

An Efficient 2D Curve Matching Algorithm under Affine Transformations

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Keywords: 2D Curve Matching, Affine Transformation, Partially Occluded, Motion Estimation, Affine Arc Length, Pseudo-inverse Matrix.

Abstract: Most of the existing works on partially occluded shape recognition are suited for Euclidean transformations. As a result, the performance would be degraded in the affine and perspective transformation. This paper presents a new estimation and matching method of the 2D partially occluded recognition under affine transformation including translation, rotation, scaling, and shearing. The proposed algorithm is designed to estimate the motion between two open 2D shapes based on an affine curve matching algorithms (ACMA). This ACMA considers the normalized affine arc length coordinated to the 2D contour. Then, it will correlate them in order to minimize the L_2 distance according to any planar affine transformation by means of a method based upon a pseudo-inverse matrix. Experiments are carried on the Multiview Curve Dataset (MCD). They demonstrate that our algorithm outperforms other methods proposed in the state-of-the-art.

1 INTRODUCTION

The motion estimation and the matching of planar shapes that are subjected to certain deformation and viewing transformations is one of the most important goals in computer vision and pattern recognition, done through different applications such as robotic vision, Medical Image Registration (Bronstein et al., 2006), 3D reconstruction, Optical Character Recognition (Belongie et al., 2002), Object Classification (Adamek and O'Connor, 2004) (Alajlan et al., 2008) (Baseski et al., 2009), and content-based image retrieval (Bronstein et al., 2008). However, despite the progress of the research, remains a challenging task that makes shape recognition more complicated. It is presented by two critical factors: (a) images taken from different viewpoints of the same object suffer from perspective distortions and (b) the partially occluded shapes sometimes make the recognition problem more challenging (Turney et al., 1985). So the matching methods should have the ability to handle the different cases.

For example, the silhouette tracking application which records the movement of objects or people, consist of matching curves extracted from two successive images at two different instants which would lead to many problems due to several factors such as local deformations, articulations, missed and extrane-

ous contour portions owing to errors in shape extraction. Under these conditions, it is known that a perspective transformation between two images of an object can be approximated by a two-dimensional affine transformation (Forsyth et al., 1991) when the object is far from the camera-since the slight distortion that may result from the more general projection-can be regarded as part of a deformation. Therefore, local deformations have been treated in the literature by allowing some leeway in the matching of curve points via methods like Chamfer and Hausdorff distance. Also, local geometric corrections of affine transformation have been applied to handle more severe distortions and articulations. However, the issue is to specify which portions of the shape should be used for the geometric corrections, although some methods have been tried to solve this problem, presented in the next section.

Towards the solution of this challenging problem, our contribution aims to recognizing and curve matching of partially occluded 2D shape under affine transformations. The ACMA algorithm is applied to estimate the motion of two contours and matching them. First, a curve re-parameterization is defined, inspired by the expression of the normalized affine arc length (Spivak, 1981),(Ghorbel, 1998). Subsequently, sampling this part of curves at constant equivalence lengths which is represented by a sufficiently large

set of points that makes the number of equations higher than the unknowns. Finally, an affine part-to-part curve matching is obtained by the computation of the pseudo-inverse matrix which makes it possible to minimize the L_2 distance. This algorithm ACMA has the ability to handle object recognition under affine distortions, partial occlusions and outperforms other methods in terms of registration and recognition accuracy.

The remainder of the paper is organized as follows: section 2 introduce the related work to our approach. Then the detailed descriptions of ACMA and the new curve matching algorithm will be presented in the next section where we will briefly recall the affine arc length reparameterization method and calculate the pseudo-inverse matrix. Section 4 investigates the effectiveness of the proposed approach through experiments and analyses. Finally, the last section gives the conclusion.

2 RELATED WORK

In this section, we focus on work that serves to place this paper relative to the state of the art. Therefore, various affine invariant shape matching methods have been developed (Latecki et al., 2000), (Ma-weheb et al., 2016), (Chaieb and Ghorbel, 2008). They are able to address difficult problems like matching under noise condition, affine transformations and so on. In this context, the most well-known-researched shape description and shape matching methods include affine invariant Fourier descriptors method (Arbter et al., 1990), (Osowski et al., 2002), (Chaker et al., 2008), affine curvature scale space (ACSS) method (Mokhtarian and Abbasi, 2001), independent component analysis (ICA) method (Huang et al., 2005), curvature tree method (Alajlan et al., 2008), shape contexts (SCs) methods (Mori et al., 2005), (Ling and Jacobs, 2007), moments invariants methods (Huang and Cohen, 1996), (Zhao and Chen, 1997), symbolic representation method (Daliri and Torre, 2008) and so on. However, they treat only the closed-to-closed shape matching and assuming that the whole shape is always visible in images. On the other side, it is possible that the shape to be recognized is only partially visible in real applications, which makes the recognition problem, far more difficult than that of closed shapes.

