Fiber Cable Network Design with Operations Administration & Maintenance Constraints

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Abstract: We introduce two specific design problems of optical fiber cable networks that differ by a practical maintenance constraint. An integer programming based method including valid inequalities is introduced for the unconstrained problem. We propose two exact solution methods to tackle the constrained problem: the first one is based on mixed integer programming including valid inequalities while the second one is built on dynamic programming. The theoretical complexities of both problems in several cases are proven and compared. Numerical results assess the efficiency of both methods in different contexts including real-life instances, and evaluate the effect of the maintenance constraint on the solution quality.

1 INTRODUCTION

Fiber To The Home (FTTH) networks are currently deployed by telecommunications operators, and require a huge capital expenditure (see (Europe, 2016), it can cost several billion euros to connect one million households). The technological architecture chosen by a majority of operators is to deploy passive optical networks, which are based on passive optical splitters. A passive optical splitter connects several fibers on one of its sides to one at the other side (divides or gathers the signal depending on its origin), which leads to a tree topology of the FTTH networks (illustrated in Fig. 1). The design of such networks includes to decide the splitter locations, the civil engineering infrastructure used (see (Bley et al., 2013), (Chardy et al., 2013), (Gollowitzer et al., 2013), (Contreras and Fernandez, 2012)). Finally, the fiber cable network has to be designed to connect these equipment (see Fig. 1). These decisions are usually taken in different steps.

This paper focuses on the problem of fiber cable network design. This problem is highlighted in the survey (Grötschel et al., 2013) as an incomplete field of study, especially when cable separation techniques are considered. The work from (Angilella et al., 2016) tackles the issue including the selection of civil engineering infrastructure, but faces computational limits on real-life instances. The paper (Mateus et al., 2000) excludes weld costs, which are a significant expense source. The work from (Angilella et al., 2017) deals with the issue of cable backfeed, specific to the problem, but restricts the possible ways to serve the demand. In the following we include several ways to serve the demand (with fiber cables or fiber modules), and introduce a maintenance constraint which, to our knowledge, is novel.

The next section introduces two problems which differ by the introduction of an Operation Administration & Maintenance constraint. We introduce an algorithm based on integer programming for the unconstrained problem in Section 3.1. Two solution methods are then proposed for the constrained problem, an integer programming based solution in Section 3.2, and a dynamic programming based solution in Section 4. The theoretical complexities of the problems are argued in Section 5. All solution methods are assessed numerically in Section 6.

2 PROBLEM DESCRIPTION

The general problem tackled in this paper consists in connecting one splitter location to several client groups, using fiber cables, with minimal cost. It arises several times in a given FTTH network, notably once for each splitter location.

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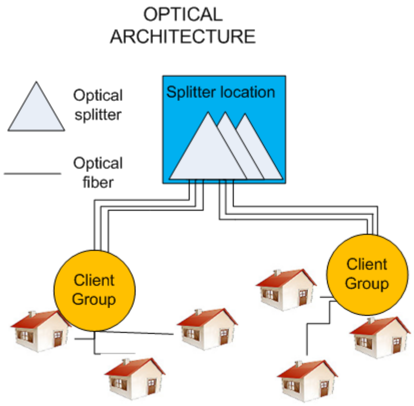

Figure 1: Underlying optical architecture example. It has a tree topology; the splitter location is connected to every client group.

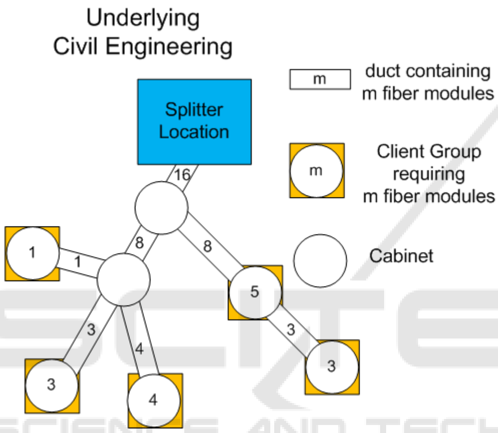

Figure 2: Underlying civil engineering tree example. The ducts, cabinets, demands and number of fiber modules are known.

2.1 Unconstrained Problem

Cables are to be laid out in a civil engineering infrastructure (usually the one used for the legacy copper network) with a tree topology, assumed chosen within previous decision steps. The cables have an arborescent structure from the splitter location to the client groups. Along the ducts of this infrastructure are located street cabinets, in which the demand lies. The civil engineering structure used is supposed to be known due to previous decision making, as well as the demand in each cabinet.

Fiber cables contain several fiber modules, and each fiber module contains several fibers. Due to operational constraints, modules are not dividable, and all modules on a given network are supposed to be identical. This allows us to consider only fiber modules, and ignore the fiber level. Some of the modules are connected to the fiber source on one of their ends, and on the fiber demand on the other end. These are actually used, and are called ”active modules”, the other ones are called ”dead modules”. The latter can arise due to cables not matching exactly the demand or in the operations described below (example: a 4 module cable serving a cabinet which requires 3 modules). Since all the demand is known and there is only one path from the source to a given demand point, the number of active modules that must be deployed through a given duct is known (see Fig 2).

At a cabinet, cables can endure a splicing operation, which leads to two basic configurations (see Fig. 3):

- All cables are continued. One only has to pay for the cost of laying out cables.
- One cable is spliced. It is cut at the cabinet, and its active modules are welded to active modules of new cables, referred to as ”born cables”. A protective box, the size of which depends on the spliced cable size, is installed. One has to pay for the cables, the box and the welds.

There are two different ways to serve the demand that cannot be combined (see Fig. 4):

- Cable-served. In this case, a single cable brings all the required active modules to the demand cabinet.
- Module-served. In this case, a splicing operation is done in the cabinet, and some modules from the spliced cable are used to serve the demand. No welds are done on these modules.

Additional engineering rules have to be taken into account:

- At most one cable can be spliced at a street cabinet. This is due to space restrictions and regulatory constraints (protective boxes are large).
- The demand of a given cabinet must be served by at most one cable.

The cost elements are as follows:

- The cost of a cable is linear with respect to its length, and concave with respect to its size (i.e. its number of modules). This derives from the catalogues of cable manufacturers, who propose a fixed price per length unit for each cable size.
- The cost of a protective box depends on the size of the cable being spliced. It is a piecewise constant function. This derives from the number of different boxes sold by manufacturers.
- The cost of welds depends on the number of welds to be done in a given cabinet. It is piecewise linear concave, and derives from manpower cost considerations.

