The Calculation of Educational Indicators by Uncertain Gates

Guillaume Petiot
CERES, Catholic Institute of Toulouse, 31 rue de la Fonderie, Toulouse, France

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Abstract: Learning Management Systems allow us to retrieve a large scale of data about learners in order to better understand them and how they learn. Thus, it is possible to suggest educational differentiated approaches which take into account the students' specific needs. The knowledge about the behavior of learners can be extracted by datamining or can be provided by teachers. The available data is often imprecise and incomplete. The possibility theory provides a solution to these problems. The modeling of knowledge can be performed by a possibilistic network but it requires the definition of all Conditional Possibility distributions. This constitutes a limitation for complex knowledge modeling. Uncertain Gates allow, as Noisy Gates in the probability theory, the automatic calculation of Conditional Possibility Tables. The existing Uncertain MIN and Uncertain MAX connectors are not sufficient for applications which need a compromise between both connectors. Therefore we have developed new Uncertain Compromise connectors. In this paper, we will present an experimentation of educational indicator calculation for a decision support system using Uncertain Gates.

1 INTRODUCTION

The main objective of Educational Data Mining is to better understand students and how learners learn in order to highlight the pedagogy which contributes to learning (Huebner, 2013; Baker and Yacef, 2009; Bousbia et al., 2010). The Learning Management System Moodle is a useful tool for EDM and allows us to retrieve data for further analysis in order to analyze them and adapt pedagogy to learners. Also teachers have an expert knowledge of what contributes to success at the examination. They can provide indicators which highlight students with difficulties.

In this paper, we would like to perform an experimentation of indicator calculation. For the experimentation, we have chosen a course of Spreadsheet program because of the available resources in Moodle. The available information concerns attendance to courses, groups, graduation and the result of the students at the examination. Moodle provides information about participation, results of the quiz and resources consulted. The goal of this paper is to build educational indicators in a decision support system.

Several studies on our problem have been performed in the last years. They use a Bayesian Network, Neural Networks, Support Vector Machines with the main objective to detect the students with difficulties who risk to drop out or fail at the examination (Huebner, 2013).

One of the problems is that the modeling of knowledge is often imprecise and uncertain. The possibility theory introduced by (Zadeh, 1978) is a solution to this problem. Since knowledge can be represented by a Directional Acyclic Graph, it can be evaluated by possibilistic networks. The possibilistic networks (Benferhat et al., 1999; Benferhat and Smaoui, 2004) are adaptations of the Bayesian Network (Pearl, 1988; Neapolitan, 1990) to the possibility theory. The building of the Conditional Possibility Tables (CPTs) is often complex. Indeed, the number of parameters of the conditional possibility distribution grows exponentially depending proportionally on the number of variables. So it can be more easy to use logical gates between the variables, as in the Noisy Gates of the probability theory, in order to build automatically the CPT. Moreover, it is difficult to have a perfect model of knowledge. There are often unknown variables which contribute to the spurious behavior of the models. So the solution is to add a leaky variable to the models to represent the unknown knowledge.

This paper is structured in 4 main sections. Firstly, we will present possibilistic networks and Uncertain Gates, then we will propose a new Uncertain Gate connector, which performs a weighted average, and

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present the algorithm implementation. In section 3, we will describe our experimentation and the modeling of knowledge. Finally, in section 4, we will analyze the main results of our experimentation.

2 UNCERTAIN GATES

The Uncertain Gates are an analogy to Bayesian Noisy Gates in the possibility theory. The latter was presented for the first time in (Zadeh, 1978). This theory proposes to use a possibility distribution to represent imprecision and uncertainty. For example, let X be a variable and $\pi : R \to [0,1]$ a possibility distribution with R the domain of X. Therefore $\pi(x) = 0$ means that $X = x$ is not possible and $\pi(x) = 1$ means that the value $X = x$ is possible.

