A Framework Supporting Literacy in Mathematics and Software Programming

Addressing Some Challenges in STEM Education

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Abstract: The second half of the last century was characterised by a shift from manufacturing to services, particularly in mature economies. This transformation has accelerated in the past decade, due to rapid progress in information technology. Excellence in the so-called STEM subjects (science, technology, engineering and mathematics) is crucial if countries are to remain competitive. Mathematics as a universally applicable method is of special significance, as is IT, which impacts on virtually all industries and can dramatically change economies. Literacy in mathematics and computers, therefore, is more important than ever for individuals, companies and countries. We propose a framework based on R to support the training of students in these crucial areas. We discuss its features, including platform neutrality, costs and specialization flexibility in our paper.

1 INTRODUCTION

We are experiencing a rapid change towards a data driven economy. Companies like the big social media firms or the large global online retailers that collect data on their customers enjoy strategic advantages over so-called old economy firms that lack such direct access to customer data. Buzz words like big data, business analytics, business intelligence and data science are used to describe this trend. Recently, artificial intelligence has been rediscovered; it finally seems to have reached a stage of development where it can have a significant impact on real life.

Mathematics and computer skills are of crucial importance when it comes to analyzing this data. Knowledge in these fields is relevant not only to professionals working in the area but also to ordinary people who should understand, to a certain degree, one of the major driving forces that may impact their lives. From an economic point of view, and from sociological and demographic perspectives, mathematics and computer skills are essential to maintain a functioning society.

Embedded into STEM subjects (science, technology, engineering and mathematics), skills in mathematics and software programming are already fostered in schools and universities, while corresponding apps for smartphones have gained attention. The use of information technology to support mathematics education has already been successful in various studies (Ruthven & Hennessy, 2005; Niess, 2005).

What is needed now is a tool that seamlessly covers basic as well as advanced mathematics and also addresses statistical analysis and data mining techniques. Furthermore, it would be desirable that the tool provides access through a well-designed web-based interface for beginners with an option for programming in a more technically-oriented console for advanced users.

The objective of our paper is to present a framework which is intended to support literacy in mathematics and software programming, addressing some vital needs in education today. In the further course of the paper, we give the framework the working title M*TH-FRAMEWORK.

The remainder of the paper is organized as follows. In the next section, we briefly review the

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need for mathematics literacy and programming skills and the respective performances of students in selected countries. In Section 3, we analyze the major requirements for the framework, followed by the selection of an appropriate technological platform for our framework. We briefly review important features of the selected platform in Section 4 and, in Section 5, we introduce our framework to support literacy in mathematics and programming and discuss its properties. The paper concludes with a summary in Section 6.

2 STUDENTS' PERFORMANCE IN MATHEMATICS IN SELECTED COUNTRIES

In this section, we concentrate on mathematics, stressing the need to remain competitive in a globalized economy. We summarize the performance of school students in mathematics according to the international PISA study of the OECD and also briefly address the need for programming skills.

2.1 Need for Mathematics and Software Programming

The vital importance of mathematics has been recognized by governments and industry around the world.

Leading German engineering companies are aware of the crucial importance of mathematics and actively promote it. The former chairman of Siemens, Peter Löscher, expressed it thus (cited at Neunzert & Prätzel-Wolters, 2015): "Mathematics – this is the language of science and technology. This makes it a driving force behind all high technologies and, thus, a key discipline for industrial nations. Without mathematics, there is no progress and technical innovation." Dieter Zetscher, current chairman of the automotive company Daimler, said (cited at Neunzert & Prätzel-Wolters, 2015): "As does no other science, mathematics helps us in our branch to solve the most varied sorts of problems – and it is exactly this universal applicability that makes it the royal discipline."

Moreover, related fields like data mining and artificial intelligence that are based on mathematics are becoming increasingly important. A good understanding of these fields is not only of crucial importance for professionals working in IT-related jobs like computer science and information systems but for an economy as a whole to remain competitive in a globalized world.

Regarding the importance of teaching software programming in schools, for example, the German chancellor Angela Merkel remarked recently (translated from Merkel (2017): "I am convinced [..]. that the ability to program, the easy ways of programming, that children should learn to do this. Because they will then have a basic understanding of how a robot works, how certain things work, how an app is created."