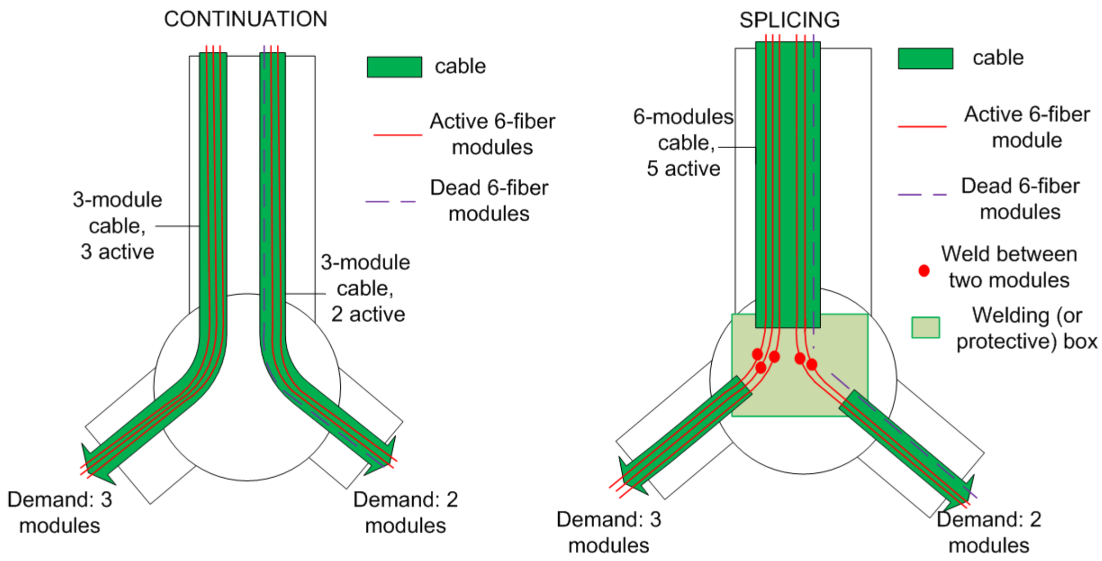

Figure 3: Left: Continued cables; Right: Splicing operation.

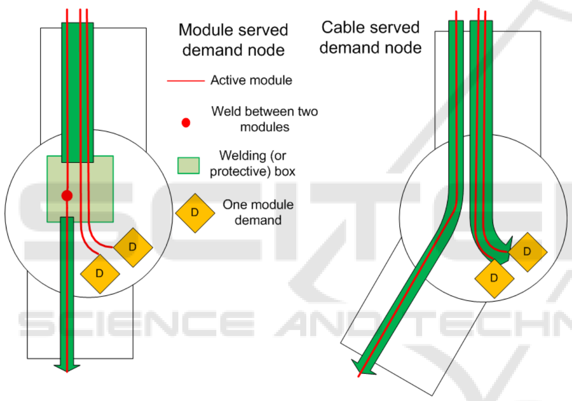

Figure 4: Left: Module-served demand node; Right: Cable-served demand node.

This decision problem, referred to as FCNDA (Fiber Cable Network Design in an Arborescence) in the following, can be formulated as follows: given a civil engineering arborescence, demand nodes, a set of available cables and the associated costs, design a minimum cost optical fiber cable network satisfying the engineering rules listed above.

Section 2.2 introduces a restriction of the FCNDA problem.

2.2 Constrained Problem

We restrict the problem by imposing that all cables going through a given duct are born in the same cabinet (eventually the fiber source). This restriction is illustrated in Fig. 5. It is motivated by operations and maintenance considerations. Indeed, assuming all the cables of a given duct are damaged, then an intervention has to be done at the cabinets where each of these cables is born. If the rule is respected, an intervention is necessary in only one cabinet.

The constrained decision problem, referred to as EFCNDA (Easy-maintenance Fiber Cable Network Design in an Arborescence) in the following consists in designing a FCNDA solution where cables on a same duct are born in the same cabinet with minimal cost.

3 INTEGER PROGRAMMING

3.1 FCNDA

3.1.1 Notation and Formulation

The following notation will also be used in section 4.

An arborescence $G = (V,A)$ describes the civil engineering infrastructure, V the cabinets and A the ducts, and its root $r \in V$ denotes the fiber source. For any $i \in V, D_i \in \mathbb{N}$ denotes the demand (number of active modules required) in node i. We define $V^* = V \setminus \{r\}$. The set of demand nodes is noted $V_D = \{v \in V, D_v > 0\}$, the set of nodes without demand $V_N = V^* \setminus V_D$. Each arc $(i,j) \in A$ has a length $d_{i,j}$ and must contain $m^{act}_{i,j}$ active modules. For $i \in V$, let $\Gamma^+(i)$ denote the set of successors of i and let $\gamma(i)$ be its predecessor.

The set of cable types is denoted by $\mathcal{L} = \{1,..,L\}$ where L is the number of different cable types available. Cables of type $l \in \mathcal{L}$ have a size of $M_l \in \mathbb{N}$ modules, and for $l \in \mathcal{L}$, we denote $\mathcal{M}_l = \{1,..,M_l\}$ the range of possible number of active modules in a cable

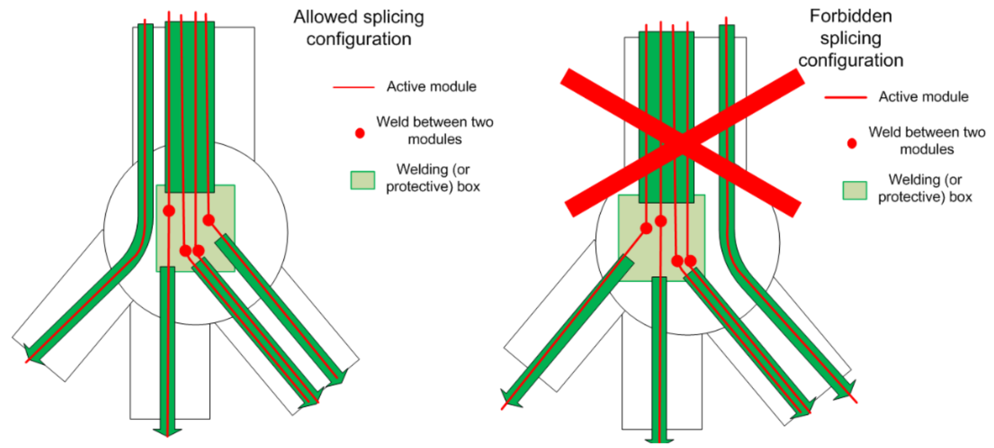

Figure 5: Left: Allowed splicing configuration for EFCNDA. On all edges, cables are born in the same cabinet; Right: Forbidden splicing configuration for EFCNDA. On the bottom-right duct, two different cables are born in different cabinets.

of type l. The cable sizes $(M_l)_l$ are supposed to be ordered with respect to l. The largest cable has a size of M_L, which corresponds to the maximal number of welds done in a node.