Authors in (Dubois and Prade, 1988) defined the possibility measure Π and the necessity measure N. The possibility measure is a function defined from the set of parts $P(X)$ of X in $[0,1]$:

$$\forall A \in P(X), \Pi(A) = \sup_{x \in A} \pi(x) \tag{1}$$

The dual necessity measure is a function on the set of parts of $P(X)$ of X, in $[0,1]$:

$$\forall A \in P(X), N(A) = 1 - \Pi(\neg A) \tag{2}$$

Let X and Y be two variables, if $x \in X$ and $y \in Y$, we can define a joint possibility distribution $\pi(x,y)$ on the Cartesian product $X \times Y$. One can define the marginal possibility $\pi_X(x)$:

$$\forall x \in X,\ \pi_X(x) = \sup_{y \in Y} \pi(x,y) \tag{3}$$

There are also conditional possibility and conditional independence in the possibility theory. As a reminder, if we have three variables X, Y, Z, we can say that X is independent of Y conditionally to Z ($X \perp Y \mid Z$), if and only if for all instances z of Z, the values of the instances x of X, never change for all instances y of Y:

$$\pi(x/y,z) = \pi(x/z) \tag{4}$$

The possibilistic networks (Benferhat et al., 1999; Borgelt et al., 2000; Caglioni et al., 2014) are like Bayesian networks in the sense that they are based on d-separation, conditional independence (Amor and Benferhat, 2005), and factoring property. The d-separation is a fundamental graphic property in causal reasoning. If there are three variables X, Y and Z and X is d-separated of Y by Z, then Z is blocking for all information between X and Y, and Z is the only known information in the graph. The factoring relation can be defined from joint possibility distribution $\Pi(V)$ for a DAG $G = (V,A)$ with V the variables and A the arcs. $\Pi(V)$ can be factorized toward the graph G:

$$\Pi(V) = \bigotimes_{X \in V} \Pi(X/Pa(X)) \tag{5}$$

With Pa the parents of the node X. The combination rule must be associative. In this study we have chosen the minimum for $\otimes$.

Let us now introduce the notion of Independence of Causal Influence as described in (Dìez and Drudzel, 2007; Heckerman and Breese, 1994). We can have a set of causal variables $X_1,\ldots,X_n$ which influence the result of another variable Y also called effect variable. So we can write the equation $Y = f(X_1,\ldots,X_n)$ where f is a deterministic function. One can introduce an intermediate variable Z_i for each X_i, and a leaky variable Z_l which represents the unknown knowledge in the model. In the probability theory we obtain the following graph:

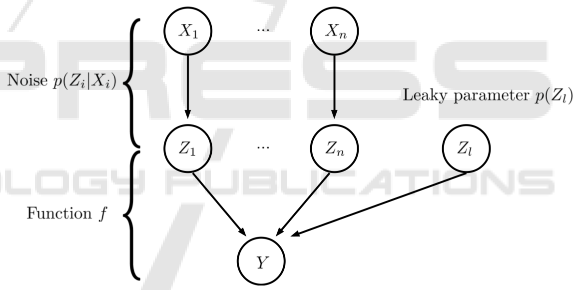


Figure 1: Causal graph.

ICI means that if the variables Z_i depend on the variables X_i and Y depend on the variables Z_i, then there is no causal interaction in the effects of the variables X_i on the variable Y. We can calculate $P(Y|X_1,\ldots,X_n)$ by marginalizing the variables Z_i (Dìez and Drudzel, 2007) as below:

$$P(Y|X_1,\ldots,X_n) = \sum_{Z_1,\ldots,Z_n} P(Y|Z_1,\ldots,Z_n) \times P(Z_1,\ldots,Z_n|X_1,\ldots,X_n)$$

$$P(Y|X_1,\ldots,X_n) = \sum_{Z_1,\ldots,Z_n} P(Y|Z_1,\ldots,Z_n) \times \prod_{i=1}^{n} P(Z_i|X_i)$$

$$\text{where } P(Y|Z_1,\ldots,Z_n) = \begin{cases} 1 & \text{if } Y = f(Z_1,\ldots,Z_n) \\ 0 & \text{else} \end{cases}$$

As a result for instanced variables we obtain:

$$P(y|x_1,\ldots,x_n) = \sum_{z_1,\ldots z_n : y = f(z_1,\ldots,z_n)} \prod_{i=1}^{n} P(z_i|x_i) \tag{6}$$

Authors in (Dìez and Drudzel, 2007) provide a description of the functions f. The most commonly used functions are the AND, OR, NOT, INV, XOR, MAX, MIN, MEAN, and linear combination. We can by analogy propose the same formula for the possibilistic model with ICI:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n:y=f(z_1,...,z_n)} \otimes_{i=1}^{n} \pi(z_i|x_i) \quad (7)$$

The $\otimes$ is the minimum. The CPT is obtained by calculation of the previous formula. For binary variables the possibility table between the variables X_i and Z_i is:

$\pi(Z_i\|X_i)$	x_i	$\neg x_i$
z_i	1	0
$\neg z_i$	κ_i	1

If we have three ordered levels of intensity such as low, medium and high, as in our application, we can encode the modality by an intensity level as in (Dubois et al., 2015). We can have 0 for low, 1 for medium and 2 for high. So we have:

$\pi(Z_i\|X_i)$	$x_i = 2$	$x_i = 1$	$x_i = 0$
$z_i = 2$	1	$\kappa_i^{2,1}$	0
$z_i = 1$	$\kappa_i^{1,2}$	1	0
$z_i = 0$	$\kappa_i^{0,2}$	$\kappa_i^{0,1}$	1

If we consider as in (Dubois et al., 2015) that a cause of weak intensity cannot produce a strong effect, then $\kappa_i^{2,1} = 0$. In our application, this parameter is greater than 0. Another constraint is that $\kappa_i^{1,2} \geq \kappa_i^{0,2}$. So we have 4 parameters per variable. If we add a leaky variable Z_l in the previous model, we obtain:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n,z_l:y=f(z_1,...,z_n,z_l)} \otimes_{i=1}^{n} \pi(z_i|x_i) \otimes \pi(z_l) \quad (8)$$

As the uncertain connectors AND, OR, MIN and MAX have already been described in (Dubois et al., 2015), we only give a reminder of these connectors below:

Table 1: Uncertain gates connectors.

Gates	$\pi(y\|x_1,...,x_n)$
AND	$\max_{z_1,...,z_n,z_l:y=(z_1\wedge...\wedge z_n)\vee z_l} \otimes_{i=1}^{n} \pi(z_i\|x_i) \otimes \pi(z_l)$
OR	$\max_{z_1,...,z_n,z_l:y=z_1\vee...\vee z_n\vee z_l} \otimes_{i=1}^{n} \pi(z_i\|x_i) \otimes \pi(z_l)$
MIN	$\max_{z_1,...,z_n,z_l:y=\max(\min(z_1,...,z_n),z_l)} \otimes_{i=1}^{n} \pi(z_i\|x_i) \otimes \pi(z_l)$
MAX	$\max_{z_1,...,z_n,z_l:y=\max(z_1,...,z_n,z_l)} \otimes_{i=1}^{n} \pi(z_i\|x_i) \otimes \pi(z_l)$

Authors in (Dubois et al., 2015) proposed an optimization of the calculation of Uncertain MIN and MAX connectors. The optimization consists in simplifying the equations of the Uncertain MAX:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n,z_l:y=\max(z_1,...,z_n,z_l)} \otimes_{i=1}^{n} \pi(z_i|x_i) \otimes \pi(z_l) \quad (9)$$

$$= \max \begin{cases} \max_{i=1}^{n} \pi(Z_i = y|x_i) \otimes \Pi(Z_l \leq y) \otimes (\otimes_{i\neq j} \Pi(Z_j \leq y|x_j)) \\ \pi(Z_l = y) \otimes (\otimes_{i=1}^{n} \Pi(Z_i \leq y|x_i)) \end{cases} \quad (10)$$

We can propose the same simplification for Uncertain MIN:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n,z_l:y=\max(\min(z_1,...,z_n),z_l)} \otimes_{i=1}^{n} \pi(z_i|x_i) \otimes \pi(z_l) \quad (11)$$

$$= \max \begin{cases} \max_{i=1}^{n} \pi(Z_i = y|x_i) \otimes \Pi(Z_l \leq y) \otimes (\otimes_{i\neq j} \Pi(Z_j \geq y|x_j)) \\ \pi(Z_l = y) \otimes (\max_{i=1}^{n} \Pi(Z_i \leq y|x_i)) \end{cases} \quad (12)$$

These connectors are useful in several applications but sometimes you need a different behavior which realizes a compromise between the conjunction and the disjunction of the intensities of the variables. Authors in (Zagorecki and Druzdzel, 2006) proposed an example of a mean Noisy Gates connector. In our study, we would like to propose a connector allowing to take into account the importance of the variables as in weighted average. To do this, we will use a linear combination (g) and a scale function (f_e). If the modalities of the variable define an ordered scale $E = \{0 < 1 < ... < m\}$, the function f of the equation $y = f(z_1,...,z_n)$ must return compatible values with the coding of the qualitative variable Y. To perform this constraint we have to use the scale function named f_e as in (van Gerven et al., 2006) which performs a threshold as below:

$$f_e(x) = \begin{cases} 0 & \text{if } x \leq \theta_0 \\ 1 & \text{if } \theta_0 < x \leq \theta_1 \\ \vdots & \vdots \\ m & \text{if } \theta_{m-1} < x \end{cases} \quad (13)$$