2.2 Performance in Mathematics and Software Programming

Nevertheless, despite excellent job prospects, it is still a challenge to motivate a sufficient number of students to step into these fields and secure a sound education in mathematics, computer science and information systems at schools and universities.

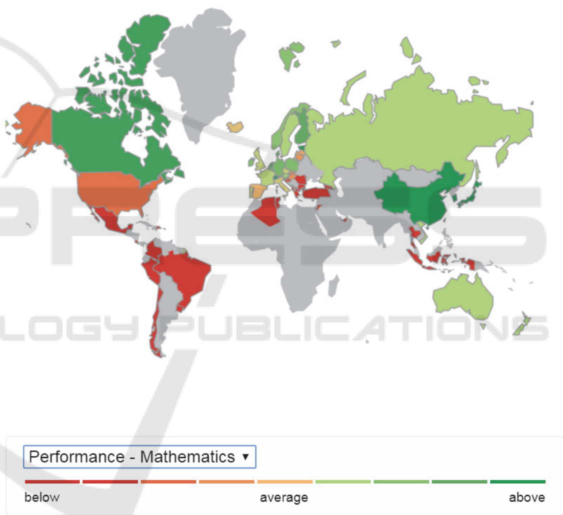


Figure 1: PISA - Performance in Mathematics in Selected Countries in 2015 (OECD, 2016b).

Germany, for instance, is poor in natural resources but has a large engineering sector, so a strong performance in mathematics at school and university is important to its economy. However, according to a recent PISA study, German school students are not amongst the top performers in mathematics. They perform only slightly above average (OECD, 2016a).

Meanwhile, in the U.S.A., interest in STEM subjects improved, but not in mathematics, according to a recent study (Neuhauser & Cook, 2016): "The 2016 U.S. News/Raytheon STEM Index recorded a slight rise in hiring, education and general interest in technology and engineering over last year, while math education and general interest in science declined."

Performance in mathematics of the countries covered by the PISA is depicted in Figure 1. The study shows that eastern Asian countries like China, South Korea and Singapore are top performers with respect to education in mathematics. The countries on the American continents, except Canada, are performing under average while most countries in Europe are ranked above average.

We are not aware of any similar study that evaluates and compares programming skills of school students in an international context. A possible reason is that computer science software programming is not a widely established subject in schools in many countries. Due to its importance this makes the need to support it even more pressing.

3 REQUIREMENTS FOR AND SELECTION OF A PLATFORM

3.1 Requirements for the Framework

In this section, we present the results of our requirements analysis for the platform that we use for the framework to support literacy in mathematics and programming.

In our analysis, we identified the following criteria as essential to a framework's success:

- Cost efficiency
- Platform independence
- Size of the platform network
- Comprehensiveness of the mathematical implementations
- Support for possible levels of user experience.

We address these criteria in more detail in the following paragraphs.

Cost Efficiency. Most educational sectors around the world suffer from budget constraints even though education is regarded as possibly the most crucial factor to address significant issues such as overcoming inequality and poverty, and strengthening the competiveness of firms and economies in a globalized world. Financing a framework, therefore, might be a considerable challenge for many educational institutions. Hence, a cost efficient, ideally free, approach is of crucial importance for the success of the framework.

Platform independence. With respect to platform independence we distinguish two factors, namely (1) independency from hardware platforms and (2) independency from software. The first is related to the possible use of the framework on smartphones, tablets and personal computers. The second corresponds to the independence from any particular operating system. Note that these two factors are beginning to overlap, for example, see the first attempts to use the Android operating system on personal laptops and computers (and, of course, the initially failed strategy of Microsoft to launch a virtually identical Windows system for all platforms, including smartphones, tablets and personal computers).

Size of the platform network. The underlying platform should be well accepted, i.e., the network of users of the platform should be large. This criterion is motivated by classic arguments well known from the theory of network externalities (Katz & Shapiro, 1985). For example, a large network has positive effects on learning since support is generally available, ranging from (online) textbooks to discussion group and forums. A large network also makes it more likely that the platform will be available for the foreseeable future. This would secure the developers' and users' investment in the framework.

Comprehensiveness of the mathematical implementations. The platform should already provide a large range of mathematical concepts and predefined solutions that can be used immediately. This avoids the cost and time of starting from scratch.