For $l \in \mathcal{L}$, let us define C_l^{le} the cost per length unit of a cable of type l, and PB_l the cost of a box of type l. For $m \in \{0,..,M_L\}$, let us define the cost of the smallest cable able to contain m active modules $C_m^{min} = C_{l_1}^{le}$ where $l_1 = \min\{l \in \mathcal{L}, m \leq M_l\}$ (which is also the cheapest as cable costs are increasing with respect to the cable size), and PW_m the cost for welding m modules.

We introduce $\mathcal{P}$ the set of oriented paths of G, and for $p \in \mathcal{P}$, we denote by $s(p)$ its source node, $t(p)$ its target node, and d_p its length (which extends d from A to $\mathcal{P}$).

We define the following variables:

- $\forall l \in \mathcal{L}, \forall p \in \mathcal{P}, k_{p,l}^{spl} \in \{0,1\}$ the binary variable equal to 1 iff there is a cable of size l on path p spliced in $t(p)$.
- $\forall p \in \mathcal{P}, k_p^{dem} \in \{0,1\}$ the binary variable equal to 1 iff there is a cable on path p serving the demand in $t(p)$ in a cable-served way; its type is known, it is $\min\{l \in \mathcal{L} | M_l \geq D_{t(p)}\}$.
- $\forall p \in \mathcal{P}, m_p^{spl} \in \{0,..,M_L\}$ the number of active modules of the cable on path p spliced in $t(p)$.
- $\forall i \in V^*, \forall m \in \mathcal{M}_L, w_{i,m}$ the binary variable equal to 1 iff m welds are done in node i.

The problem can be formulated as follows:

$$\min \sum_{p\in\mathcal{P}} d_p \cdot \left(\sum_{l\in\mathcal{L}} (C_l^{le} \cdot k_{p,l}^{spl}) + C_{D_{t(p)}}^{min} \cdot k_p^{dem} \right) + \sum_{i\in V_N} \sum_{m\in\mathcal{M}_L} PW_m \cdot w_{i,m} + \sum_{p\in\mathcal{P}} \sum_{l\in\mathcal{L}} PB_l \cdot k_{p,l}^{spl}$$

such that

$$\forall i \in V^*, \sum_{p\in\mathcal{P}|t(p)=i} \sum_{l\in\mathcal{L}} k_{p,l}^{spl} \leq 1 \quad (1)$$

$$\forall i \in V_D, \sum_{p\in\mathcal{P}|t(p)=i} k_p^{dem} \leq 1 \quad (2)$$

$$\forall p \in \mathcal{P}, \sum_{l\in\mathcal{L}} M_l \cdot k_{p,l}^{spl} \geq m_p^{spl} \quad (3)$$

$$\forall i \in V^*, \sum_{p\in\mathcal{P}|t(p)=i} m_p^{spl} = D_i \cdot (1 - \sum_{p\in\mathcal{P}|t(p)=i} k_p^{dem}) + \sum_{p\in\mathcal{P}|s(p)=i} (m_p^{spl} + D_{t(p)} k_p^{dem}) \quad (4)$$

$$\forall i \in V^*, \sum_{m\in\mathcal{M}_L} m \cdot w_{i,m} = \sum_{p\in\mathcal{P}|i=s(p)} (m_p^{spl} + D_{t(p)} \cdot k_p^{dem}) \quad (5)$$

$$\forall i \in V_N, \sum_{m\in\mathcal{M}_L} w_{i,m} \leq 1 \quad (6)$$

$$k^{dem} \in \{0,1\}^{|\mathcal{P}|}, k^{spl} \in \{0,1\}^{|\mathcal{P}|\times L}, w \in \{0,1\}^{|V^*|\times M_L}, m^{spl} \in \{0,..,M_L\}^{|\mathcal{P}|}.$$

In the cost function, the first term stands for the cost of cables, the second term for the cost of welds, and the last term for the cost of boxes. Equations (1) ensure that at most one cable is spliced in a node. Constraints (2) state that at most one cable serves the demand in a cable-served way. Equations (3) make sure that spliced cables are large enough to contain their number of active modules. Constraints (4) are active module conservation equations. The left-hand side term stands for the number of modules of the spliced cable. The first right-hand side term is the number of modules necessary to serve the demand, in case it is not cable-served. The last term is the number of active modules of born cables. Finally, constraints (5) and (6) ensure that w counts the number of welds to be done in each node.

Remark 3.1. *It is possible to fix the value of some variables. First, notice that leaf nodes are demand nodes. These nodes will be served in a cable-served way, and no operation will be done inside them. This gives, for all nodes $i \in V_D$ such that $|\Gamma^+(i)| = 0$:*

$$\forall m \in \mathcal{M}_L, w_{i,m} = 0$$

$$\forall p \in \mathcal{P} | t(p) = i, \forall l \in \mathcal{L}, k_{p,l}^{spl} = 0$$

Furthermore, the number of welds done in a node cannot exceed the number of active modules going out of this node. This gives:

$$\forall i \in V^*, \forall m \in \mathcal{M}_L, m > \sum_{j \in \Gamma^+(i)} m_{i,j}^{act} \implies w_{i,m} = 0$$

3.1.2 Valid Inequalities

We propose here several valid inequalities to tighten the linear programming continuous relaxation the formulation.

Let us define, for all $m \in \mathbb{N}$, the minimum cost per length unit of a set of cables able to contain m active modules denoted by $LB(m)$. For a given m, $LB(m) = \{\min \sum_{l \in \mathcal{L}} C_l^{le} \cdot n_l | \sum_{l \in \mathcal{L}} M_l \cdot n_l \geq m, n \in \mathbb{N}^L\}$. The following inequalities are valid for the FCNDA problem:

$$\forall (i,j) \in A, \sum_{p \in \mathcal{P} | (i,j) \in p} \Big(\sum_{l \in \mathcal{L}} (C_l^{le} \cdot k_{p,l}^{spl}) + C_{D_{t(p)}}^{min} \cdot k_p^{dem} \Big) \geq LB(m_{i,j}^{act}) \quad (7)$$

The left hand side is the cost per length unit of the cables going through arc (i,j).

