Travel Time Modeling using Spatiotemporal Speed Variation and a Mixture of Linear Regressions

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Abstract: Real-time, accurate travel time prediction algorithms are needed for individual travelers, business sectors, and government agencies. They help commuters make better travel decisions, avert traffic congestion, help the environment by reducing carbon emissions, and improve traffic efficiency. Travel time prediction has begun to attract more attention with the rapid development of intelligent transportation systems (ITSs), and is considered one of the more important elements required for successful ITS subsystems deployment. However, the stochastic nature of travel time makes accurate prediction a difficult task. This paper proposes travel time modeling using a mixture of linear regressions. The proposed model consists of two normal components. The first component models the congested regime while the other models the free-flow regime. The means of the two components are modeled by two linear regression equations. The predictors used in the linear regression equation are selected out of the spatiotemporal speed matrix using a random forest machine-learning algorithm. The proposed model is tested using archived data from a 74.4-mile freeway stretch of I-66 eastbound connecting I-81 and Washington, D.C. The experimental results show the ability of the model to capture the stochastic nature of travel time and to predict travel time accurately.

1 INTRODUCTION

Minimizing drivers' travel times from their origins to their destinations is a major Intelligent Transportation Systems (ITSs) objective. However, it is also extremely challenging due to the dynamic nature of traffic flow, which is, in most cases, highly unpredictable. One straightforward strategy involves directing vehicles or guiding drivers to follow routes that avoid congested paths. A critical step for this route planning or guidance to be effective is the ability to accurately predict travel times of different alternative routes from source to destination.

In addition, travel time represents an important performance measure for traffic system evaluation. It is easily understood by drivers and operators of traffic management systems, and can be viewed as a simple summary of a traffic system's complex behavior. In order for an ITS to accurately predict the travel time, it must have the following capabilities, each of which comes with associated difficulties:

1. Sensing and acquiring the current state of the transportation network of interest where a number of data values need to be detected and collected, including traffic conditions and parameters at different parts of the network, whether some roads are currently congested, current weather conditions, time of day, whether there is an incident on any road in the network, etc. Gathering such data on every road and intersection with the quality that allows accurate forecasting of travel time between two points in the network may be fairly expensive.
2. Storing a long history of traffic parameters for the transportation network of interest to support future prediction of travel times. This historical dataset may be large and difficult to use and manage.

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3. Feeding the current state of the network along with its traffic history to some type of model that predicts travel time if a trip will start from some point and end in another in the network at some specific time. Designing such a model is challenging, as is finding a set of current or historical parameters with real prediction power. The most useful model may be road dependent, and even for a single road, it has been shown that different models may describe the traffic behavior more accurately at different traffic conditions. For instance, one model may be more useful when the road is congested, while another model may be more accurate when vehicles are flowing freely, etc.

In short, accurate traffic time prediction is challenging due to the high cost of sensing and collecting enough useful current and historical traffic data. Even when such data is available, it is still difficult to determine which type of model best describes the traffic behavior, and which traffic parameters should be fed to the model for the best predictions. Moreover, the best course of action may be to use two or more models and switch between them depending on current traffic conditions. This option adds a new challenge, as it is necessary to decide which model from the set of models will be used for some specific input data, or whether different models will be used for prediction with some weight applied to each output prediction to reach a final travel time prediction.

In this paper, a new method for travel time prediction is proposed. This method uses a mixture of linear regressions motivated by the fact that travel time distribution is not unimodal, since two modes or regimes of traffic can exist—one at congestion state, and the other at free-flow state. The proposed model was built and tested using probe data provided by INRIX and supplemented with traditional road sensor data as well as mobile devices and other sources. The dataset was collected from a freeway stretch of I-66 eastbound connecting I-81 and Washington, D.C. The traffic on this stretch is often extremely heavy, which makes travel time prediction more challenging, but also makes the data more valuable and helps create a more realistic model.