Possible Levels of User Experience. Here we refer to the different needs of a diverse range of users, including absolute beginners and experts. The advantages of a diverse range of users are twofold. First, the different groups can learn from each other: the beginners learn from the advice of the experts and the experts benefit from explaining and answering questions from the beginners. Second, when a beginner improves her/his knowledge she/he can continue to use the same platform by gradually increasing her/his level of expertise within the framework.

3.2 Selection of a Platform for the Framework

In our analysis, we distinguish between three categories of possible platforms for the framework:

- General purpose programming languages
- Mathematics solution apps, webpages and software
- Numerical computing environments.

Note that in general some applications do not fit into one single category but offer functionalities across these categories.

General purpose programming languages. General purpose programming languages like C/C++ or Java provide excellent frameworks for virtually any kind of task. In particular, due to its efficiency, C/C++ has been a leading platform for the implementation of mathematical/numerical problems in industry and academia over the past decades. Take-up by a wide range of respected libraries is an impressive indicator of its success.

However, we wish to establish a framework that suits beginners as well as advanced learners. Programming languages like C/C++ are already too "sophisticated" for beginners and will possibly lead to frustration. On the other hand, implementing a framework including the mathematical algorithms from scratch would possibly exceed our resources, even considering the advanced apps and websites tools that are available.

Thus, general purpose programming languages do not satisfactorily address the levels of user experience that will be covered and also violate our cost efficiency criteria.

Mathematics solution apps, software or webpages. Examples of mathematics solution apps are Mathway (mathway.com), MATH 42 (math-42.com), MalMath (malmath.com) and Wolfram Alpha (wolframalpha.com). All run on Android, and Mathway, MATH 42 and Wolfram Alpha are also available for further operating systems. Mathway and Wolfram Alpha offer websites for mathematical problem solving. Wolfram Alpha's website even goes beyond mathematics towards a structured general purpose search engine.

While the apps and webpages are excellent for addressing mathematical problems, they lack a programming environment for advanced learners who are wanting individually designed software that perfectly fits their needs. Therefore, as with general purpose programming languages, mathematics solution apps, software or webpages do not satisfactorily address our criterion about levels of user experience.

Numerical computing environments. Leading numerical computing environments include Mathematica (wolfram.com/mathematica) and MATLAB (mathworks.com/products/matlab.html). While MATLAB has had a leading role in engineering with many corresponding libraries, Mathematica has a strong position in the education sector and is also frequently used as a problem solving tool in industry. Its particular strength is that it integrates mathematical symbolic computation functionality; it can, for example, symbolically integrate or differentiate (note the same applies to the related website Wolfram Alpha).

While Mathematica and MATLAB are commercial products there are free alternatives available, in particular to MATLAB. For example, the numerical computing environment GNU Octave (gnu.org/software/octave) claims to be compatible with MATLAB; another popular environment, Scilab (scilab.org/), is similar/partly compatible with MATLAB.

R-Project (r-project.org) has gained increasing popularity in the past decade. R started as a free alternative to S as an environment for statistical computing. Since then, its range of applications has gone beyond statistics and now covers wide parts of numerical analysis, data mining and mathematics. According to the TIOBE Index (tiobe.com/tiobe-index/), which ranks the popularity of programming languages, it is leading the field of special purpose languages for numerical computing, slightly ahead of MATLAB: in August 2017, R is ranked #15 and MATLAB #17 among all programming languages.

The core features of these numerical computing environments include integrated development environments (IDEs), purpose-built programming languages, graphing utilities and respective substantial libraries. Numerical computing environments, therefore, generally address our criterion about possible levels of user experience. All these environments are excellent bases for the framework.

Mathematica, with its mathematical symbolic computation power, is an outstanding product and leads the field in the educational sector. However, one of our essential criteria in the framework selection process was that the framework can be accessed for free (cost efficiency criterion). Firstly, schools and universities are under budget restraints that may make it difficult or even impossible to obtain commercial software and, secondly, students should be able to get the framework for free and independent of their enrolment status (for example whether they are a current or former student). The cost efficiency criterion also leads to the exclusion of MATLAB.

