Hurst Exponent and Trading Signals Derived from Market Time Series

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Abstract: In this contribution, we investigate whether it is possible to use chaotic properties of time series in forecasting. Time series of market data have components of white noise without any trend, and they have components of brown noise containing trends. We constructed a new technical indicator MH (Moving Hurst) based on Hurst exponent that describes chaotic properties of time series. Further, we stated and proved a hypothesis that this indicator can bring more profit than the very well known indicator MACD (Moving Averages Convergence Divergence) that is based on moving averages of time series values. In our experiments, we tested and evaluated our proposal using hypothesis testing. We argue that Hurst exponent can be used as an indicator of technical analysis under considerations discussed in our paper.

1 INTRODUCTION

The economists dispute over the problem how much randomness influences markets and stock prices. There are the following competing hypothesis trying to explain it.

Efficient Market Hypothesis (Fama, 1970) states that markets are efficient in the sense that the current stock prices reflect completely all currently known information that could anticipate future market, i.e., there is no information hidden that could be used to predict future market development. This model is based on the assumption that market changes can be represented by a normal distribution.

Inefficient Market Hypothesis (Shleifer, 2000) was formulated later because some anomalies in market development have been found that cannot be explained as being caused by efficient markets. In (Peters, 1996) and (Lo and MacKinlay, 1999), a strong statistical evidence is provided that the market does not follow a normal, Gaussian random walk.

Fractal Market Hypothesis (Peters, 1994) represents a new framework for modeling the conflicting randomness and deterministic characteristic of capital markets. We follow ideas of this hypothesis in our paper.

Our motivation was to answer the question whether it is possible to construct a new technical indicator based on chaos theory which would bring more profit than some of standard technical indicators, e.g., MACD, that are often used.

We developed a new technical indicator MH (abr. Moving Hurst) based on fractal dimension of time series, i.e., on Hurst exponent, and we evaluated the hypothesis that Hurst exponent of market time series can be successfully used as a technical indicator in a trading strategy. Exactly, it was used on existing data, and it brought more profit than using of MACD.

The main idea of our approach is that changes in fractal dimension of a time series, which describe the history of prices, invoke changes in behavior of investors and traders. They buy or sell, and the feedback can be either negative, i.e., the fluctuation of prices decreases (a trend appears or continues), or positive, i.e., the fluctuation of prices increases.

The Hurst exponent derived from fractal dimension describes chaotic properties of time series, and it tells us whether there is a long memory process in a time series or not. We discuss the values of Hurst exponent in Section 3.6.

In contrast to the works cited in Section 2, we do not agree with the commonly accepted conclusion that Hurst exponent has no practical value in trading forecast. Our original contribution is that we found, implemented, and tested a construction supporting the idea of Hurst exponent applicability to trading.

Our paper is structured as follows. In Section 2, we discuss related works. We present the problems of

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chaotic markets in Section 3. Then we briefly explain fractal dimension and methods how Hurst exponent can be estimated in Sections 3.4, 3.6. In Section 4, we describe the trading strategy we used in our experiments. Our contribution is presented in Sections 4.1. Our implementation, used data, experiments, and results are described in Sections 5. Section 5.1 contains testing of our hypothesis. In Section 6, we conclude.

2 RELATED WORKS

The topic Fractals and Markets is covered by many interesting and famous publications. Mandelbrot (Mandelbrot, 1963) has shown that financial data have a fractal nature, i.e., that time series of prices in 15 minutes interval have a very similar shape like time series of daily close prices.

In his book (Mandelbrot, 1997), Mandelbrot collected his papers on the application of the Hurst exponent to financial time series. Unfortunately, he does not describe how fractals might be applied to financial data to achieve more profit. In (Kroha and Lauschke, 2012), fractal dimension was used in fuzzy approach to market forecasting.

Both Peters's books (Peters, 1994), (Peters, 1996) explain and discuss the Hurst exponent and its calculation using the rescaled range analysis (R/S analysis). Conclusions of Peters support the idea that there is indeed some local randomness and a global structure in the financial market. Unfortunately, Peters only applies Hurst exponent estimation to a few time series and does not discuss the accuracy of Hurst exponent calculation for sets of stock prices.

