}$ - the phase shift calculated according to the above algorithm, $\Delta\varphi$ - the real phase shift (at the numerical experiment illustrated by Table 1 the value of the real phase shift was equal to 1,318), $SNR = 0,5(A_1 + A_2)/\sigma$ - the value that characterizes the signal-to-noise ratio, n - the number of measurements in a sample.

Table 1 presents the results of the technique's numerical simulation, i.e. the calculated values of the absolute error modulus are provided, at averaging by $N_{av} = 10^3$ measurements.

Table 1: Numerically calculated magnitude of the absolute error modulus $err = \left|\Delta\varphi_{calc} - \Delta\varphi\right|$ as dependent on the signal-to-noise ratio SNR and the sample length. n.

	SNR=10^2	*SNR*=0.25 10^3	*SNR*=0.5 10^3	*SNR*=10^3
n=8	$4,2\cdot10^{-4}$	$1,4\cdot10^{-4}$	$5,8\cdot10^{-5}$	$1,2\cdot10^{-5}$
n=16	$1,8\cdot10^{-4}$	$0,9\cdot10^{-4}$	$4,9\cdot10^{-5}$	$1,0\cdot10^{-5}$

4 CONCLUSIONS

The paper presents an original technique of measuring the phase difference between two quasi-harmonic optical signals based upon the statistical processing of the amplitudes values of the following three signals: the two compared signals and their sum. The theoretical consideration of the problem is provided. The amplitudes of the three signals to be analyzed are shown to obey the Rice statistical distribution. The algorithm of the proposed technique implementation consists in the joint reconstruction of the undistorted signals' amplitudes against the noise background. Therefore the sough for phase shift is obtained as a result of the amplitude measurements only what significantly decreases the demands to the equipment and simplifies the realization of the proposed method in a wide circle of applied tasks to be solved in numerous ranging and communication systems. The digital experiments confirm the theoretical conclusions on the feasibility and efficiency of the proposed technique.

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