The graphic of this function is as follows:

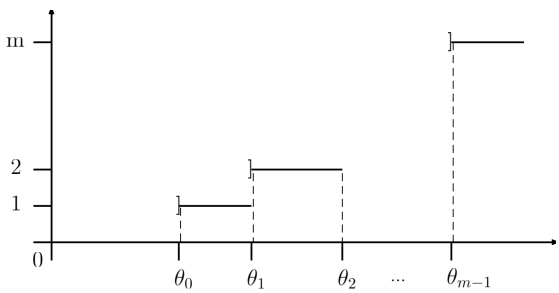


Figure 2: Scale function.

In the figure above, the coefficient θ_i defines the expected behavior of the threshold. Thus, if the values of θ_i are well defined $\theta_i = i + \frac{1}{2}$, then we perform a rounding to the nearest value.

The function f is now $f = f_e \circ g$ where g is a function. The possibility $\pi(y|x_1,...,x_n)$ is:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n: y=(f_e \circ g)(z_1,...,z_n)} \otimes_{i=1}^{n} \pi(z_i|x_i) \quad (14)$$

So we can now define a new connector to build CPTs with an intermediate behavior between MAX and MIN uncertain connectors. For that, the function g must be a linear combination : $g(z_1,...,z_n) = \omega_1 z_1 + ... + \omega_n z_n$. The scheme of the new connector is as follows:

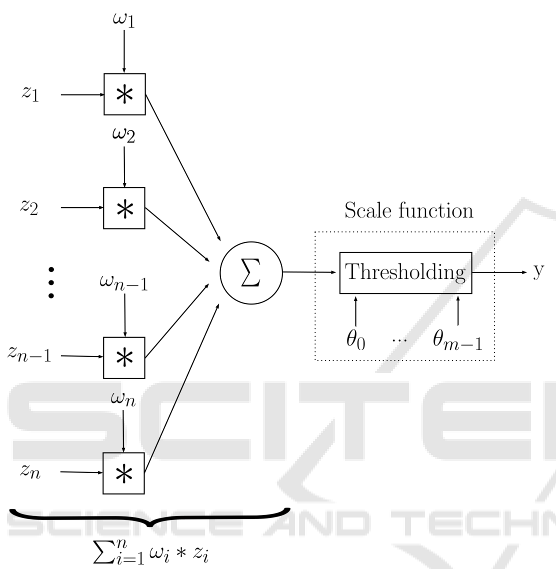


Figure 3: The compromise connector.

The function g has n parameters which are the weights of the linear combination. If all the weights are the same and equal to $\frac{1}{n}$, then we calculate the mean of the intensity of the variables and we return the value of Y closest to the mean. Of course the values θ_i must be $\theta_i = i + \frac{1}{2}$. If $\forall_{i \in [1,n]} \omega_i = 1$, then we calculate the sum of the causal variable intensities. We can associate for each value of Y a threshold for the sum of intensities. If the weights are different for all variables, then we can take into account the importance of each variable. We can perform in this case a weighted average. The constraint is that the sum of the weights must be equal to 1. We can also divide the result by $\sum_{j=1}^{n} \omega_j$. The general expression of the connector is:

$$\pi(y|x_1,...,x_n) = \max_{z_1,...,z_n: y=f_e\left(\frac{\omega_1 z_1 + ... + \omega_n z_n}{\sum_{j=1}^{n} \omega_j}\right)} \otimes_{i=1}^{n} \pi(z_i|x_i) \quad (15)$$

The algorithm of the Uncertain Weighted Average connector calculation is as follows:

Algorithm 1: Uncertain Weighted Average.

Input :
Y: CPT to calculate.
$X_1,...,X_n$: the n parents of Y.
ω: a weighted vector $(\omega_1,...,\omega_n)$.
$\kappa[i][Z][X]$: the coefficients $\pi(Z_i|X_i)$.
f_e: a threshold function.