All remaining free numerical computing environments run on all major operating systems for personal computers. So, in terms of our platform independence criterion they are equally suitable for our framework. We would also consider the criterion about comprehensiveness of the mathematical implementations as fulfilled. However, a detailed comparison is practically impossible: virtually everyone can upload improved and new packages to the R repository anytime. This has led to a puzzlingly high number of packages, which is one of the biggest advantages - and disadvantages - of R.

Our final decision between GNU Octave, R and Scilab, therefore, is based on network aspects (size of the network of the platform criterion) and the possibility of integrating different levels of user experience (possible levels of user experience criterion).

In relation to the size of the network, we have the impression that R has a strong momentum which puts it significantly ahead of GNU Octave and Scilab. R and MATLAB family (in particular GNU Octave, Scilab and MATLAB itself) are similarly popular according to the TIOBE Index (see above). With respect to the possibility of integrating different levels of user experience, we found that R provides excellent packages, particularly Shiny (https://shiny.rstudio.com, see further down for examples), that support the development of interactive webpages to suit the needs of beginners.

The evaluation is summarised in Table 1. Note that platforms with a minus (–) are excluded (essential criteria) and the remaining criteria are not further investigated. Therefore, general purpose programming languages and mathematics solution apps, software or webpages are excluded from any further evaluation. The same applies to the commercial numerical computing environments Mathematica and MATLAB. Among the rest, we consider R stronger with respect to its network size and the possible levels of user experience that can be addressed in comparison to GNU Octave and Scilab. Hence R was selected as the platform for the M*TH-FRAMEWORK as it addresses most of the criteria well. Its only considerable weakness is that it does not offer symbolic computation functionalities like Mathematica.

4 THE R-PROJECT ENVIRONMENT

In this section, we briefly review the major features of R-Project. Firstly, we describe its origin; then we give a very brief impression about its power to deal with mathematics and statistics. Lastly, we discuss the package Shiny which supports the generation of dynamic websites.

Table 1: Summary of the Evaluation of the Platforms.

	Cost Efficiency	*Platform Independency*	*Size of Network*	*Comprehensiveness of Math. Implementation*	*Possible Levels of User Experience*
General purpose programming language					
C/C++, Java etc.					– not suitable for beginners
Mathematics solution apps, software or webpages					
MATH 42, MalMath etc.					– limited flexibility for experts
Numerical computing environments					
Mathematica	– commercial tool				
MATLAB	– commercial tool				
GNU Octave		+ runs on all major platforms	= good	= sufficient, no symbolic comp.	=
R-Project		+ runs on all major platforms	+ excellent	= sufficient, no symbolic comp.	+ console+packages for webpages
Scilab		+ runs on all major platforms	= good	= sufficient, no symbolic comp.	=

Legend: + fine, = acceptable, – essential criterion, leads to exclusion.

4.1 Fundamentals about R-Project

R-Project (r-project.org) was founded as an alternative to S at the University of Auckland, New Zealand, in 1993 (Ihaka, 1993). Initially its focus was on statistical computing, but it has since become a universal platform for mathematics, with a particular strength in data analysis. In the past decade, it has rapidly gained attention and is now used by both academics and company professionals.

A core team organizes and maintains R-Project. In addition to its core functions the user can write packages and upload them to R repository. This means there is a large community contributing packages to virtually any area of mathematical analysis.

4.2 R's Programming Environment

The R software is console-based and can be accessed by R's standard console or by third party IDEs, with RStudio (rstudio.org) probably the most popular (see Figure 2 for a simple mathematics example using RStudio IDE). R's programming language is optimized for statistical analysis and therefore provides convenient commands to manipulate data structures like vectors and matrices as well as more complex structures like R's data frames.

For example, the command for addressing the 5th column of matrix `a` is:

```
a[,5]
```

or the multiplication of matrixes `a` and `b` is just one command:

```
a %*% b
```

R not only provides virtually all standard statistical functions, for example `mean()` or standard deviation `sd()`, but also provides advanced statistical methods in corresponding packages. Besides statistics, several other areas of mathematics are covered by R packages, for example operations research with classic algorithms for linear programming, or linear algebra, with linear equation systems packages. Moreover, R has sophisticated graphics and plotting functionalities (Murrell, 2011) such as 2D and 3D plots, pie chart, histograms and contour lines. R provides the possibility of writing one's own programs in R. The inclusion of code writing in, e.g., C/C++ is also possible.