2 RELATED WORK

Various methods and algorithms have been proposed in the literature for travel time prediction. These methods can roughly be classified into two main categories: statistical-based data-driven methods and simulation-based methods. This section focuses on the statistical-based methods since the proposed solution in this paper falls under this class of methods, and because more research in the literature uses statistical methods.

Several researchers fit different regression models to predict travel time. A typical approach is to fit a multiple linear regression (MLR) model using explanatory variables representing instantaneous traffic state and historical traffic data, as, for example, (Rice and van Zwet, 2004, Zhang and Rice, 2003) . The model proposed in (1) was even able to use a single linear regression (SLR) to successfully provide acceptable travel time predictions. Some researchers developed hybrid methods where a regression model was used in conjunction with other advanced statistical methods. For example, (Kwon et al., 2000) used regression with statistical tree methods. Another approach (Chakroborty and Kikuchi, 2004) proposed an SLR model using bus travel time to predict automobile travel time.

Regression models are generally powerful in predicting travel time for short-term prediction, whereas long-term predictions are less accurate. Regression models are also reported to be more suitable for use in free-flow rather than congested traffic, and fail to accurately predict when incidents have occurred (Guin et al., 2013).

The idea of using a mixture models for different traffic regimes has also previously been explored (Guo et al., 2012). The model developed in this paper attempts to overcome the drawbacks of previous work that used mixture models of two or three components to model travel time reliability, which suffer from the following limitations:

1. The mean of each component is not modeled as a function of the available predictors.
2. The proportion variable is fixed at each time slot, which limits the model's flexibility.
3. Information provided given the time slot of the day is the probability of each component (fixed) and the 90th percentile.

Another class of statistical-based methods in literature uses time series models for travel time prediction, using, for example, auto-regressive prediction models (Oda, 1990, Iwasaki and Shirao, 1996, D'Angelo et al., 1999), multivariate time series models (Al-Deek et al., 1998), and the auto-regressive integrated moving average (ARIMA) technique (Williams and Hoel, 2003). Similar to regression models, time series models are more suitable for free-flow traffic than for congested traffic, may fail with unusual incidents, and are more accurate for short-term predictions (Guin et al., 2013).

Another common technique used for travel time prediction is the use of artificial neural networks. A feed-forward neural network is used in (A Cherrett et al., 1996) to predict journey time. Later, more advanced neural network techniques were used to model and predict travel time (Rilett and Park, 2001, Matsui, 1998, You and Kim, 2000, Guiyan and Ruoqi, 2003, Guiyan and Ruoqi, 2001, Wei et al., 2003, . Kisgyorgy and Rilett, 2002). Accurate predictions were achieved for most proposed models; for example, in (Kisgyörgy and Rilett, 2002) the prediction error was only 4%.

3 METHODS

In this section, we present a brief introduction of the powerful modeling techniques used in this paper. The random forest machine-learning algorithm (RF) is used to select a subset of important predictors for travel time modeling. Expectation-maximization (EM) is used to fit the mixture of linear regression models to the historical data. The techniques used are among a number of machine learning and statistical learning techniques representative of the wide variety of algorithms that can be used by transportation practitioners.

3.1 Variables (Predictors) Selection

The I-66 stretch of the freeway section used for this research consists of 64 segments. The dataset comprises the spatiotemporal speed matrices for every day in 2013. The default approach for modeling and predicting travel time was to take all the speeds within a window starting right before the departure time t_0 and covering L past time slots back to time $t_0 - L$. Setting L=30 minutes for example, the number of predictors will be 64*6 at 5 minutes time aggregation. In order to reduce the dimensions of the predictors' vector, RF is used to select the most important predictors for the travel time model. Steps to select the most important predictors are as follows (Breiman, 2001):

1. For each month, build an RF consisting of 100 trees and find the out-of-bag samples that are not used in the training for each tree.
2. Find the mean square error $MSE_{\text{out of bag}}$ of the RF using the out-of-bag samples.
3. Randomly permute the value for each predictor x_i among the out-of-bag samples and calculate the mean square error $MSE_{\text{out of bag}}^{\text{permuted } x_i}$ of the RF.
4. Finally, rank the predictors in descending order based on the $\frac{1}{12}\sum_{\text{month}=1}^{12}\left(MSE_{\text{out of bag}}^{\text{permuted } x_i} - MSE_{\text{out of bag}}\right)$ and choose the top m ranked predictors.