In the book (Lo and MacKinlay, 1999), long-memory processes in stock market prices are discussed. But the authors do not find long-term memory in stock market return data sets they examined. Methods that compute fractal dimension or Hurst exponent are described in overview in (Gneiting et al., 2012). The problem is that all of them are estimators and deliver values that differ.

Another promising approach has been presented in (Selvaratnam and Kirley, 2006), (Qian and Rasheed, 2004) where Hurst exponent is used as an input parameter in neuronal networks applied to predict time series.

We investigated the topic of fractal dimension in markets in our paper (Kroha and Lauschke, 2012) and compared the fuzzy and fractal technology.

In paper (Mitra, 2012), the correlation between the Hurst exponent of a time series and 1-day profit has been measured, but because the 1-day profit has its Hurst exponent near 0.5, only weak positive correlation coefficients have been found.

Our goal is to investigate the impact of changes of the Hurst exponent on trading strategies. This topic is discussed in (Vantuch, 2014), where Hurst exponent is used for supposed improvement of performance of trading strategies based on technical indicators RSI and CCI. However, no improvement has been found.

3 PROBLEMS OF CHAOTIC MARKETS

3.1 Deterministic Chaotic Systems and Non-deterministic Random Systems

The behavior of many physical systems is strongly given by physical laws and initial conditions, i.e., an initial state given by values of input parameters at the start time. Often, we suppose we can restart such systems using the same initial state. In simple systems (e.g., some computer programs), we really can do it. In complex systems (e.g., computer programs cooperating with Web), we practically cannot restart the system twice under the same initial conditions. We do not know the state of the global network exactly enough, because of ever-present changes and unexpected events. This phenomenon is not a new one. Old Greeks knew the saying "No man ever steps in the same river twice" (Heraclitus of Ephesus).

There are two kinds of such systems:

- Deterministic chaotic systems have the property that their final behavior is extremely dependent on any imprecision in the initial conditions (Lorenz, 1963). Determinism means that rules of behavior do not involve probabilities. These systems are described by nonlinear differential equations, whose solutions behave irregularly (Casdagli, 1991). Their properties are discussed in many publications, e.g., in (Gleick, 1987).
- Stochastic nonlinear systems are affected not only by small differences of input parameters but also by unpredictable, random, external events having unpredictable impact on system behavior. They are indeterministic because their rules of behavior involve probabilities.

Financial markets can be seen as a complex mixture of deterministic chaotic systems and stochastic non-linear systems, even though fractal market hypothesis stresses the chaotic part. Processes behind markets have their weak deterministic component (some deterministic rules exist, e.g., 1-day returns have Gaussian distribution), but they have a strong built-in randomness component, because the main changes are reactions on unpredictable, random events in the world, e.g., volcano eruption, terrorist attack on World Trade Center, floods in Thailand, some political decisions.

Additionally, compared with deterministic chaotic systems in physics (e.g., in meteorology, etc.), markets are nonlinear feedback systems, because they contain a component including psychology of human investors called behavior finance. This component brings reflexivity into the system, i.e., circular relationships between cause and effect. For example, when we would predict weather very exactly, weather were not change because of it. On the other hand, a well-known, precise market prediction would change markets completely.

Before trading, investors try to analyze the market. There are two main methods of stock analysis.

- Fundamental analysis assumes that the markets prices in the future can be derived from the economic results of companies today.
- Technical analysis assumes that market prices are affected mainly by behavior and sentiment of investors, because investors interpret the economic results. Components of investor behavior, like greed, are regarded as being stable, and they specify some patterns usable for price prediction.

Often, methods of fundamental and technical analysis are combined.

3.2 The Problem of Volatility

To measure the chaotic behavior of markets is not a new idea, because the chaotic behavior is very obvious in every observation. There was an instrument used by traders before any knowledge about fractal dimension. It has been denoted as volatility, and it is the standard deviation of prices. It is defined as a measure for variation of price of a financial instrument over time, i.e., volatility is simply the range in which a day trader operates. Volatility is investigated mainly for purpose of risk management (Brooks and Persand, 2003). Volatility forecasting is described in (Northington, 2009), and all aspects of using volatility are critically discussed in (Goldstein and Taleb, 2007).