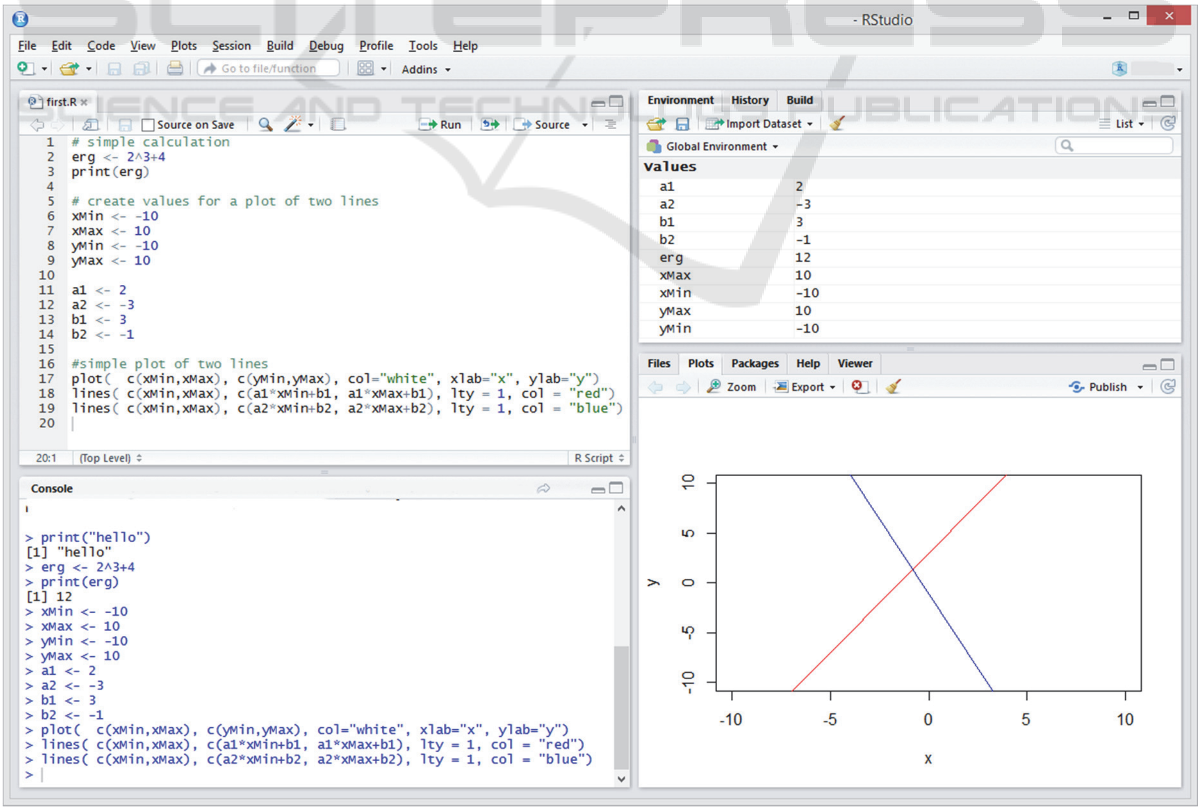


Figure 2: Mathematics Example in RStudio IDE (rstudio.com).

Global companies also integrate R in their products. Microsoft, for example, recently took over Revolution Analytics, a company which specialises in R. It maintains Microsoft R Open, an enhanced R distribution (https://mran.microsoft.com/), and also supports R in its cloud computing service Azure (Olavsrud, 2015). Another example of the commitment of major companies to R is Oracle, which provides Oracle R Enterprise to support R (Oracle, 2017). Hence, R is not only used in academia but is also employed in and supported by companies throughout the world as a professional statistical and numerical computing environment (Heller, 2017; Vance, 2009).

This popularity has generated comprehensive literature about R. There are several text books on R, for example Matloff (2011) and Teetor (2011), many online resources, the most notable being "Introduction to R" (Venables, Smith, R Core Team, 2017) by some of the main drivers of R, and a range of discussion groups and blogs.

In our context so far, R is an excellent platform for advanced learners in mathematics and data analysis who can develop their mathematical skills as well as their programming experience. However, this console-driven programming environment would probably overwhelm most beginners in mathematics.

For this user group app- or web-based interfaces like MATH 42, MalMath and others would provide a better starting point. However, as we discuss in the next section, R has also excellent packages that support the development of user-friendly interactive interfaces.