The higher the predictor's rank in step 4, the more important that predictor. The ranking result shows that, most of the important predictors are speeds of recent segments $(t_0 - 5)$. In addition to speed predictors chosen by RF, the historical average travel time at t_0 given the day of the week is added as a predictor.

3.2 Mixture of Linear Regressions

A mixture of linear regressions was studied carefully (De Veaux, 1989, Faria and Soromenho, 2009). It can be used to model travel time under different traffic regimes. The mixture of linear regression can be written as:

$$f(y|X) = \sum_{j=1}^{m} \frac{\lambda_j}{\sigma_j\sqrt{2\pi}} e^{-\frac{\left(y - x^T\beta_j\right)^2}{2\sigma_j^2}} \quad (1)$$

where y_i is the response corresponding to a vector p of predictors; x_i^T ,β_j is the vector of regression coefficients for the j^{th} component and λ_j is mixing probability of the j^{th} component.

The model parameters $\psi=\{\beta_1, \beta_2, \ldots, \beta_m, \sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2, \lambda_1, \lambda_2, \ldots, \lambda_m\}$ can be estimated by maximizing the log-likelihood of equation (1) given a set of response predictor pairs $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ using an EM algorithm. The EM algorithm iteratively finds the maximum likelihood estimates by alternating the E-step and M-step. Let $\psi^{(k)}$ be the parameters' estimates after the k^{th} iteration. In the E-step, the posterior probability of the i^{th} observation from component j is computed using equation (2).

$$w_{ij}^{(k+1)} = \frac{\lambda_j^{(k)}\phi_j\left(y_i|x_i, \psi^{(k)}\right)}{\sum_{j=1}^{m}\lambda_j^{(k)}\phi_j\left(y_i|x_i, \psi^{(k)}\right)} \quad (2)$$

where $\phi_j(y_i|x_i, \psi^{(k)})$ is the probability density function of the j^{th} component.

In the M-step, the new parameters' estimates $\psi^{(k+1)}$ that maximize the log-likelihood function in equation (1) are calculated using equations (3-5)

$$\lambda_j^{(k+1)} = \frac{\sum_{i=1}^{n} w_{ij}^{(k+1)}}{n} \tag{3}$$

$$\hat{\beta}_j^{(k+1)} = (X^T W_j X)^{-1} X^T W_j Y \tag{4}$$

where X is the predictors' matrix with n rows and $(p+1)$ columns, Y is the corresponding nx1 response vector, and W is a nxn diagonal matrix which has $w_{ij}^{(k+1)}$ on its diagonal.

$$\hat{\sigma}_j^{2(k+1)} = \frac{\sum_{i=1}^{n} w_{ij}^{(k+1)} (y_i - x_i^T \hat{\beta}_j^{(k+1)})^2}{\sum_{i=1}^{n} w_{ij}^{(k+1)}} \tag{5}$$

The E-step and M-step are alternated repeatedly until the change in the incomplete log-likelihood is arbitrarily small as shown in equation (6).

$$\left| \prod_{i=1}^{n} \sum_{j=1}^{m} \lambda_j^{(k+1)} \phi_j(y_i|x_i, \psi^{(k+1)}) - \prod_{i=1}^{n} \sum_{j=1}^{m} \lambda_j^{(k)} \phi_j(y_i|x_i, \psi^{(k)}) \right| < \xi \tag{6}$$

where ξ is a small number.

4 DATA DESCRIPTION

The freeway stretch of I-66 eastbound connecting I-81 and Washington, D.C. was selected as the test site for this study. High traffic volumes are usually observed during morning and afternoon peak hours on I-66 heading towards Washington, D.C., making it an excellent environment to test travel time models.