Let us consider a path $p \in \mathcal{P}$ such that $t(p) \in V_D$ and $s(p) \neq r$. If there is a cable deployed on p, born in $s(p)$ and serving the demand in $t(p)$, then we know that there is a splicing operation done in $s(p)$. Furthermore, there are at least $D_{t(p)}$ welds in this operation, since the cable serving $t(p)$ contains $D_{t(p)}$ active modules. This gives the following valid inequalities for the FCNDA problem:

$$\forall p \in \mathcal{P} | t(p) \in V_D \text{ and } s(p) \neq r, \quad k_p^{dem} \leq \sum_{m \geq D_{t(p)}} w_{s(p),m} \quad (8)$$

3.2 EFCNDA

EFCNDA can be solved by using the same variables as in Section 3.1. The cost function is the same, the set of feasible solutions is described by constraints (1) to (6) to which we add the maintenance constraints described below:

$$\forall (p,p') \in \mathcal{P}^2 \text{ such that } s(p) \neq s(p') \text{ and } \exists a \in A, a \in p, a \in p'$$

$$k_p^{dem} + k_{p'}^{dem} \leq 1 \quad (9)$$

$$\sum_{l \in \mathcal{L}} k_{p,l}^{spl} + \sum_{l \in \mathcal{L}} k_{p',l}^{spl} \leq 1 \quad (10)$$

$$\sum_{l \in \mathcal{L}} k_{p,l}^{spl} + k_{p'}^{dem} \leq 1 \quad (11)$$

These constraints ensure that on two paths which have different origins but an edge in common, there can be only one cable.

The next section introduces an alternative mixed integer programming approach for EFCNDA, based on arcs rather than paths. It uses the properties of the problem, and has less variables and less constraints.

3.2.1 Notation and Formulation

We keep the same notation for the problem instance. In addition, let us define for $(i,j) \in A, U_{i,j}$ an upper bound of the cost per length unit of the cables going through (i,j).

We define the following variables:

- $\forall (i,j) \in A, x_{i,j} \in \{0,1\}$ the binary variable equal to 1 iff the cables on edge (i,j) are born in i.
- $\forall (i,j) \in A, c_{i,j} \in \mathbb{R}$ the continuous variable equal to the cost per length unit of the cables on edge (i,j) (when all constraints are satisfied, including integrality constraints).
- $\forall (i,j) \in A, z_{i,j} \in \mathbb{R}$ the continuous variable equal to $x_{i,j} \cdot c_{i,j}$.
- $\forall i \in V_D, u_i \in \{0,1\}$ the binary variable equal to 1 iff the node i is module-served.
- $\forall i \in V^*, \forall m \in \mathcal{M}_L, w_{i,m}$ the binary variable equal to 1 iff m welds are done in node i (since its meaning is identical to Section 3.1, we keep the same name).
- $\forall i \in V^*, \forall l \in \mathcal{L}, y_{i,l}$ the binary variable equal to 1 iff a cable of size l is spliced in i.

The problem can be formulated as follows:

$$\min \sum_{i \in V^*} \sum_{m \in \mathcal{M}_L} PW_m \cdot w_{i,m} + \sum_{(i,j) \in A} d_{i,j} \cdot c_{i,j} + \sum_{i \in V^*} \sum_{l \in \mathcal{L}} PB_l \cdot y_{i,l}$$

such that $\forall i \in V_D$,

$$c_{\gamma(i),i} = \sum_{l\in\mathcal{L}} C_l^{le} y_{i,l} + \sum_{j\in\Gamma^+(i)} c_{i,j} - \sum_{j\in\Gamma^+(i)} z_{i,j} + (1-u_i)\cdot C_{D_i}^{min} \quad (12)$$

$$\forall i \in V_N, c_{\gamma(i),i} = \sum_{l\in\mathcal{L}} C_l^{le} y_{i,l} + \sum_{j\in\Gamma^+(i)} c_{i,j} - \sum_{j\in\Gamma^+(i)} z_{i,j} \quad (13)$$

$$\forall i \in V_D, \sum_{l\in\mathcal{L}} M_l \cdot y_{i,l} \geq D_i \cdot u_i + \sum_{j\in\Gamma^+(i)} m_{i,j}^{act} \cdot x_{i,j} \quad (14)$$

$$\forall i \in V^*, \sum_{l\in\mathcal{L}} M_l \cdot y_{i,l} \geq \sum_{j\in\Gamma^+(i)} m_{i,j}^{act} \cdot x_{i,j} \quad (15)$$

$$\forall i \in V^*, \sum_{l\in\mathcal{L}} y_{i,l} \leq 1 \quad (16)$$

$$\forall i \in V^*, \sum_{m\in\mathcal{M}_L} m \cdot w_{i,m} = \sum_{j\in\Gamma^+(i)} m_{i,j}^{act} \cdot x_{i,j} \quad (17)$$

$$\forall i \in V^*, \sum_{m\in\mathcal{M}_L} w_{i,m} \leq 1 \quad (18)$$

$$\forall (i,j) \in A, z_{i,j} \geq c_{i,j} - U_{i,j} \cdot (1 - x_{i,j}) \quad (19)$$

$$\forall (i,j) \in A, z_{i,j} \leq U_{i,j} \cdot x_{i,j} \quad (20)$$

$$\forall (i,j) \in A, z_{i,j} \leq c_{i,j} \quad (21)$$

$$u \in \{0,1\}^{|V_D|}, w \in \{0,1\}^{|V^*|\times M_L}, x \in \{0,1\}^{|A|}, y \in \{0,1\}^{|V^*|\times L}, c \in \mathbb{R}^{|A|}, z \in \mathbb{R}^{|A|}.$$

The first term of the cost function denotes the cost of welds, the second term stands for the cost of cables, and the last term stands for the cost of boxes. Equations (12) ensure that the cost per length unit of any arc is properly counted. The term $\sum_{l\in\mathcal{L}} C_l^{le} y_{i,l}$ stands for the cost of the cable spliced in i, if any. If for some arc $(i,j) \in A$ such that $j \in \Gamma^+(i)$ we have $x_{i,j} = 0$, then the cables on arc (i,j) come from arc $(\gamma(i),i)$ unchanged. Otherwise, they come from the splicing operation done in i. The last term stands for the cost of the cable serving the demand in i. Equations (13) are the equivalent concerning nodes without demand. Equations (14), (15) and (16) ensure that the cable spliced in i is large enough to contain its active modules. The first term of the right-hand side of (14) stands for modules serving the demand, the second term for modules of born cables. Constraints (17) and (18) ensure that the variable $w_{i,m}$ is equal to 1 iff there are m welds done in node i. Finally, constraints (19), (20) and (21) ensure that $\forall (i,j) \in A, z_{i,j} = x_{i,j} \cdot c_{i,j}$ (these are Mc Cormick linearisation equations).