Output:
The result is $\pi(Y|X_1,...,X_n)$.

forall $(y, x_1,...,x_n) \in Y \times X_1 \times ... \times X_n$ **do**
  $\pi(y|x_1,...,x_n) \longleftarrow 0$
  forall $(z_1,...,z_n) \in Z_1 \times ... \times Z_n$ **do**
    $Sum \longleftarrow 0$
    for $i = 1$ **to** n **do**
      $Sum \longleftarrow Sum + (z_i \times \frac{\omega_i}{\sum_{j=1}^{n} \omega_j})$
    $V[z_1,...,z_n] \longleftarrow f_e(Sum)$
  $K \longleftarrow \{(z_1,...,z_n) \in Z_1 \times ... \times Z_n | V[z_1,...,z_n] = y\}$
  $\gamma \longleftarrow 0$
  forall $(z_1,...,z_n) \in K$ **do**
    $\beta \longleftarrow \min_{i \in [1,n]} \kappa[i][z_i][x_i]$
    $\gamma \longleftarrow \max(\gamma, \beta)$
  $\pi(y|x_1,...,x_n) \longleftarrow \gamma$

3 KNOWLEDGE MODELING

In our experimentation we focused our interest on a Spreadsheet course at bachelor studies proposed in face-to-face enriched. This means that the course is face-to-face but with resources on Moodle. The knowledge about the indicators is provided by the teachers and extracted with datamining methods. Indeed, we can take datamining approach to extract knowledge or highlight some special behavior of the students. The result is a set of rules which describe what contributes to success at the examination. The pedagogical indicators are evaluated through quantitative data such as the use of Moodle resources, results of the quiz, attendance, etc. The questions of the quiz are categorized by skills. If there is missing data, we perform an imputation of the missing data by iterative PCA (Audigier et al., 2015). To represent the knowledge, we use a Directional Acyclic Graph (DAG). This graph encompasses both kinds of knowledge: the one extracted by a datamining approach and the one provided by teachers. The graph is as follows:

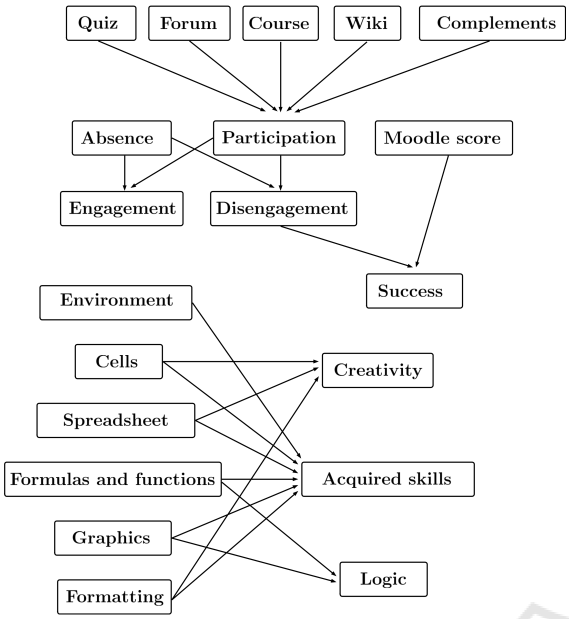


Figure 4: The modeling of knowledge by a DAG.

The qualitative variables are not binary. In fact, there are 3 ordered modalities (low, medium, high). As often happens in human descriptions, knowledge is uncertain and imprecise. So we can use a possibility distribution for each modality to tackle these problems. The possibilistic network can be used to evaluate the indicators but it requires the definition of all CPTs. This is time-consuming. For example, for the participation indicator which has 5 parent variables, we have $3^{5+1} = 729$ parameters. We cannot elicit all these parameters easily. The use of Uncertain Gates needs fewer parameters, so it is more adapted to the modeling of complex knowledge. Therefore, we can merge information on the consultation of the resources to build a participation indicator which takes into account the importance of the variables linked to the Moodle resources. The Uncertain Weighted Average connector can be used. The weights are provided by the teachers and shown below:

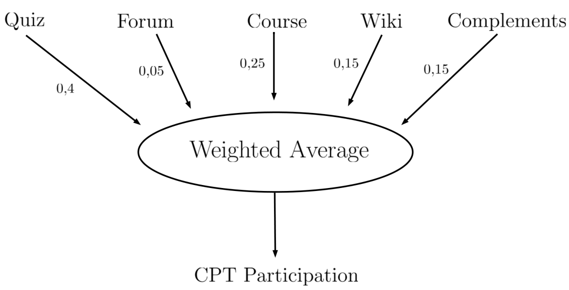


Figure 5: The weights of the Weighted Average.