4.3 Interfaces of R

Besides the console, R has strong export facilities including several packages that help the user report the results of an analysis. These include Sweave (Leisch, 2016) that generates LaTeX documents from R code.

Another package that supports the reporting is Markdown (rmarkdown.rstudio.com/). It can generate Word, HMTL or LaTeX/PDF files. The basic workflow of the generation of such documents is shown in Figure 3.

For the M*TH-FRAMEWORK, we use R's Shiny package which is a web application framework (Chang, 2017) that generates interactive and dynamic webpages from R. Due to limited space we refrain from a detailed technical discussion of the Shiny web application framework and refer the reader to the respective webpages and documents for further information. The homepage of the Shiny framework at https://shiny.rstudio.com/ is an especially rich resource, providing impressive examples for websites generated by Shiny and detailed documentations on the framework.

An example for a webpage generated by the framework is depicted in Figure 4. In the next section, we discuss the concept and the layout of these webpages.

5 A COMPUTER-BASED FRAMEWORK FOR MATHEMATICAL EDUCATION

In this section, we present a computer-based framework for mathematical education based on the R-Project.

5.1 Layout of the Web-based Interface

As already mentioned, an example for the M*TH-FRAMEWORK is depicted in Figure 4. In this case, the screenshot was taken from the framework running in web browser on a personal computer. However, on smartphones and tablets the layouts are virtually identical, allowing the user to switch seamlessly between her/his devices. Therefore, it is sufficient to discuss the layout as shown in Figure 4.

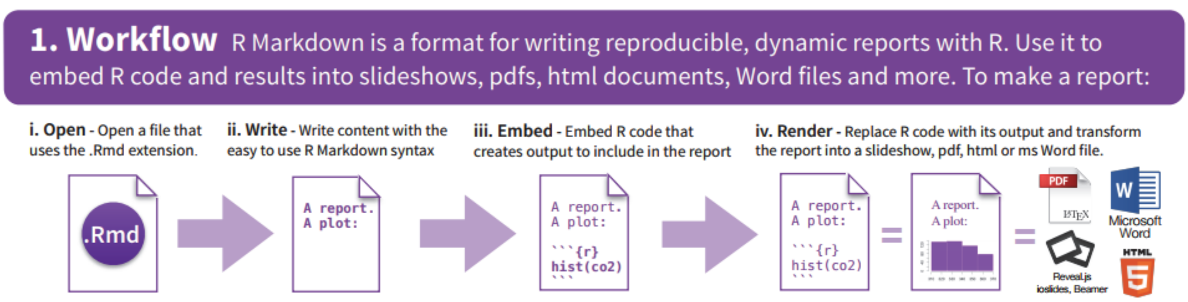


Figure 3: Workflow of R Markdown to Generate Reports (RStudio, 2014).