Remark 3.2. *It is possible to fix the value of some variables. Assuming there exists $i \in V^*$ and $m_1 \in \mathcal{M}_L$ such that $w_{i,m_1} = 1$, then by* (17)*, we know that there exists $S \subseteq \Gamma^+(i)$ such that $m_1 = \sum_{j\in S} m_{i,j}^{act}$. This gives by contraposition $\forall i \in V^*$, $\forall m \in \mathcal{M}_L$ if $m \notin \{\sum_{j\in S} m_{i,j}^{act} | S \subseteq \Gamma^+(i)\}$ then $w_{i,m} = 0$. It can be computed either in $O(2^{|\Gamma^+(i)|})$ or in $O(|\Gamma^+(i)| \times M_L)$ (which is not a polynomial with respect to the instance size, provided M_L is not coded in an unary system).*

3.2.2 Valid Inequalities

The continuous relaxation of the formulation above does not seem to be very tight in practice (see Table 4 from Section 6). We propose here several valid inequalities to tighten the continuous relaxation of the formulation.

In nodes without demand, if a cable of size l is spliced, then it has a number of active modules between M_l and $M_{l-1}+1$; otherwise one could install a smaller cable and obtain a cheaper solution. With the convention $M_0 = 0$ and $\mathcal{M}_0 = \emptyset$, every optimal solution of the EFCNDA problem verifies the following:

$$\forall i \in V_N, \forall l \in \mathcal{L}, y_{i,l} = \sum_{m\in\mathcal{M}_l\setminus\mathcal{M}_{l-1}} w_{i,m} \quad (22)$$

With a reasoning similar to the one from 3.1.2 (see definition of LB), we can get a lower bound of the cost per length unit of the cables on each arc. The following inequalities are valid for the EFCNDA problem:

$$\forall (i,j) \in A, c_{i,j} \geq LB(m_{i,j}^{act}) \quad (23)$$

If the cables on some arc $(i,j) \in A$ are born in i, then at least $m_{i,j}^{act}$ welds are done in node i. This implies that the following inequalities are valid for the EFCNDA problem.

$$\forall (i,j) \in A, x_{i,j} \leq \sum_{m\in\mathcal{M}_L | m \geq m_{i,j}^{act}} w_{i,m} \quad (24)$$

4 DYNAMIC PROGRAMMING FOR EFCNDA

For any node $i \in V^*$, we introduce the additional notation $V^{pr}(i)$, which refers to the set of nodes on the path from the root to i, excluding i and including r.

The EFCNDA problem can be solved by Algorithm 1. To each node $i \in V^*$, and for each node $j \in V^{pr}(i)$, we associate to i a label $< j, C(i,j) > \in V^{pr}(i) \times \mathbb{R}$ where $C(i,j)$ is the minimum cost of the

Algorithm 1: Exact Resolution Algorithm for EFCNDA.

```
1:  procedure INITIALISATION()
2:      for i ∈ V_D | Γ⁺(i) = ∅ do
3:          for j ∈ V^pr(i) do
4:              Add to i the label < j, C^min_{D_i} · d_p > where p ∈ 𝒫 is the only path s.t. s(p) = j and t(p) = i.
5:          end for
6:          Declare i labeled.
7:      end for
8:  end procedure
9:  procedure RECURSION()
10:     while ∃r' ∈ Γ⁺(r) such that r' has not been labeled do
11:         for every node i ∈ V* such that all nodes in Γ⁺(i) have been labeled do
12:             for j ∈ V^pr(i) do
13:                                 ▷ We select the operation in i minimizing the network cost.
14:                 Add the label < j, C(i,j) > to node i where
```

$$\begin{aligned} C(i,j) \quad &= \min_{S \subseteq \Gamma^+(i), b \in \{0,1\}} \sum_{k \in S} C(k,i) + \sum_{k \in \Gamma^+(i) \setminus S} C(k,j) \\ &\quad + PW_m + d_p \cdot C^{le}_{l_1} + d_p \cdot C^{min}_{D_i} \cdot (1-b) \\ \text{with} \quad & \begin{cases} m = \sum_{k \in S} m^{act}_{i,k}; l_1 = \min\{l \in \mathcal{L} | M_l \geq b \cdot D_i + \sum_{k \in S} m^{act}_{i,k}\} \\ p \in \mathcal{P} \text{ is the only path such that } s(p) = j, t(p) = i \end{cases} \end{aligned} \tag{25}$$

```
15:             end for
16:             Declare i labeled.
17:         end for
18:     end while
19: end procedure
20: procedure TERMINATION()
21:     return ∑_{r' ∈ Γ⁺(r)} C(r', r)
22: end procedure
```


network rooted in i plus the cost of the cables on the path from j to i, assuming these are born in node j.

The algorithm is initialized at leaf nodes (line 4), which are cable-served demand nodes, and where the size of the cable serving the demand is known.

For a node i such that all nodes in $\Gamma^+(i)$ have been labeled, and for $j \in V^{pr}(i)$, (25) computes the minimum cost operation if the next operation is done in j. For $i \in V^*$ and $k \in \Gamma^+(i)$, $k \in S$ iff the cables going through arc (i,k) are born in node i. Similarly, the boolean b is equal to 1 iff the node i is module-served (its meaning is similar than the variable u_i in Section 3.2).

We propose to compute it with a brute-search algorithm on the set S and on b. For given nodes $i \in V^*, j \in V^{pr}(i)$, it can be done in $O(P^*(\ln(M_L), L) \times 2^{|\Gamma^+(i)|+1})$ where P^* is a time sufficient to compute C^* (sums and minimums can be computed in polynomial time).

Lemma 4.1. *Algorithm 1 runs in time $O(P^*(\ln(M_L), L) \times 2^{\max \Gamma} \times |V|^2)$ where $\max \Gamma$ denotes the maximal degree (number of successors) of a node in the graph and P^* is a polynomial.*

Indeed, in each loop a number of iterations smaller than V^* is done.

Proposition 4.2. *Let us consider $i \in V^*$. When i is declared labeled in algorithm 1, there exists a node $j \in V^{pr}(i)$ such that in the label $< j, C(i,j) >$, $C(i,j)$ describes the cost of the minimum cost network in the arborescence rooted in node i plus the cost of the cables on the path from j to i.*

This proposition shows the validity of the algorithm. We will start to prove it for leaf nodes, then recursively on higher nodes.