Only some approaches of shape matching under partially occluded 2D shapes have been suggested. However, the most of them work only for shapes up to a similarity transform. The work presented in (Demirci, 2010) proposes a new indexing structure under

partial matching. Shan (Shan et al., 2006) proposes a method to present model objects using histograms and then matches the histogram between model and object to be recognized. Their method can match partial occluded objects. Orrite (Orrite and Herrero, 2004) estimates projective transform using alignment approach and extractes the invariant points bitangents, this method is able to deal with partial occluded and perspective transform. However it requires a complete searching match so that it is time consuming. Zhang (Zhang et al., 2015) presents a method dealing with recognition of partially occluded and affine distortion objects. Their method was designed for objects with planar polygon shapes, but many objects cannot be approximated by polygons.

In order to handle local affine changes, Gopalan et al. (Gopalan et al., 2010) proposed a shape-decomposition technique that divides a shape into convex parts using Normalized Cuts. These parts were then individually affines normalized and combined into a single shape that was matched using the Inner Distance Shape Context (IDSC). As a result, this method is able to capture more deformations of local portions, such as a 3D part articulation that may be modeled by a 2D affine transformation of its projection. It nevertheless assumes an a-priori shape decomposition from a single shape that may be inconsistent in the presence of occlusions or noise in shape extraction. Furthermore, the matching is still global and hence we will be unable to handle partial occlusions of the shapes. Also, Mai et al. (Mai et al., 2010) proposed a method for partial matching and affine distortions shapes, where the shape is described by a sequence of ordered affine-invariant segments based on the properties of curvature scale space (CSS) shape descriptor. Then Smith-Waterman algorithm is applied to match these sequences. This idea is developed by Huijing et al. (Fu et al., 2013) where an affine-invariant curve descriptor (AICD) based on a new-defined affine-invariant signature and its unsigned sum is proposed to represent the local shape of a curve with high distinctiveness. The comparison of our method to these methods will be highlighted in Section 4.

3 AFFINE MOTION ESTIMATION AND CURVE MATCHING

In this section, we will present a contour matching based on the affine curves matching algorithm (ACMA), Fig.1, we will illustrate the main steps performed to obtain the proposed algorithm. First, an affine arc-length re-parametrization is performed to meet invari-

ant parametrization. Then, the L_2 distance is minimized by the calculation of pseudo-inverse matrix to estimate the translation vector b and the linear transformation A . Finally, Affine curve matching algorithm is obtained.

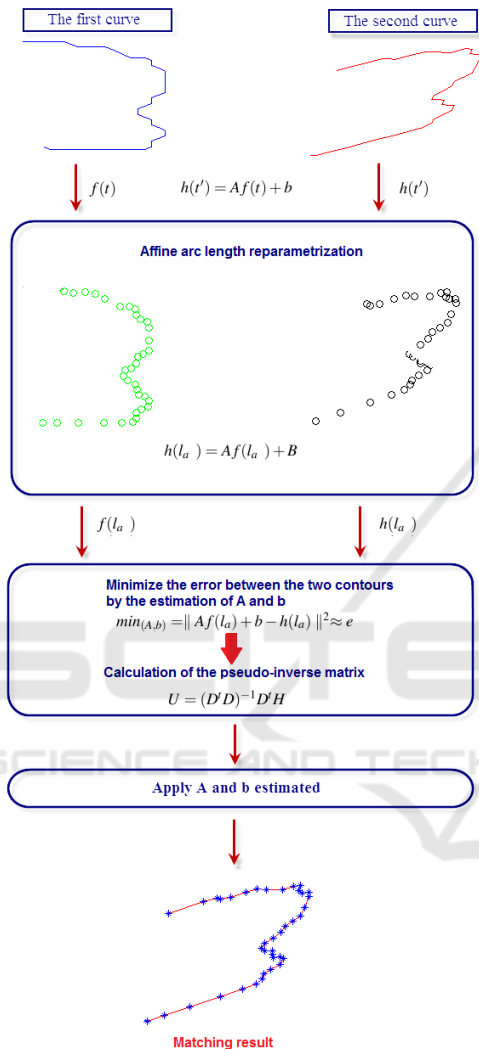


Figure 1: Block diagram of ACMA algorithm.