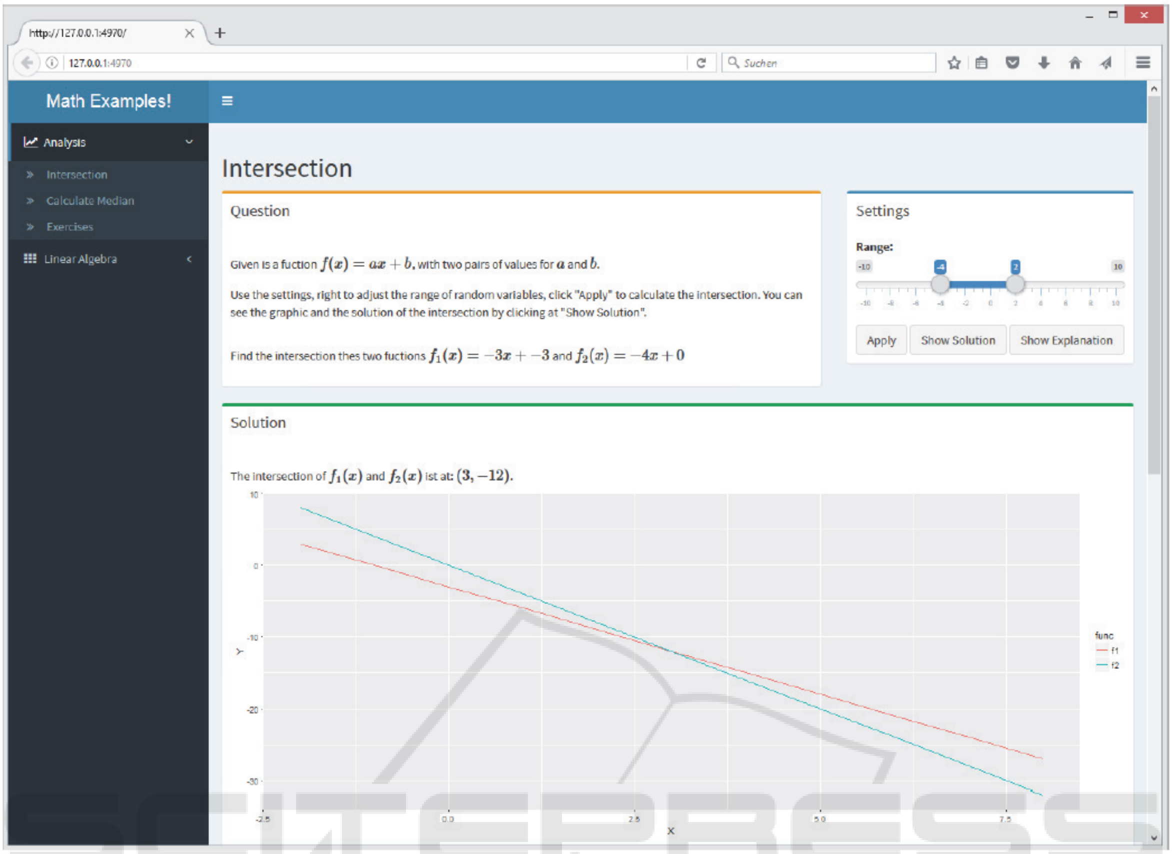


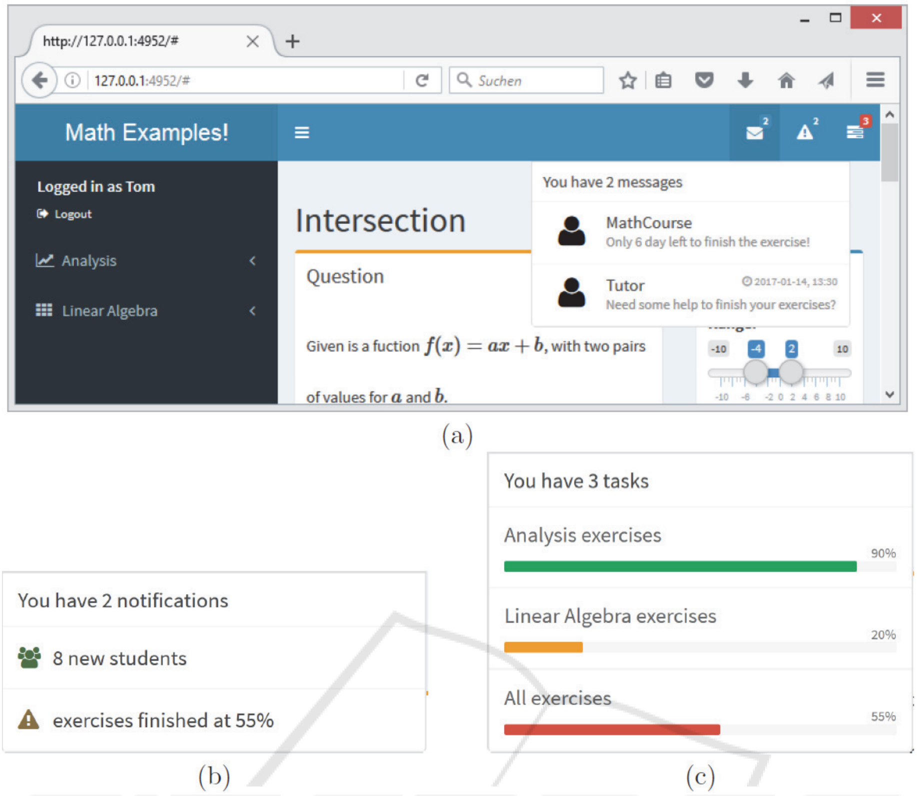
Figure 4: Mathematics Example for a Webpage of the Framework.