Another useful indicator is the indicator of acquired skills. We propose to use for this indicator a connector Uncertain Sum $\sum_{f_e}$ which calculates the sum of the intensities of the causal variables and performs a threshold, as illustrated below:

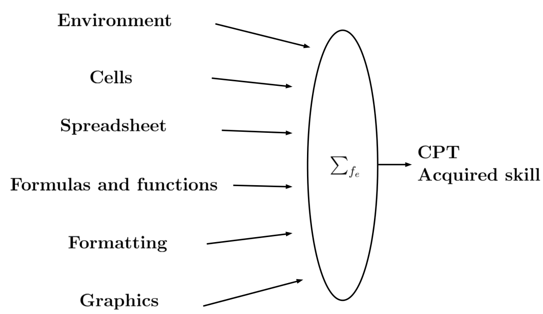


Figure 6: CPT computation of the indicator of acquired skills.

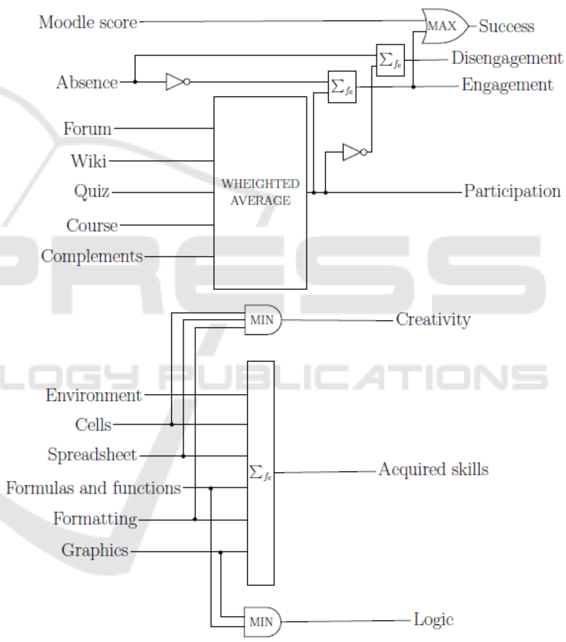


Figure 7: Knowledge modeling with Uncertain connectors.

We will use the Uncertain MIN connector for conjunctive behavior and the Uncertain MAX connector for disjunctive behavior. As a result, we obtain the model seen in figure 7.

Before the propagation of the new information, we have to build the CPT of all the Uncertain Gates. Then, we can apply the junction tree algorithm (Lauritzen and Spiegelhalter, 1988) of Bayesian networks adapted to possibilistic networks. The junction tree is composed of cliques and separators. The cliques are extracted by using the Kruskal algorithm (Kruskal, 1956) after the generation of the moral graph and the triangulated graph (Kjaerulff, 1994). Therefore, we can propagate the new information. The propagation algorithm can be resumed in three steps:

1. The initialization phase with the injection of evidence (new information).
2. The collection phase with the propagation of evidence from the leaf to the root.
3. The distribution phase with the propagation of evidence from the root to the leaf.

4 RESULTS

We can present the result of a more interesting indicator which is the success indicator. This indicator represents the synthesis of several variables and the prediction of the students' success at the examination. We can compare the modalities of the indicator and the successful result. We present below the results of this indicator with missing data:

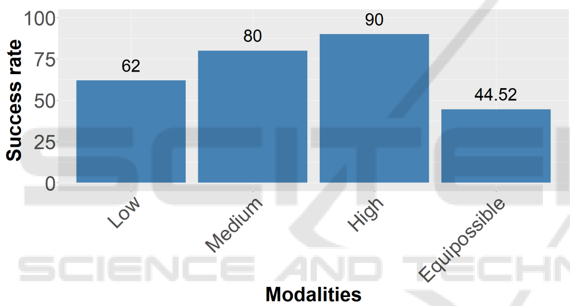


Figure 8: The result of the success indicator with missing data.