3.3 The Problem of Stationarity

Both kinds of analysis use the hypothesis that we can apply our experience coming from the past to predict the future. The hypothesis is based on the assumption that the same set and structure of patterns, which occurred in the past, will occur again in the future, i.e., the statistical characteristics of data in the past are the same as they will be in the future. This property is called stationarity.

We cannot exactly answer the question whether time series representing market data stay stationary in the future. We can only investigate whether time series were stationary in the past. However, if they were, it does not mean that they stay stationary for ever. More or less, we cannot see any reason for that, because there are unique events that never occurred in the past.

In (Taleb, 2007), the story of a turkey is presented as an excellent example of expected (but not well-founded) stationarity. A turkey is regularly fed by a farmer for 1,000 days. It derived a simple pattern saying that it will be fed at the Day 1001, too. But it was two days before Thanksgiving, and the turkey was served in the next days as a dinner. We can see that we can never know whether our patterns from the past will be valid in the future. In (Nava et al., 2016), the problem of stationarity of patterns in high frequency financial data is discussed.

3.4 Fractal Dimension and Its Estimators

The concept of fractal dimension started with Hausdorff dimension. It was introduced in 1919 (Hausdorff, 1919) as a measure of smoothness of spatial data. The Hausdorff method completely covers the given set X by $N(r)$ balls (circles for E2) of radius at most r. The Hausdorff dimension is the unique number d such that $N(r)$ grows as $1/r^d$ as r approaches zero (Falconer, 1990), (Gneiting et al., 2012). The similar idea in used in Minkowski's box-counting dimension D. It uses an evenly spaced grid (building boxes) to cover the set. Then, it counts how many boxes are required to cover all set elements. This number changes as we make the grid finer by applying a box-counting algorithm.

Fortunately, we can use a simple relation $H = 2 - D$ (Peters, 1994) between fractal dimension D (estimation of d) and Hurst exponent H for our purpose. However, it can be used only for so-called self-affine processes, because the local properties are reflected in the global ones in them, and this is the case of markets. More generally, fractal dimension is a local property, while the long-memory dependence characterized by the Hurst exponent is a global characteristic (Gneiting and Schlather, 2004).

3.5 The Rescaled Range Analysis - R/S Analysis Method

The rescaled range analysis (R/S analysis) is the original method invented and used by Hurst (Hurst, 1951). Briefly, the R/S analysis is the range of partial sums of deviations of time series parts from their means, rescaled by their standard deviations. We use here the definition following (Peters, 1994), (Krištoufek, 2010).

First, we start with processing of the complete time series of length N and divide it into $2^k(k = 0,1,\dots)$ adjacent sub-periods of the same length n, so that $2^k * n = N$. This means, we obtain 1 sub-period of length N, 2 sub-periods of length $N/2$, 4 sub-periods of length $N/4$, etc. For each sub-period, we calculate the arithmetic mean x_{mean} and construct a new series $Z_r = x_r - x_{mean},\ r = 1,\dots,n$, and in the next step we create a next new series $Y_m, m = 1,\dots,n$ of cumulated deviations from the arithmetic mean values, i.e., we create a sum of all deviations from the mean in the given sub-period. Then, we calculate an adjusted range R_n, which is defined as a difference between a maximum and a minimum value of the cumulated deviations Y_r , i.e., $R_n = \max(Y_1,...Y_n) - \min(Y_1,...Y_n)$, and finally, we compute a standard deviation S_n of original elements of each sub-period. Each adjusted range R_n is then standardized by the corresponding standard deviation S_n and forms a rescaled range as R_n/S_n. Then, we calculate an average rescaled range $(R/S)_n$ for all sub-periods of fixed length n (i.e., average of all sub-periods having the same length).

Second, we repeat the process iteratively using $k = 0,1,2,\dots$ for each length $n = N/2^k$ of sub-periods. They scale as described more formal in the formula for the estimation of the Hurst exponent (Peters, 1994):

$$E\left[\frac{R_n}{S_n}\right] = Cn^H \text{ as } n = 1,\dots, \text{ as } n \to \infty \qquad (1)$$

where:

- R_n is the adjusted range as explained above
- S_n is its standard deviation
- $E[x]$ is the expected value - in our case, we used arithmetic average
- n corresponds to the time span of the observation
- C is a constant
- H is the Hurst exponent

The Hurst exponent H is the slope of the plot of each ranges $log((R/S)_n)$ versus each ranges $log(n)$. Analysis of Hurst exponent estimation and its accuracy is given in (Resta, 2012).