Proof. $\star$ Let us consider a leaf node i. In the minimum cost network, it is served in a cable-served way with a cable of type $l_1 = \min\{l \in \mathcal{L} | M_l \geq D_i\}$. This cable is born in some node $j \in V^{pr}(i)$, eventually the root. The minimum cost network has a cost 0 in the arborescence rooted in j. Let us call $p \in \mathcal{P}$ the only path such that $s(p) = j$ and $t(p) = i$. The label $< j, C(i,j) >$ of i has a cost of $C^{min}_{D_i} \cdot d_p$.

$\star$ Let us consider a non-leaf node $i \in V^*$ such that all nodes in $\Gamma^+(i)$ have been labeled. In the minimal cost network, the cables going through arc $(\gamma(i), i)$ are all born in a node $j \in V^{pr}(i)$. Thanks to the maintenance constraint, we know that they are all born in the same node. Since all nodes $k \in \Gamma^+(i)$ have been labeled, for each of these nodes, there is a node $j_k \in V^{pr}(k)$ such that in the label $< j_k, C(k, j_k) >$, $C(k, j_k)$ describes the cost of the minimum cost network in the arborescence rooted in k plus the cost of the cables on the path from j_k to k. Furthermore, since the cables going through arc $(\gamma(i), i)$ are all born in j, we have either $j_k = j$ or $j_k = i$. Let us consider the label $< j, C(i, j) >$ of node i. If in the minimal network i is module-served, then we will have $b = 0$ in the computation of (25). Furthermore, let us consider $k \in \Gamma^+(i)$. If $j_k = i$, we will have $k \in S$ in the computation of (25), and $k \in \Gamma^+(i) \setminus S$ otherwise. Hence the result. $\square$

The termination of the algorithm derives from Proposition 4.2. For each node $r' \in \Gamma^+(r)$, we have $V^{pr}(r') = \{r\}$. This implies, using this proposition, that in the label $< r, C(r', r) >$, $C(r', r)$ is the cost of the minimum network cost in the arborescence rooted in r' plus the cost of the cables on (r, r'). Summing these values gives the minimum network cost.

The next section assesses the complexity of FCNDA and EFCNDA.

5 COMPLEXITY

We show in Section 5.1 that FCNDA is NP-hard even with 1 cable size and an upper bound on the node degree of 2, and in Section 5.2 that EFCNDA is NP-hard.

5.1 FCNDA

Let us consider the Number Partitioning Problem (NPP), which is shown to be NP-complete in (Karp, 1972).

Instance: A set of N strictly positive integers $\{n_i \in \mathbb{N} | i \in \{1, .., N\}\}$.

Question: Is there a subset $S \subseteq \{1, .., N\}$ such that $\sum_{i \in S} n_i = \sum_{i \notin S} n_i$?

We consider an instance of the NPP and reduce it to the following FCNDA instance. Let $G = (V, A)$ be an arborescence describing the civil engineering structure, $(V = \{r, 0, 1\} \cup \{v_i | i \in \{1, .., N\}\}, A = \{(r, 0); (0, 1); (1, v_1); (v_{i-1}, v_i) | i \in \{2, .., N\}\})$ (G is a chain graph), r is the root. The demand nodes are $\{v_i, i \in \{1, .., N\}\}$ and have respective demands $n_i, i \in \{1, .., N\}$ modules. Only one type of cable is available, with size $M_1 = \frac{1}{2} \sum_{i \in \{1,...,N\}} n_i$. Its cost per length unit is $C_1 = 1$. The lengths of all arcs of the arborescence are null, except $(r, 0)$ which is of length 1. This means the cost of a cable born in r is 1, and the cost of the other ones is 0. The cost of welds and boxes is null.

The question associated to this FCNDA instance is ”Is there a cabling solution cheaper than 2 ?”.

$\star$ If NPP is feasible: $\exists S \subseteq \{1, .., N\}$ such that $\sum_{i \in S} n_i = \sum_{i \notin S} n_i$. We then build the following cabling solution:

- Two cables holding only active modules are installed on link $(r, 0)$.
- In node 0, one incoming cable is spliced into $N - |S|$ born cables. The born cables have a number of active modules $n_i, i \notin S$ and serve respectively the demand nodes $(v_i)_{i \notin S}$.
- In node 1, the cable coming from the root with only active modules is spliced into $|S|$ born cables. The born cables have n_i active modules and serve the demand nodes $(v_i)_{i \in S}$.

Since the number of active modules is conserved in each splicing, the cabling solution described above is feasible (it is illustrated in Fig. 6, as well as the instance). Its cost is equal to 2.

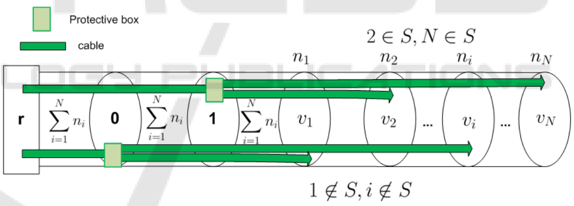

Figure 6: Instance and solution used in the complexity proof.

$\star$ If NPP is not feasible. Then, the solution described above is not possible anymore. One cable is not large enough to cover link $(r, 0)$. Two cables cannot cover $(r, 0)$ either, since they would both have only active modules, which would mean that the NPP problem was feasible. Consequently, at least 3 cables need to be installed on arc $(r, 0)$, and such a solution has a cost of a least 3.

Remark 5.1. *The solution illustrated in Fig. 6 is not valid for EFCNDA, the maintenance rule is not respected in nodes* 0 *and* 1.

5.2 EFCNDA

EFCNDA can be shown to be NP-complete by reduction from the NPP. With the same notation, let us consider an instance of the NPP and reduce it to the following EFCNDA instance. The civil engineering structure is described by the set of nodes is $V = \{r, 0\} \cup \{v_i | i \in \{1,..,N\}\}$; the set of arcs $A = \{(0, v_i) | i \in \{1,..,N\}\} \cup \{(r, 0)\}$; r is the root, the nodes $\{v_i | i \in \{1,..,N\}\}$ have a demand of n_i modules. The length of all arcs except $(r, 0)$ is null. We have $N+1$ available cables:

- N cables of sizes n_i modules and cost per length unit n_i
- A cable of size $\frac{1}{2}\sum_{i=1}^{N} n_i$ and cost per length unit $\frac{1}{2}\sum_{i=1}^{N} n_i - 1$

The cost of welds and boxes is null.