The traffic data was provided by INRIX, which mainly collects probe data by GPS-equipped vehicles, supplemented with traditional road sensor data, along with mobile devices and other sources (INRIX, 2012). The probe data covers 64 freeway segments with a total length of 74.4 miles. The average segment length is 1.16 miles, and the length of each segment is unevenly divided in the raw data from 0.1 to 8.22 miles. Figure 1 shows the study site and deployment of roadway segments. The raw data provides average speed for each roadway segment and was collected at 1-minute intervals.

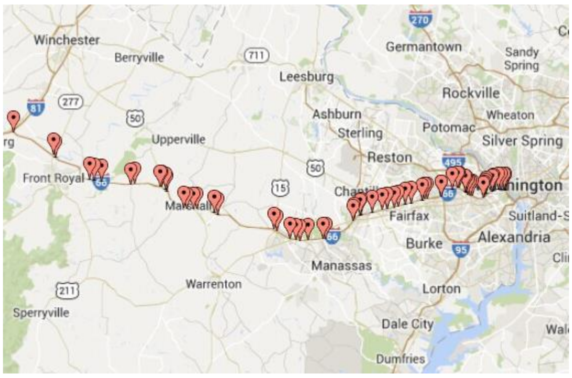

Figure 1: The study site on I-66 eastbound (source: Google Maps).

We sorted the raw data was the roadway direction according to each TMC station's geographic information (e.g., towards eastbound of I-66). Data was examined to check any overlapping or inconsistent stations along the route. Afterward, speed data was aggregated by time intervals (5 minutes in this study) to reduce noise and smooth measurement errors. This way, the raw data was aggregated to the form of the daily data matrix along spatial and temporal intervals. Data was missing in the developed data matrix, so data input methods were conducted to estimate the missing data using values of neighboring cells. Finally, the daily spatiotemporal traffic state matrix was generated to model travel time.

5 EXPERIMENTAL WORK

The experimental work is divided into three parts. The first part is travel time modeling using a mixture of two linear regressions with fixed proportions (λ_1, λ_2) and comparing the proposed model with the linear regression model. The second part is travel time modeling using a mixture of two linear regressions with a variable proportions function of the same predictors used in the linear regression equations. The last part explains how the proposed model can be used to convey travel time reliability to users.

5.1 Modeling Travel Time using a Mixture of Linear Regressions with Fixed Proportions

The purpose of this section is to experimentally prove that a model using a mixture of two linear regressions is better than the one component linear regression model. To show that, the proposed model is fitted to four months of the data then compared to the linear regression model. Three measures are used to compare the two models. The Mean Absolute Percentage Error (MAPE) and the Mean Absolute Error (MAE) are used to quantify the errors of both models with respect to the ground truth. MAPE is the average absolute percentage change between the predicted $\widehat{y_i^j}$ and the true values y_i^j. MAE is the absolute difference between the predicted and the true values.

$$\mathrm{MAPE} = \frac{100}{\mathrm{IxJ}} \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\left|y_i^j - \widehat{y_i^j}\right|}{y_i^j} \quad (7)$$

$$\mathrm{MAE} = \frac{1}{\mathrm{IxJ}} \sum_{j=1}^{J} \sum_{i=1}^{I} \left|y_i^j - \widehat{y_i^j}\right| \quad (8)$$

Here, J is the total number of days in the testing dataset; I is the total number of time intervals in a single day; and y and $\hat{y}$ denote the ground truth and the predicted value, respectively, of the travel time for the time interval on the day. The lower the value of these error measures, the better the model.

The other measure used for comparison is the histogram intersection. It measures how much the histogram of the predicted travel time, using a certain model, is similar to the histogram of ground truth travel time. The higher the value of the histogram intersection, the better the model,

$$\mathrm{H}(y) \cap \mathrm{H}(\hat{y}) = \frac{1}{Q} \sum_{q=1}^{Q} \min\left(\mathrm{H}_q(y), \mathrm{H}_q(\hat{y})\right) \quad (9)$$

where $\mathrm{H}(y)$ and $\mathrm{H}(\hat{y})$ are the histograms of ground truth travel time and the predicted travel time, respectively. Table 1 shows values for the MAE, MAPE and the histogram intersection for models using a different number of top ranked predictors. As shown in Table 1, for all models that are built using a different number of predictors, the models built using the proposed mixture of regressions are better than the linear regression models with smaller MAE, MAPE and greater histogram intersection.