3.6 Hurst Exponent and Market Time Series

In the sections above, we resumed briefly some basics about fractal dimension and Hurst exponent of time series. Now, we explain what is a relationship between Hurst exponent and market trends. This problem has been investigated since Mandelbrots first papers in 60-ties, and it is presented in Peters books (Peters, 1994) and (Peters, 1996).

The Hausdorff definition of fractal dimension and its relation to Hurst exponent specify that values of the Hurst exponent range between 0 and 1 because our objects have fractal dimension between 1 (a line) and 2 (a line covering an E2-surface completely).

It is known (Peters, 1994) that a value of 0.5 indicates a true random process (a Brownian time series). A Hurst exponent value H, $0.5 < H < 1$ indicates persistent behavior (e.g., a positive autocorrelation - a trend). A Hurst exponent value $0 < H < 0.5$ indicates anti-persistent behavior (or negative autocorrelation - change of a trend). An H closer to one indicates a high risk of large and abrupt changes.

It is also known (Peters, 1994) that the value of Hurst exponent changes with the length of time period used. For example, we measured for DAX index 0.54 for 1-day return time series and 0.82 for 50-days return time series. This means that 1-day returns correspond practically to white noise (Hurst exponent of white noise = 0.5), but the 50-days returns contain some trends.

Another known fact is (Mandelbrot, 1963) that time series gains a long memory character when the return period increases. Moreover, their distributions move away from the Gaussian normal to so-called fat-tailed distributions. Mandelbrot (Mandelbrot and Hudson, 2004) has shown that prices change in financial markets did not follow a Gaussian distribution, but rather more general Lévy stable distributions.

4 TRADING STRATEGY

Trading strategy is a set of rules used by traders to buy or sell their investments. An important part of it are BUY-signals and SELL-signals denoting the corresponding time points.

Buy&Hold is a passive investment strategy. An investor selects stocks at the beginning, buys them and holds them for a long period of time, regardless of fluctuations in the market.

MACD (Moving average convergence divergence) is a basic trend indicator that represents the relationship between two moving averages of prices.

Moving averages smooth the price data. MACD is usually calculated by subtracting the 26-day exponential moving average (EMA) of the price data from the 12-day EMA of the price data. The result is called a MACD line. A 9-day EMA of the MACD is then used as the "signal line". In a graph of MACD and 9-day EMA of MACD, we observe the crossings as triggers for buy and sell signals (Murphy, 1999). This construction is denoted as MACD(26,12, 9).

4.1 Our Non-linear Method for Generating Signals - Indicator MH

We correlated fractal dimension with investment risk. A simple model is given by the following situation. Experienced traders being in market, i.e., having an opened position after a BUY-signal of their trading strategy signalized them a starting trend, see or feel the growing risk, expect the end of trend, and start selling. They start to sell earlier than the others (not so experienced), who continue to buy because they believe that the market will carry on growing. Prices can continue growing. But after a specific time period, more and more traders start selling their positions, and also the unexperienced traders begin to understand the situation and start to sell, too. At this moment, prices start to fall down.

Our hypothesis is that we can estimate the time point at which the clever traders start to sell. There is the possibility that their feeling correlate with fractal dimension changes. Standard technical indicators react on changes of prices, but it is likely that the fact that clever traders start to sell does not impact prices strongly enough to influence the standard technical indicators. We suppose that chaotic properties can react before prices change. Because of that, we expect more profit when using a strategy based on fractal dimension changes.

The idea of our non-linear method is similar to the mechanism of MACD. However, instead of moving averages computed from daily values of time series in a given box size, we used moving Hurst exponents computed from fractal dimension of daily returns in a given box size.

We experimented with different moving box sizes for moving Hurst exponents and with different return periods R, i.e., for each time series of values, we built a time series of returns for a given period R and searched a maximum of a profit-function of three variables - H-fast, H-slow, and R.