The question we ask is "is there a solution of cost at most $\sum_{i=1}^{N} n_i - 1$"?

$\star$ If NPP is feasible. Then, we have $S \subseteq \{1,..,N\}$ such that $\sum_{i \in S} n_i = \sum_{i \notin S} n_i$. We consider the solution of EFCNDA where

- For $i \in \{1,..,N\}$, on each arc $(0, v_i)$, we lay down a cable of size n_i
- In the node 0, a cable of size $\frac{1}{2}\sum_{i \in \{1,..,N\}} n_i$ is spliced. Cables of size $n_i, i \in S$ are born, and serve the demand of nodes $v_i, i \in S$.
- On the arc $(r, 0)$, a cable of size $\frac{1}{2}\sum_{i \in \{1,..,N\}} n_i$ holding only active modules is deployed (the one spliced in 0); as well as $N - |S|$ cables of sizes $n_i, i \notin S$ which serve the demand in nodes $v_i, i \notin S$.

The cost of this solution is the cost of cables on arc $(r, 0)$ which is $\sum_{i \in \{1,..,N\}} n_i - 1$. It is illustrated in Fig. 7.

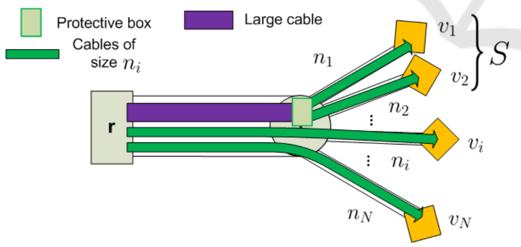

Figure 7: Instance and solution used in the complexity proof for EFCNDA.

$\star$ If NPP is not feasible. In a minimal cost solution, the size of cables serving the demand is known. For a given $i \in \{1,..,N\}$, v_i is served by a cable of size n_i. Which leaves three types of solutions to consider.

The solution without splicing has a cost $\sum_{i \in \{1,..,N\}} n_i$. Each demand node is served by a cable coming directly from the root r.

Any solution where a cable of size $\frac{1}{2}\sum_{i \in \{1,..,N\}} n_i$ is spliced in 0 has a cost at least equal to $\sum_{i \in \{1,..,N\}} n_i$. Indeed, let us note $E \subseteq \{1,..,N\}$ the set such that cables of sizes $n_i, i \in E$ are born in 0. Since the NPP instance is not feasible, we have $\sum_{i \in E} n_i < \frac{1}{2}\sum_{i \in \{1,..,N\}} n_i$, so the cost of cables which are continued in 0 is $\sum_{i \notin E} n_i > \frac{1}{2}\sum_{i \in \{1,..,N\}} n_i$, and the total cost of the network is $\sum_{i \notin E} n_i + \frac{1}{2}\sum_{i \in \{1,..,N\}} n_i - 1 \geq \sum_{i \in \{1,..,N\}} n_i$.

Any solution where a smaller cable is spliced in 0 has a cost at least equal to $\sum_{i \in \{1,..,N\}} n_i$. Indeed, in any splicing of a cable of size n_i for a given $i \in \{1,..,N\}$, the spliced cable is at least as expensive as the born cables.

5.3 Synthesis

To the results proven in Sections 5.1 and 5.2, we can add those deducible from Section 4. The restriction of EFCNDA where there is an upper bound on the node degree can be solved in polynomial time, since in that case the computation of (25) can be done in polynomial time (straightforward consequence of Lemma 4.1). This implies that it is also polynomial when more parameters are fixed. As for FCNDA, its NP-hardness in a restricted setting implies its NP-hardness in the more general cases. These results are summed up in Table 1 (recall that L is the number of cable sizes available, and $\max\Gamma$ stands for the maximum degree of a node in the graph).

Table 1: Complexity of the two problems in different contexts.

Fixed elements	none	$\max\Gamma$	$\max\Gamma$ and L
EFCNDA	NP-hard	Polynomial	Polynomial
FCNDA	NP-hard	NP-hard	NP-hard

Table 1 shows a theoretical difference in the complexities of the two problems EFCNDA and FCNDA. Fixing the maximum degree of the node in the instances makes EFCNDA polynomial. It seems like a very important factor of the complexity of this problem. Meanwhile, FCNDA stays NP-complete even with a fixed maximum degree and number of cable sizes.

We assess the numerical aspect of this complexity difference in the next section.

6 RESULTS

We assessed the solution methods on real-life instances taken from the city of Arles (France). Some of their features are displayed in Table 2.

The cables available have a size of 1, 2, 4, 6, 8, 12, 18 or 24 modules. The resolution algorithm for the MIPs was the Cplex 12.6 default branch-and-bound algorithm. The dynamic algorithm was implemented in Java. Both were run on a machine composed of 16 processors Intel Xeon of CPU 5110 and clocked at 1.6 GHz each.

Table 2: Key features of the real-life instances.

instance	features			
	max degree	arcs	demand nodes	total demand
Ar 1	4	113	45	61
Ar 2	6	103	38	55
Ar 3	5	103	35	66
Ar 4	6	123	43	80
Ar 5	7	129	44	68
Ar 6	6	137	43	67
Ar 7	4	139	35	68
Ar 8	5	163	41	63
Ar 9	4	219	68	78

6.1 Models Comparison

The results of the numerical experiments regarding the FCNDA and EFCNDA problems are displayed respectively in Tables 3 and 4, "base model" always refers to the MIP without valid inequalities, and "enhanced model" to the MIP with valid inequalities. The columns of both tables are labeled as follows: "time" stands for the computation time; "CR" stands for the ratio between the continuous relaxation value and the optimal solution; "DP" stands for dynamic programming (Table 4 only).

Table 3: Results for FCNDA.

instance	base model		enhanced model	
	time (s)	CR (%)	time (s)	CR (%)
Ar 1	8	90.3	16	91.0
Ar 2	9	83.7	24	92.4
Ar 3	17	92.2	22	93.3
Ar 4	19	89.2	46	90.0
Ar 5	1	94.9	2	95.2
Ar 6	2	92.5	3	94.7
Ar 7	13	92.4	29	93.7
Ar 8	8	89.6	12	91.7
Ar 9	4837	89.4	408	91.6

Regarding FCNDA (Table 3), the valid inequalities have had a positive effect on the average computation time, which went down from 546 to 62 seconds. However, on most instances (8 out of 9), the MIP is solved faster without the valid inequalities. This suggest that they are more useful for instances that are hard to solve. Regarding the algorithm, ratio CR goes from an average of 90.5 % to 92.6 %. The relatively high CR of the base model can explain the mitigated impact of the inequalities on the performances.