On the left side of the browser a navigation bar shows the tasks that are available. On the right side the selected task is presented. In the example, the task is to determine the intersection point of two linear functions. The frame of the task has an orange bar at the top. To the right there is another frame, with a blue bar at the top. In this frame settings can be chosen, such as the range of the parameters of the two linear functions. Within this range the parameters are set randomly to provide new tasks every time. Furthermore, the command buttons are located in this frame. The standard ones are "Apply", "Show Solution" and "Show Explanation". While the latter two are self-explanatory, "Apply" means that a new task is generated based on the range of the parameters as defined in the frame. In a future release of the framework, we are considering implementing a fourth standard button ("Show Code") that bridges the gap between the interface and the underlying R code.

In Figure 4, the solution is shown in a separate frame with a green bar at the top. Besides the numerical values of the intersection, a graph is also shown to illustrate the solution. The mathematical background of the solution can be found when the user scrolls further down in the window; it is not presented here due to space restrictions.

The presentation of the M*TH-FRAMEWORK on a smartphone is virtually the same since it is a website-based framework with no separate app for mobiles. This has the advantage that the framework runs on personal computers/laptops and smartphones independently of an operating system. This means that the development and maintenance costs are minimal in comparison to a solution where the software needs to be adapted to PC and smartphone operating systems. In particular, for smartphones there is no need to develop and maintain different apps for different mobile environments.

Finally, besides the core application involving mathematics tasks, it is intended that the framework will also support functions that let students interact with each other and communicate with their instructors (see Figure 5a). It should have a memo board (see Figure 5b) where messages are displayed, for example new students joining a course or the proportion of exercises completed. It will also provide basic tools to manage and monitor the progress of a student from both the student and the



(a) (b) (c)

Figure 5: Dashboard and Messaging Functionality of the Framework.

tutor's perspective (see Figure 5c).

A feasibility study confirmed that these functionalities can be implemented in R. They have not been implemented yet as our current focus is on extending and completing the portfolio of mathematics tasks and developing tasks in the area of data mining. They are scheduled for future versions of the framework.

5.2 Discussion

The project so far has confirmed that R is an excellent platform for the development of the framework.

Its specialized programming language makes it a perfect environment to develop respective code very quickly. Furthermore, CRAN (Comprehensive R Archive Network at https://cran.r-project.org/), with its large number of packages, is a rich repository for "off the shelf" applications. These packages not only support solutions for mathematical algorithms but also support exporting functions and frameworks for interactive dynamic websites.

R's large network, which embraces both academia and major international companies, has led to an active community that provides support, including IDE and special purpose packages. Many blogs and tutorials can be found free of charge on the web.

Therefore, the framework is not only a tutorial in mathematics but it also introduces beginners to R, one of the leading platforms for statistical analysis and data mining. In this context, it is a very suitable environment to gain and enhance programming experience. Furthermore, as many applications in academia and industry show, it can be used to address real life problems. Last but not least, there are practically no entry barriers as it runs on virtually any operating system and is free.

6 CONCLUSIONS

In this paper, we presented a framework for mathematics education using R-Project. The framework also supports the acquisition of knowledge in computer science and information systems, in particular in programming.

Crucial advantages of R are that it is free and runs on virtually any platform, so it can be used regardless of budget constraints or a given computer infrastructure. Another important advantage is that it provides facilities that support beginners (for example, through its interactive Shiny web framework) as well as experts (by its programming facilities in the console).

Finally, R already offers a portfolio of sound mathematical functions. Many packages, often written by senior experts in their field or even by the people who proposed the algorithms, offer a wide range of mathematical tools. Although R's main strength is in the field of statistics and data mining, several other fields of mathematics are also covered.

Therefore, R is an ideal platform for the framework to support mathematics and computer literacy. Presently, we are extending the portfolio of mathematical tasks within the Shiny web framework and working on pathways to bridge the gap between web- and console-based access to mathematics. Dashboards, messaging and monitoring tools are planned for a future version.

We are also intending to form a group of educators who use and contribute to the framework to further advance its coverage of important fields of mathematics.

In the long run, a portfolio of applications in data mining would be desirable to address one of the hottest areas in computer science and information systems today.

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TRADEMARKS