Finally, after time consuming brute force computations, we obtained the best profit results for the fast moving Hurst exponent H-fast = H16 (i.e., Hurst exponent is computed for a time window of the last 16 days), the slow moving Hurst exponent H-slow = H32 (i.e., Hurst exponent is computed for a time window of the last 32 days), and for the return period of one day R1. So, we generate BUY- or SELL-signals at each crossing of H16 and H32 as shown in Fig. 1. In the following formulas, n denotes the index of a time series member.

$(H16-H32)_n > 0$ and $(H16-H32)_{n+1} < 0 \Longrightarrow$ signal BUY

$(H16-H32)_n < 0$ and $(H16-H32)_{n+1} > 0 \Longrightarrow$ signal SELL

When H16 is crossing down the H32 then we generate a BUY-signal. We suppose that chaotic properties will decrease. When H16 is crossing up the H32 then we generate a SELL-signal. We suppose that chaotic properties will increase.

Of course, we can imagine a very large number of other constructions of this kind covering every thinkable combination of Hurst exponent and any of some hundreds existing technical indicators. However, experiments are time consuming because of the enormous number of combinations and because of input data volume (about 3,000,000 data elements).

5 IMPLEMENTATION, DATA, EXPERIMENTS, AND RESULTS

Our prototype was implemented in MATLAB. However, the Hurst function had to be reimplemented from C++ according to the version written by Kaplan (Kaplan, 2003). We decided to use it because of its performance. We also tried to use the Hurst function written by Weron (Weron, 2011) that was more accurate, but it was slower by grade and unusable for bunch of tests we needed.

For simulations, we used all daily close prices of all stocks (about 31 stocks and DAX value) from DAX index between 1995-2013 and all daily close prices of all stocks (about 100 stocks) from NASDAQ index between 1996-2014. Derived from these close prices, we used returns R_i (i.e., profits) of the last i days $(i = 1,...,10)$ as time series data elements. Exactly, it was more complex because the membership of a company in an index is not fixed forever. It depends on capitalization of companies. Some companies are growing and replace companies with decreasing capitalization. Our data are exactly described in (Skoula, 2017).

Altogether, we used 728,190 data elements for DAX and 2,349,000 data elements for NASDAQ. The processing of these data was very time consuming. We found out that the indicator MH is able to

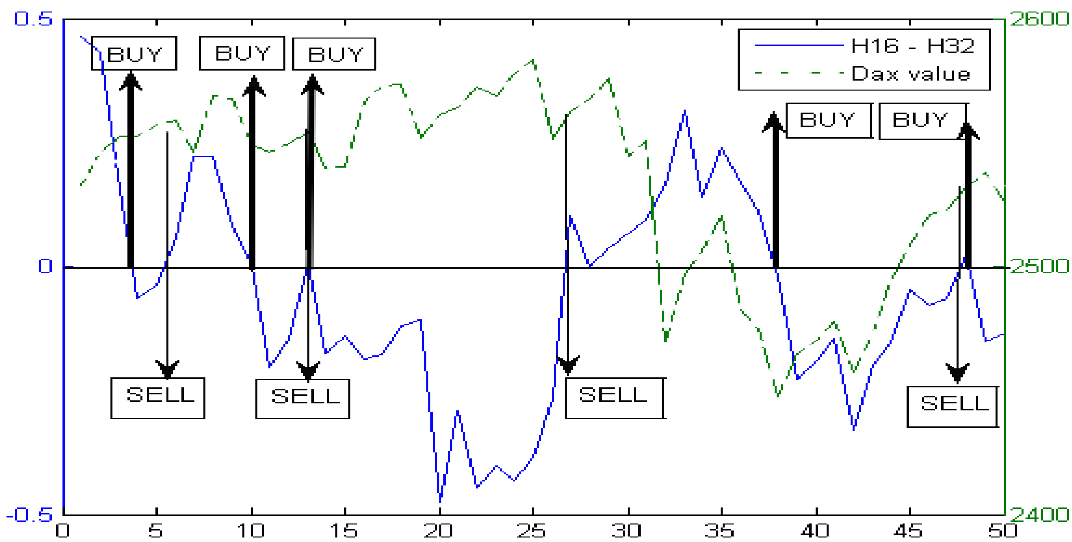

Figure 1: The non-linear MH indicator used for DAX.

produce about three to four time better results than Buy & Hold strategy in our experiments.