Regarding EFCNDA (Table 4), all instances were easier to solve (computation times are displayed in milliseconds). The valid inequalities have had a beneficial effect on the computation time, all instances are solved faster with the enhanced formulation. The average computation time goes from 1730 to 329 ms. On an algorithmic level, the ratio CR goes from an average of 13.2 % to 87.3 % of the optimal solution cost. The dynamic programming approach was more efficient than the enhanced integer programming formulation, it solved 7 out of 9 instances faster.

Table 4: Results for EFCNDA.

instance	base model		enhanced model		DP
	time (ms)	CR (%)	time (ms)	CR (%)	time (ms)
Ar 1	1457	14.0	305	89.2	324
Ar 2	1174	17.8	239	86.6	239
Ar 3	1317	13.6	318	81.7	66
Ar 4	742	15.7	268	86.8	87
Ar 5	746	18.2	477	89.2	88
Ar 6	1477	15.5	238	91.8	110
Ar 7	1667	9.7	190	80.1	121
Ar 8	1786	9.4	344	89.8	103
Ar 9	5204	5.3	507	90.8	306

6.2 Sensitivity Analysis

Section 5 highlights the maximal node degree as a key element of the problems complexity. Since the highest node degree of all real-life instances is between 4 and 7, we used fictive (simulated) instances to assess the performances of each resolution technique when some of the nodes have a high degree. Their features are displayed in Table 5.

Table 5: Key features of the fictive instances.

instance	features			
	max degree	arcs	demand nodes	total demand
Fi 10	11	20	15	71
Fi 11	12	22	16	84
Fi 12	13	24	18	97
Fi 13	14	26	19	112
Fi 14	15	28	21	112
Fi 15	16	30	22	127
Fi 16	17	32	24	144

As expected, the dynamic programming algorithm was very sensitive to the node degree, the computation time growing exponentially (see Table 6). The enhanced MIP formulation for EFCNDA was able to solve all instances in less than one second, with an average of 200 ms. This is the opposite of the results obtained on real-life instances, where the dynamic pro-

Table 6: Computation time on fictive instances (ms).

instance	enhanced model FCNDA	enhanced model EFCNDA	dynamic programming
Fi 10	205	166	322
Fi 11	327	77	652
Fi 12	993	332	1409
Fi 13	1130	120	3800
Fi 14	1369	347	12 403
Fi 15	1450	98	39 654
Fi 16	2691	280	164 243

gramming was more efficient. As for FCNDA, the MIP formulation proved to be efficient, with an average computation time of 900 ms. Although the instances with a higher degree are harder to solve, these instances stay tractable in practice. One should favor a MIP based approach, regardless of the problem, when dealing with high degree nodes. As the fictive instances have less arcs, the MIP approaches seem more sensible to the overall number of arcs than to the maximum degree of the instances.

6.3 Operational Considerations

We compared the optimal solutions of both problems. Results are displayed in Table 7, the column labeled "arcs with rule broken" denotes the number of arcs where the maintenance rule (illustrated in figure 5) is violated when FCNDA is solved.

Table 7: Optimal solution costs and characteristics.

instance	Solution EFCNDA	Solution FCNDA	arcs with rule broken
Ar 1	6156.6	6087.3	6
Ar 2	10 357.3	9870.0	8
Ar 3	6546.2	6125.8	14
Ar 4	6720.8	6461.9	14
Ar 5	5081.8	5081.8	0
Ar 6	6546.5	6544.2	1
Ar 7	9348.0	8638.6	18
Ar 8	12 328.3	12 248.4	4
Ar 9	25 619.1	24 422.8	15

An optimal EFCNDA solution is on average 3.7 % more expensive than a FCNDA optimal solution (see Table 7). This can be seen as an acceptable capital expenditure over-cost if it is compensated by future easier maintenance activities.

The maintenance rule is violated in almost every real-life instance we tried (8 out of 9). On average, it is not respected in 6.2 % of the arcs, which is significant. This suggests that optimal FCNDA solutions will be much harder to repair in case of failure on one duct.

7 CONCLUSION

We introduced two combinatorial problems related to FTTH network design, one unconstrained by maintenance consideration and the other one constrained. Regarding the unconstrained problem, one integer programming based solving algorithm was proposed. Adding valid inequalities leads to a more tractable problem. We proposed two solution methods for the constrained problem. These methods are complementary, as they prove efficient in different contexts: the dynamic programming approach is generally faster in graphs where nodes have a small degree, whereas the mixed integer programming, embedding efficient valid inequalities, is generally faster otherwise.

On a complexity level, the unconstrained problem seems harder to solve than the constrained problem. Our numerical experiments confirmed this tendency on real-life instances. From the operational point of view, the maintenance rule can be considered as a reasonable compromise between capital expenditure over-costs for the network deployment and maintenance savings.

REFERENCES

Angilella, V., Chardy, M., and Ben-Ameur, W. (2016). Cables network design optimisation for the fiber to the home. *Design of Reliable Communication Networks, Paris, France*.

Angilella, V., Chardy, M., and Ben-Ameur, W. (2017). Design of fiber cable tree networks for the fiber to the home. *International Networks Optimisation Conference, Lisboa, Portugal*.

Bley, A., Ljubic, I., and Maurer, O. (2013). Lagrangian decompositions for the two-level fttx network design problem. *European Journal of Computational Optimisation*, 1(3):221–252.

Chardy, M., Costa, M.-C., Faye, A., and Trampont, M. (2013). Optimising splitter and fiber location in a multilevel optical ftth network. *European Journal on Operational Research*, 222(3):430–440.

Contreras, I. and Fernandez, E. (2012). General network design: A unified view of combined location and network design problems. *European Journal of Operational Research*, 219(3):680–697.

Europe, F. T. T. H. C. (2016). *FTTH Handbook, Edition 7*. Wettelijk Depot.

Gollowitzer, S., Gouveia, L., and Ljubic, I. (2013). Enhanced formulations and branch and cut for the two level network design problem with transition facilities. *European Journal of Operational Research*, (2):211–222.

Grötschel, M., Raack, C., and Werner, A. (2013). Towards optimising the deployment of optical access networks. *European Journal on Computational Optmisation*, 2(1-2):17–53.

Karp, R. M. (1972). Reducibility among combinatorial problems. In *Complexity of Computer Computations*, pages 85–103.

Mateus, G. R., Luna, H. P., and Sirihal, A. B. (2000). Heuristics for distribution network design in telecommunication. *Journal of Heuristics*, 6:131–148.