5.2 Travel Time Prediction

Modeling travel time allows for travel time prediction, and conveying this information to travellers helps them make better decisions. If we are interested in providing travel time information, we usually convey the expected travel time as one value and sometimes also we provide upper and lower travel time bounds.

Table 1: Comparison between One and Two Components Models.

	MAE		MAPE		Similarity	
p	m=1	m=2	m=1	m=2	m=1	m=2
6	6.57	5.22	7.19	5.69	189	217
11	6.39	5.10	6.99	5.63	192	223
16	6.36	5.05	6.96	5.57	196	224
21	6.32	5.04	6.89	5.56	200	225
26	6.31	5.06	6.90	5.59	199	227
31	6.32	5.09	6.90	5.64	198	228
36	6.30	5.08	6.88	5.62	199	228
41	6.30	5.13	6.88	5.69	200	231
46	6.29	5.12	6.87	5.68	200	232
51	6.23	5.13	6.80	5.70	208	232
56	6.24	5.12	6.82	5.69	207	232
61	6.18	5.16	6.77	5.74	215	233
66	6.18	5.16	6.76	5.74	215	234
71	6.20	5.15	6.79	5.73	216	234
76	6.19	5.15	6.78	5.73	215	233
81	6.20	5.16	6.79	5.74	215	233
86	6.18	5.19	6.78	5.78	215	233
91	6.19	5.21	6.79	5.80	214	234
96	6.19	5.22	6.79	5.80	214	234

Conveying travel time as an interval makes more sense because it reflects the travel time uncertainty In this work, for a given unseen new vector of predictors, the mean of each component is determined and then travel time is predicted as a weighted average of the travel time means. The weights used are the λ_j's. The travel time interval for the unseen predictors' vector is calculated as the weighted average of the 95% confidence interval for each component. To evaluate the proposed model in travel time prediction, the two regression mixture models are tested using four unseen months. MAPE and MAE are used to measure expected travel time accuracy. To evaluate the travel time interval, a hitting rate measure is defined as the ratio of the number of ground truth travel times within the calculated interval to the total number of ground truth travel times. Table 2 shows the MAPE, MAE, hitting rate, and travel time width at different number of predictors. As shown in the table, the models built using 16 or more predictors have almost the same accuracy. The parameters' estimates for the model using a predictor vector of 16 dimensions are shown in Table 3. Figure 2 gives a better idea of how good the predicted travel times and intervals are.

Table 2: Travel Time Accuracy in Terms of MAPE and MAE, Travel Time Interval's Width and Hitting Rate.

p	MAPE	MAE	% Hitting rate	Interval width in minutes
6	7.97	7.64	81.9	26.4
11	7.74	7.43	81.1	24.7
16	7.70	7.39	81.0	24.5
21	7.70	7.37	80.9	24.4
26	7.69	7.36	80.8	24.3
31	7.68	7.35	80.6	24.1
36	7.67	7.34	80.6	24.1
41	7.67	7.34	80.4	23.9
46	7.68	7.34	80.5	23.8
51	7.68	7.33	80.4	23.8
56	7.70	7.34	80.5	23.8
61	7.69	7.32	80.4	23.7
66	7.69	7.32	80.4	23.7
71	7.69	7.32	80.4	23.6
76	7.70	7.32	80.3	23.6
81	7.71	7.33	80.4	23.5
86	7.74	7.34	80.2	23.5
91	7.73	7.33	80.1	23.4
96	7.73	7.33	80.1	23.4

Table 3: Parameters' Estimates for Mixture of Two Regressions*.