5.1 Hypothesis Testing

Since the differences between profits from the usage of MACD and MH were fluctuating, we used paired t-test statistics to test our hypothesis that MH brings more profit than MACD. We used the time series of profits generated by MH and the corresponding time series of profits generated by MACD for all stocks of DAX and for all stocks of NASDAQ as described in Section 5. We stated the following two hypotheses:

$H_0 : \mu_{MH} = \mu_{MACD}$

and

$H_A : \mu_{MH} > \mu_{MACD}$

The function used in MatLab was:

```
alpha = 0.01
test = t.test(macd, y=mh,
                alternative='less',
                paired=TRUE,
                conf.level=1-alpha)
```

The t.test procedure answered for DAX:

```
t = -6.9046, df = 27, p-value = 1.014e-07
alternative hypothesis:
true difference in means is less than 0
99 percent confidence interval:
-Inf -102.9352
sample estimates:
mean of the differences:
-160.3643
```

The t.test procedure answered for NASDAQ (data omitted here):

```
t = -8.8001, df = 87, p-value = 5.764e-14
alternative hypothesis:
true difference in means is less than 0
99 percent confidence interval:
-Inf -86.42111
sample estimates:
mean of the differences:
-118.2739
```

So, we reject the theory H_0 with possible error $\alpha = 1\%$ for both data collection (DAX and NASDAQ). The winning theory is H_A, i.e., MH has a greater income rate than MACD.

Similarly, we tested the hypothesis that the using of MH indicator brings more profit than the using of Buy & Hold strategy.

5.2 Practical Aspects

However, it must be said that this profit is purely theoretical.

First, there is the problem of stationarity explained in detail in Section 3.3, i.e., the patterns that brought profit in the past can bring loss in the future, because the stationarity of market time series is not guaranteed, and it is very probably that it is an oversimplification.

Second, we have shown that the using of the MH indicator generated more profit than the using of the MACD or Buy & Hold strategy, but it can also mean that the using of both indicators generates a loss, even though MH generates a smaller loss.

Third, there is the problem of transaction frequency and fees. Usually, at least for small investors, transactions on stock exchange are charged by fees and taxes. Unfortunately, the using of the MH indicator generates a large number of transactions.

We simulated a real investment of 2000 into NASDAQ for a start, and we used the fee of 1% for each transaction (BUY or SELL) because it corresponds with the reality. After the whole period (i.e.,1996-2014), we finished with 7819.8, and we spent 34497.3 on fees. To make a picture clear, using Buy & Hold on NASDAQ, we would finish with 16648.1, and we would pay only 187.7 for fees.

Tax rules are different in different countries, e.g., between $15\% - 25\%$, but it is evident that they reduce the profit, too. Usually, the tax has to be paid immediately after the transaction is finished. So, the reinvestment of profit is reduced.

However, the transaction fees depend on the stock exchange provider. Big investors can use the strategy of scalping that represents many thousands transactions in a day. Such investors are classified as market makers, and they are not charged by transactions fees. The more detailed explanation is out of the scope of our paper.

6 CONCLUSIONS

The goal of our investigation was to develop and test a new indicator MH for technical analysis based on chaos measure represented by Hurst exponent of the underlying time series of prices.

We found a construction described in Section 4.1, and we evaluated it in comparison to the strategy using MACD or Buy & Hold for data described in Section 5.

Using hypothesis testing, we proved our hypothesis that the new MH indicator developed in this work, i.e., our non-linear method described in Section 4.1, generates more profit compared to the MACD technical indicator and to the Buy & Hold investment strategy.

On DAX, MH was 4.5 times better than Buy & Hold and 7.2 times better than MACD. On NASDAQ, it was 2.9 times better than Buy & Hold and 16.8 times better than MACD.

In Subsection 5.2, we explained why the indicator MH cannot be used as a money generating machine.

However, we believe that complex, non-linear systems with problematic stationarity are an important research topic. More research has to be done to answer questions about filtering of BUY- and SELL-signals. In our future research, we will apply methods of genetic programming to improve it like in our previous work (Kroha and Friedrich, 2014).

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