3.1 Affine Arc-Length Reparametrization

We focus on planar shapes represented by an open 2D continuous curves Γ_1 and Γ_2 which we can obtain one of them from other by a planar affine transformation. Lets consider $f(t)$ and $h(t')$ two respective parametrizations of curves Γ_1 and Γ_2 where their relation is defined by:

$$h(t') = Af(t) + b$$

with b is a translation vector and A is a linear transformation.

It is obvious that a given curve can be represented with various parameterizations. So, we can't compare different views of a planar contour and assume that the parameterizations are the same. To avoid this problem we must ensure that the parameterization is independent of transformations.

For this aim, we need to normalize the number of sampled points of the curves. The underlying idea is to do an affine re-parameterization of these curves by applying an affine arc length function $L(t)$ defined by:

$$L(t) = \frac{1}{l_a} \int_0^t \sqrt[3]{|\det(f'(u), f''(u))|} du \quad (1)$$

Where the total affine arc length L_a of the considered curve presented by:

$$L_a = \int_0^T \sqrt[3]{|\det(f'(u), f''(u))|} du \quad (2)$$

With f' and f'' denote, respectively, first and second derivative of f , while \det represents the determinant operator.

3.2 Calculation of the Pseudo-inverse Matrix

After re-parametrization by the affine arc length, the estimate of the apparent motion is equivalent to extracting the parameters of A and the translation vector b .

$$\begin{cases} h(l_{a1}) = Af(l_{a1}) + B \\ h(l_{a2}) = Af(l_{a2}) + B \\ \dots \\ h(l_{aN}) = f(l_{aN}) + B \end{cases}$$

with $f(l_a)$ and $h(l_a)$ are the reparametrization, respectively, of two contours $f(t)$ and $h(t')$. Our goal is to minimize the error between the two contours by the estimation of A and b which is defined by:

$$\min_{(A,b)} = ||Af(l_a) + b - h(l_a)||^2 \approx e$$

Explaining this system of $2N$ equations and 6 unknowns we obtain the following set of systems:

$$\begin{cases} h^x(l_{a1}) = f^x(l_{a1})a_{11} + f^y(l_{a1})a_{12} + B^x \\ h^y(l_{a1}) = f^x(l_{a1})a_{21} + f^y(l_{a1})a_{22} + B^y \\ \dots \\ h^x(l_{aN}) = f^x(l_{aN})a_{11} + f^y(l_{aN})a_{12} + B^x \\ h^y(l_{aN}) = f^x(l_{aN})a_{21} + f^y(l_{aN})a_{22} + B^y \end{cases}$$

This system can be written in matrix notation:

$$H = DU$$

with $U = [a_{11}a_{12}a_{21}a_{22}B^xB^y]^t, H = [h_1^x h_1^y h_2^x h_2^y \dots h_N^x h_N^y]$ and

$$D = \begin{pmatrix} f^x(l_{a1}) & f^y(l_{a1}) & 0 & 0 & 1 & 0 \\ 0 & 0 & f^x(l_{a1}) & f^y(l_{a1}) & 0 & 1 \\ f^x(l_{a2}) & f^y(l_{a2}) & 0 & 0 & 1 & 0 \\ 0 & 0 & f^x(l_{a2}) & f^y(l_{a2}) & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f^x(l_{aN}) & f^y(l_{aN}) & 0 & 0 & 1 & 0 \\ 0 & 0 & f^x(l_{aN}) & f^y(l_{aN}) & 0 & 1 \end{pmatrix} \quad (3)$$

The idea of the method of least squares is to solve the overdetermined system of linear equations when the numbers of equations are more than unknowns. So, the resolution of this rectangular system can be done by minimizing the error via inverting the system by using pseudo-inverse of the matrix D .

$$U = (D^t D)^{-1} D^t H$$

The instability of the reconstruction of the movement sometimes arises from the poor conditioning of the normal matrix $(D^t D)$. There are stabilization methods to reduce the effect of poor conditioning (when the conditioning value of the inverted matrix becomes high). We suggest in this case:

- To use the classical method which is obtained by means of multiplication by appropriately chosen diagonal matrices.
- To realize the best choice of the set pairs of points in correspondence by reducing to the best conditioning.