		1st component	2nd component
β	$intercept$	79.4354	96.5943
	$x_{29,t0-1}$	-0.0153	-0.0148
	$x_{2,t0-1}$	-0.0903	-0.0250
	$x_{28,t0-1}$	-0.0668	0.0061
	$x_{18,t0-1}$	-0.0912	-0.0519
	$x_{27,t0-1}$	0.0187	-0.0449
	$x_{40,t0-1}$	-0.2107	-0.1107
	$x_{25,t0-1}$	-0.0652	-0.0603
	$x_{14,t0-1}$	-0.0245	-0.0136
	$x_{29,t0-2}$	-0.0106	-0.0224
	$x_{1,t0-1}$	-0.0745	-0.0150
	$x_{39,t0-1}$	-0.0174	-0.0331
	$x_{21,t0-1}$	-0.0203	-0.0252
	$x_{24,t0-1}$	-0.0742	-0.0239
	$x_{19,t0-1}$	0.0075	-0.0078
	$x_{30,t0-1}$	-0.1269	-0.0558
	$x_{13,t0-1}$	0.6767	0.0834
σ^2		11.8066	1.7746
λ		0.4466	0.5534

*(In this table x_(seg#,time) is the speed at certain segment and time)

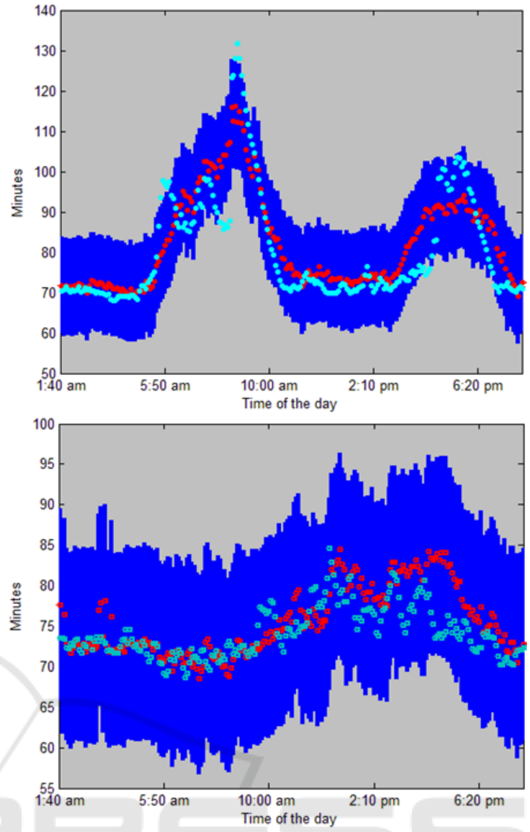

Figure 2: Travel time ground truth (red), predicted travel time (cyan), and travel time interval (blue).

5.3 Travel Time Reliability

Travel time reliability is another piece of information that can be conveyed to drivers using the travel time model. Using the proposed model, a traveler can be informed of probabilities for congestion and free-flow. Moreover, the expected and 90th percentile travel times for each regime can be provided. In order to get good estimates for the above quantities, the proportions should be functions of the predictors. Revisiting the EM algorithm, it estimates the posterior probabilities w_{ij} and model parameters ψ, and returns only ψ at convergence without using w_{ij}. As shown in equation (4), the returned λ_j is the average of the posterior probabilities w_{ij}. In the two components model, if w_{ij} is modeled using logistic regression at the convergence of the EM, this means that λ_j becomes a function of the predictors as well as the components' means. Values of w_{ij} are used, which result from fitting the model shown in Table 3 to build a logistic regression. This logistic regression models the probability of the predictor vector being drawn from component number two. Then, using simple algebraic manipulation, equation (10) is derived for λ_2 . The new model is the same model in Table 3 but with variable λ_2 and λ_1.

$$\lambda_2 = 1/1 - \exp([1\ x_{29,t0-1}\ x_{18,t0-1}\ x_{40,t0-1}\ x_{25,t0-1}\ x_{14,t0-1}\ x_{1,t0-1}\ x_{21,t0-1}\ x_{30,t0-1}\ x_{13,t0-1}] \begin{bmatrix} -1.8828 \\ -0.0249 \\ -0.0062 \\ -0.0114 \\ -0.0305 \\ -0.0155 \\ -0.0141 \\ -0.0131 \\ 0.0042 \\ 0.0896 \end{bmatrix}) \qquad (10)$$