We can see in the graph a lot of equipossible results. This means that we cannot decide which modality is more possible. To reduce the number of equipossible variables, we propose to perform an imputation of missing data using an iterative PCA algorithm (Audigier et al., 2015). Therefore the results are now as follows:

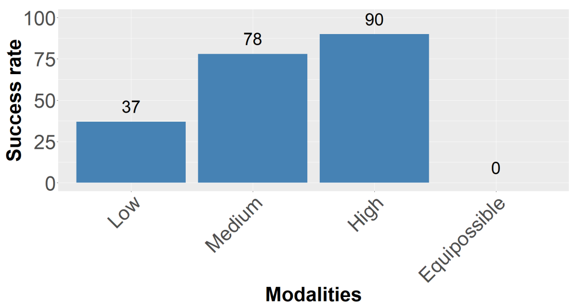


Figure 9: The result of the success indicator with missing data imputation.

The estimation of the missing data allowed us to solve the problem. We can now compare the results of the Uncertain Gates approach and the traditional possibilistic network:

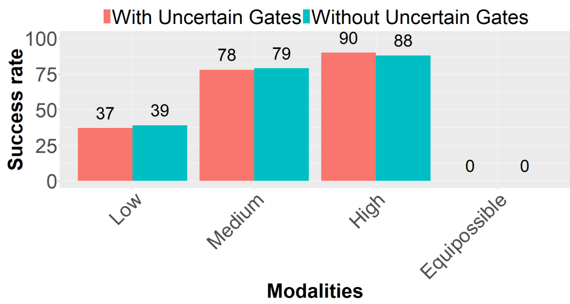


Figure 10: Comparison of the success indicator with Uncertain Gates and without Uncertain Gates.

The results are very close because modeling is well performed with Uncertain Gates. Nevertheless Uncertain Gates require fewer parameters than CPTs elicited by a human expert. We can compare the number of parameters for the two approaches:

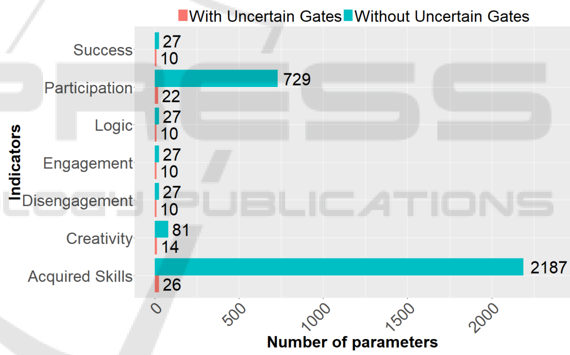


Figure 11: Number of parameters to define the CPTs with and without Uncertain Gates.

The number of parameters is very small for Uncertain Gates compared with the CPTs elicited by a human expert. We have also measured the performance of the Uncertain connectors. The computation time is better for Uncertain MAX and Uncertain MIN because of the mathematical simplification demonstrated in (Dubois et al., 2015). To realize the performance measure, we have developed small networks with the number of parents from 2 to 6 and we have generated the CPT. The results are the following:

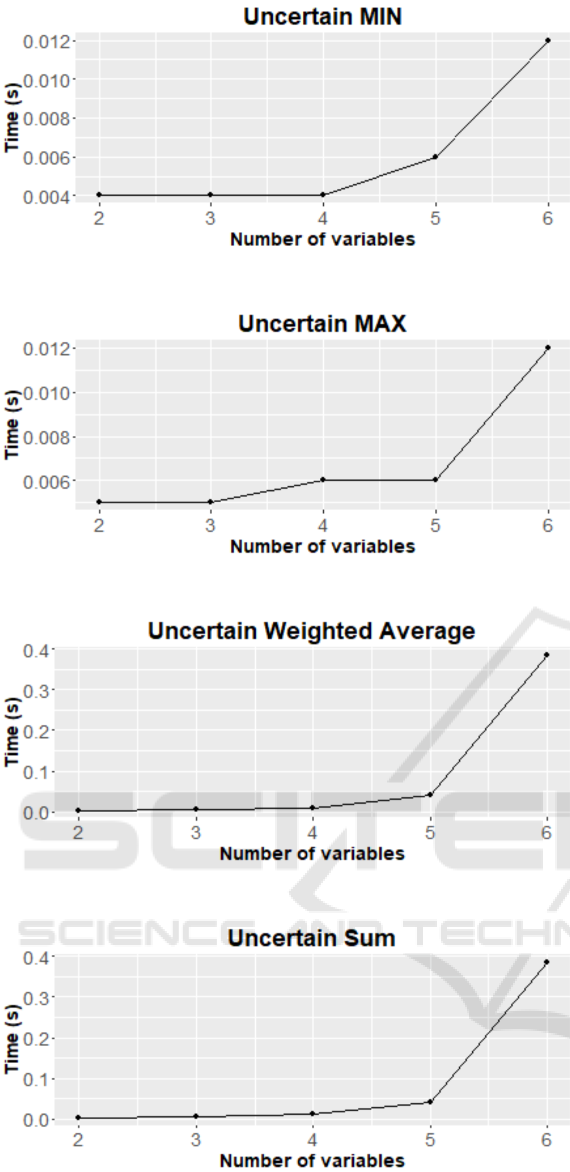
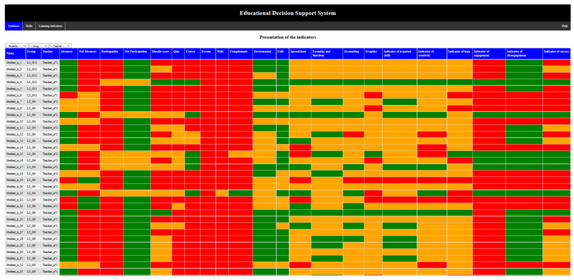