The matrix to be inverted is a normal matrix whose expression is:

$$D^t D = N^6 \begin{pmatrix} \bar{X}^2 & \bar{X}\bar{Y} & 0 & 0 & \bar{X} & 0 \\ \bar{X}\bar{Y} & \bar{Y}^2 & 0 & 0 & \bar{Y} & 0 \\ 0 & 0 & \bar{X}^2 & \bar{X}\bar{Y} & 0 & \bar{X} \\ 0 & 0 & \bar{X}\bar{Y} & \bar{Y}^2 & 0 & \bar{Y} \\ \bar{X} & \bar{Y} & 0 & 0 & 1 & 0 \\ 0 & 0 & \bar{X} & \bar{Y} & 0 & 1 \end{pmatrix} \quad (4)$$

and

$$\begin{aligned} \bar{X} &= \frac{1}{N} \sum_{i=1}^N (f^x(l_i)), \bar{Y} = \frac{1}{N} \sum_{i=1}^N (f^y(l_i)) \\ \bar{X}^2 &= \frac{1}{N} \sum_{i=1}^N (f^x(l_i))^2, \bar{Y}^2 = \frac{1}{N} \sum_{i=1}^N (f^y(l_i))^2 \\ \bar{X}\bar{Y} &= \frac{1}{N} \sum_{i=1}^N (f^x(l_i)f^y(l_i)) \end{aligned}$$

For mathematical proof the reader can be referred to (Ghorbel, 2013).

3.3 ACMA Algorithm

the procedure of matching the apparent affine partially occluded curves can be described by the following algorithm:

Algorithm

- Step1: take two contours of partially affine shape.
 - Step2: re-parametrize the two contours by the normalized affine arc length f^* and h^* .
 - Step3: sample at constant equivalence lengths in N points.
 - Step4: calculate $\bar{X}^2, \bar{Y}^2, \bar{X}\bar{Y}, \bar{X}, \bar{Y}$.
 - Step5: write the matrices $D, D^t D$ and H .
 - Step6: reverse $D^t D$.
 - Step7: calculate U by performing $(D^t D)^{-1} D^t H$.
 - Step8: reconstruct \hat{A} and \hat{B} from U .
 - Step9: apply \hat{A} and \hat{B} to f^* to obtain $(\hat{A}f^* + \hat{B})$.
 - Step10: superimpose $\hat{A}f^* + \hat{B}$ a h^* by the maximization of the correlation.
 - Step11: calculate the distance L^2 between $\hat{A}f^* + \hat{B}$ and h^* .
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4 EXPERIMENTS

In this section, we test our algorithm for shape matching and estimation where shapes are represented only by contours for the task of shapes recognition and retrieval. Our experimentations were conducted on the Multiview Curve Dataset (MCD) (Zuliani et al., 2004) which is composed of 40 shape classes taken from MPEG-7 database. Each class contains 14 curve samples that correspond to different perspective distortions of the original curve. Samples of shapes from MCD databases are shown in figure 2.



Figure 2: Different shape images from the MCD dataset, two images from each class.

In the initial MCD database, all shapes are presented by closed curves. So, to make it open and partially visible, we remove a few parts of the contour. In our experiments, we make three types of test to improve

the performance of our algorithm in matching: (a) Whole-to-whole matching(Fig.3), (b) whole-to-part (Fig.4) and (c) Part-to-part matching (Fig.5)

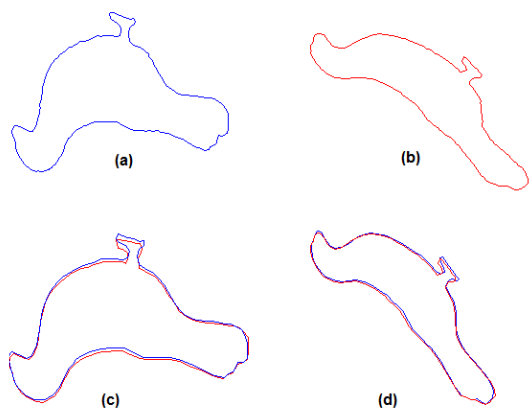


Figure 3: Bird pair: (a) and (b) are the initial curves; (c) and (d) show the original shape overlaid with the matched shape.