This model is tested by calculating the mean, 90th percentile, and probabilities of congestion and free-flow for each predictor vector in each day of May 2013. Then a day is divided into four time intervals and the mean of the above quantities is calculated within each time interval given the day. The results shown in Table 4 are consistent with the travel time pattern observed in Table 3, where the probability of the congestion component increases at the congestion time of the day. Also, the model shows that the probability values of morning congestion during weekends are lower than on weekdays.

6 CONCLUSIONS

In this paper, we demonstrated the effectiveness of a travel time model based on a two component mixture of linear regressions. The proposed model captures the stochastic nature of travel time, and assigns one component for the free-flow regime and the other component for the congested regime. The means of the components are a function of the input predictors, which are chosen using a random forest algorithm. The proposed model can be used to predict the travel time and the upper and lower bounds for the travel time as well. Moreover, the proposed model can be used to provide travel time reliability information at any time on any day. The experimental results show the proposed algorithm's performance to be promising. The current model does not consider weather conditions, incidents, or work zones; however, this model can easily integrate these factors if a dataset including them is available.

Table 4: Testing the Model for Travel Time Reliability Using May 2013 Data.

		1:40 a.m.–4:55 a.m.		5:00 a.m.–10:00 a.m.		10:05 a.m.–3:00 p.m.		3:05 a.m.–7:00 p.m.	
		congested	free-flow	congested	free-flow	congested	free-flow	congested	free-flow
Tues	Mean (min)	87.07	73.07	127.66	85.51	94.88	75.53	120.96	81.44
	90th percentile (min)	71.94	70.80	112.53	83.23	79.75	73.25	105.83	79.17
	probability	0.0046	0.9954	0.8241	0.1759	0.1334	0.8666	0.8516	0.1484
Wed	Mean (min)	87.09	73.09	127.71	85.65	95.45	75.91	121.44	81.85
	90th percentile (min)	71.96	70.82	112.58	83.37	80.32	73.63	106.31	79.57
	probability	0.0051	0.9949	0.8114	0.1886	0.1488	0.8512	0.8684	0.1316
Thurs	Mean (min)	87.41	73.28	127.01	85.02	96.26	76.23	122.50	82.55
	90th percentile (min)	72.28	71.00	111.87	82.75	81.13	73.96	107.37	80.28
	probability	0.0050	0.9950	0.8035	0.1965	0.1581	0.8419	0.9057	0.0943
Fri	Mean (min)	87.24	73.12	119.99	81.62	95.53	75.95	122.95	82.96
	90th percentile (min)	72.11	70.84	104.86	79.34	80.40	73.67	107.82	80.69
	probability	0.0045	0.9955	0.7499	0.2501	0.1432	0.8568	0.9146	0.0854
Sat	Mean (min)	87.47	73.30	109.75	75.64	98.78	78.32	123.52	83.33
	90th percentile (min)	72.34	71.03	94.62	73.36	83.65	76.05	108.39	81.06
	probability	0.0048	0.9952	0.5760	0.4240	0.3129	0.6871	0.9588	0.0412
Sun	Mean (min)	86.84	73.07	110.00	76.00	99.38	78.38	120.64	81.81
	90th percentile (min)	71.71	70.80	94.87	73.73	84.25	76.11	105.51	79.54
	Probability	0.0038	0.9962	0.5908	0.4092	0.3237	0.6763	0.9145	0.0855
Mon	Mean (min)	87.19	73.18	122.06	82.46	93.21	74.46	117.66	79.51
	90th percentile (min)	72.06	70.90	106.93	80.18	78.08	72.19	102.53	77.24
	Probability	0.0046	0.9954	0.7524	0.2476	0.0738	0.9262	0.8304	0.1696

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