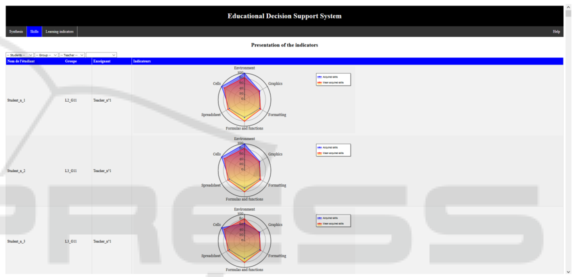
Figure 12: Computation time of Uncertain Gates (Intel I7 5500U processor, 8Go of RAM, OS 64 bits Windows 10).

In the previous figure, one can remark that the calculation time is growing exponentially when the number of variables is increased. This is a big limitation for complex systems. The solution to improve the performance of our system is to calculate only once the CPT and save the results.

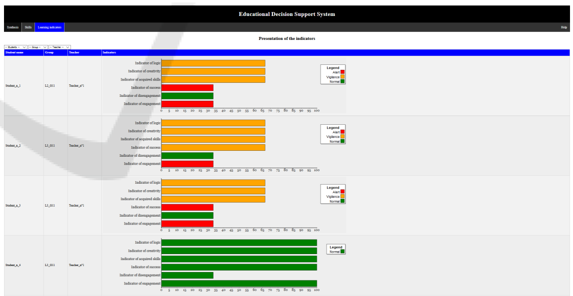
The system displays the indicators in a Pedagogical Information System. We have opted for a system easy to interpret and chosen to lose some information. Indeed, the possibilistic results are transformed to present only the more possible modality of indicators, the one with the highest necessity. The PIS system is generated automatically at the end of calculation as a PHP web site with three tabs. The first tab is the synthesis of all information and allows us to sort data for all columns. The second tab gathers all skills information. We can compare the results for the year group and the teacher. The last tab concerns all educational indicators and uses a color code to highlight the students with difficulties. The PIS system is presented below:



(a) First tab.



(b) Second tab.



(c) Third tab.

Figure 13: The tabs of the Pedagogical Information System.

5 CONCLUSION

The use of the Uncertain Gates has allowed complex knowledge modeling which would not have been performed if we had been obliged to define all conditional possibility values. The behavior of the Uncertain MIN and Uncertain MAX gates is not sufficient to model all the knowledge of our application. Indeed, we need for our application a compromise behavior between the two connectors. So we have proposed a new connector which allows a combination of the variables taking into account the importance of the variable as with weighted average. We used this connector to merge the information about the consultation of the Moodle resources to elaborate the participation indicator. We have also proposed a connector which takes into account the reinforcement of the intensity of the variables. There are good results which highlight the students with difficulties. But the main problem is the performance limitation of the approach for complex system modeling. We have to continue the improvement of the performance and to find new solutions to reduce the calculation time. We have to complement the toolbox of Uncertain connectors with new connectors which can be used for information fusion. A vast experimentation is also needed to evaluate the pedagogical impact of this approach on students and teachers but also on skill attainment. We also have to perform comparative studies with other approaches for further investigation.

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