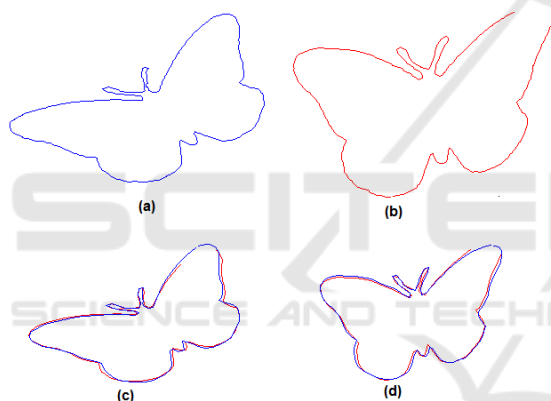


Figure 4: Butterfly pair:(a) and (b) are the initial curves; (c) and (d) show the original shape overlaid with the matched shape.

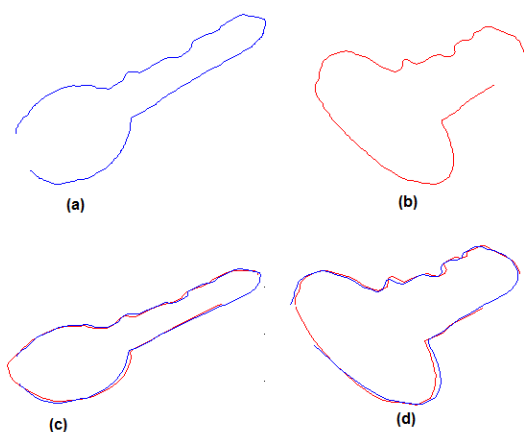


Figure 5: Key pair: (a) and (b) are the initial curves; (c) and (d) show the original shape overlaid with the matched shape.

So from this result we can conclude that our algorithm works well for all the three cases and it is robust to both partial occlusions and affine transformation.

4.1 Alignment Error Calculation

The shape registration is one of the important applications to evaluate the robustness of our algorithm under partial occlusion, where the estimated affine transformation align the two different curves of the same shape. So, we calculate the percentage of non-overlapping areas to obtain the alignment error between the common part of these two 2D open curves. Then, we compare our result with different methods presented in the literature. Our average alignment error is 8.95 % which is smaller than 12.13 % , 13.12 % and 49.41 % , respectively, the average error of the reference approach (Fu et al., 2013), (Mai et al., 2010) and (Pettrakis et al., 2002).

4.2 Image Database Retrieval

Another significant application to test our algorithm is the shape retrieval. Several techniques for shape database retrieval exist in the literature, among which FD is one of the most well-known descriptors and a state-of-the-art algorithm for affine-invariant shape retrieval . Therefore, we select two Affine-Invariant Fourier Descriptors (Chaker 's FD (Chaker et al., 2007) and Arber's FD (Arbter et al., 1990)) as reference methods. Besides, we select another three reference methods for shape retrieval: wavelet-based method (El Rube et al., 2006) , ICA-based method (Huang et al., 2005) and Mai 's method (Mai et al., 2010). Table 1 compares the retrieval average rates for the first 10 shapes (apple, bell, bone, bird, butterfly, bottle, bat, brick, camel and insect) of the MCD dataset using our suggest with the five reference methods. The different methods results for this dataset are collected from the respective papers. In terms of the average rates performance, our approach performs reasonably well as compared to many other techniques. Figure 6 , shows the retrieval results of 10 random queries from MCD databases based on our algorithm.

Table 1: Retrieval results on the entire MCD dataset.

Methods	Average
Arber (Arbter et al., 1990)	41 %
Huang (Huang et al., 2005)	71 %
Chaker (Chaker et al., 2007)	76 %
Rube (El Rube et al., 2006)	79 %
Mai (Mai et al., 2010)	89 %
Our algorithm	94 %



Figure 6: 10 random retrieval results from MCD database.

5 CONCLUSIONS

This paper presents a general affine motion estimation algorithm based on Affine Curve Matching Algorithm (ACMA). The re-parameterization of the contours based on the affine arc length is indispensable when the movement is assumed affines. Under this hypothesis, we recover the affine parameters by the computation of the pseudo-inverse matrix which minimizes the error. Our experiments indicate that our algorithm works well on the MCD database compared to many existing techniques, particularly in the case of partial occlusions that might arise in many situations. While the results on this dataset are interesting, but there is no guarantee that the same ordering of the methods would be obtained with other datasets or other methods. So, in the future, we intend to compare our method with other approaches and other datasets in terms of both performances under perspective distortion and complexity.

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