# Monte Carlo Simulation of Non-stationary Air Temperature Time-Series

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Abstract: Two numerical stochastic models of air temperature time-series are considered in this paper. The first model is constructed under the assumption that time-series are nonstationary. In the second model air temperature time-series are considered as a periodically correlated random processes. Data from real observations on weather stations was used for estimation of models' parameters. On the basis of simulated trajectories, some statistical properties of rare meteorological events, like sharp temperature drops or long-term temperature decreases in summer, are studied.

## 1 INTRODUCTION

The study of statistical properties of atmospheric processes involving adverse weather conditions (for example, long-term heavy precipitation, dry hot wind, unfavourable combination of low temperature and high relative humidity, etc.) is of great scientific and practical importance. Results of this study are crucial for solution of some problems in agroclimatology, planning of heating and conditioning systems and in many other applied areas (see, for example, Pall et al., 2013; Araya and Kisekka, 2017; Khomutskiy, 2017). Unfortunately, there are extremely few real observation data for obtaining stable statistical characteristics of rare / extreme weather events. Moreover, the behaviour of their characteristics is influenced by climatic changes, and hence it is not always possible to obtain reliable estimates only from observation data. In this regard, in recent decades a lot of scientific groups all over the world work at development of so-called "stochastic weather generators" (or short "weather generators"). At its core, " weather generators" are software packages that allow numerically simulate long sequences of random numbers having statistical properties, repeating the basic properties of real meteorological series. Using the Monte Carlo method, both the properties of specific meteorological processes and their complexes are studied (see, for example, Kleiber et al., 2013; Ailloit et al., 2015; Semenov et al., 1998, Kargapolova, 2017). Depending on the problem being solved, time-series of meteorological elements of different time scales are simulated (with hours, days, decades, etc. as a time-step). The type of simulated random processes (stationary or non-stationary, Gaussian or non-Gaussian, etc.) is determined by the properties of real meteorological processes and by the selected time step.

In this paper two numerical stochastic models of air temperature non-Gaussian time-series are considered. The first model is constructed under the assumption that time-series are nonstationary. In the second model air temperature time-series are considered as a periodically correlated random process. Both models let to simulate air temperature time-series with 3 h. time-step, taking into account daily oscillation of a real process. Parameters of both models were estimated on the basis of data from long-term real observations. On the basis of simulated trajectories, some statistical properties of rare meteorological events, like sharp temperature drops or long-term temperature decreases in summer, are studied.



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## 2 REAL DATA

In this paper a problem connected with the study of some statistical characteristics of rare and extreme behavior of air temperature is considered. In order to solve this problem, one has to construct a numerical stochastic model of the air temperature time-series based on real data collected at weather stations. To define models' parameters data collected 8 times per day (i.e. every 3 hours) during 23 years from 1993 to 2015 were used. For the sake of convenience, month-long time-series of air temperature that start on the first of a month are considered. The most noticeable feature of the temperature series at such time interval is the diurnal variation, defined by the day/night alternation. As an illustration, on the Fig. 1 temperature in Sochi (Russia) in December 1993 and 2015 is presented. All models, considered in this paper, were tested on a basis of real data from 12 weather stations situated in different climatic zones (for example, weather stations in Sochi (subtropical zone), Ekaterinburg (temperate continental zone), Tomsk (sharply continental zone), Prigranichniy (polar zone), etc.). Although all examples in the article are given only for Sochi and Tomsk, all conclusions are valid for all considered weather stations.

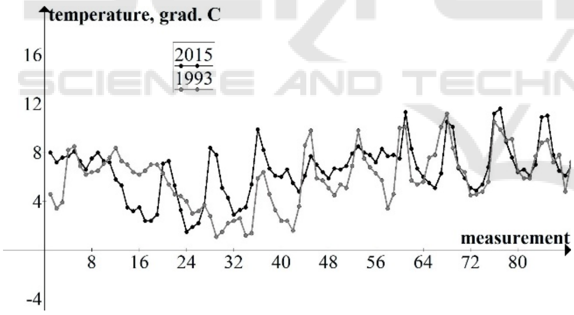


Figure 1: Air temperature. Sochi. December, 1-11.

## 3 PERIODICALLY CORRELATED MODEL

Recall that a random process $X(t)$ is a periodically correlated process with a period $T$ if its mathematical expectation, variance and correlation function are periodic functions (Gladyshev, 1961; Dragan et al., 1987):

$$EX(t) = EX(t+T),\ DX(t) = DX(t+T),$$
$$corr\left(X(t_1), X(t_2)\right) = corr\left(X(t_1+T), X(t_2+T)\right).$$

The idea of simulation of Gaussian sequences satisfying these conditions belongs to V.A. Rozhkov and was first realized in the form of a first-order autoregression vector model (Bokov et al., 1995). Later, other approaches to simulation of random processes with such properties were developed (see, for example, Hurd and Miamee, 2007; Kargapolova and Ogorodnikov, 2012; Ogorodnikov et al., 2010; Sereseva and Medvyatskaya, 2017).

The idea to consider air temperature time-series as periodically correlated processes with a period equal to 24 hours was suggested in (Derenok and Ogorodnikov, 2008). However, due to ill-considered choice of approximation of sample one-dimensional distributions, the proposed model gave acceptable results in the study of extreme temperature behavior only for weather stations located in a temperate climatic zone. In this paper a modification of a model, suggested in (Derenok and Ogorodnikov, 2008), is presented. This modification gives good results for all considered climatic zones.

Let's consider time-series $\vec{T} = (T_1, T_2, \ldots, T_{8d})$ of air temperature as a periodically correlated discrete-time random process with a period $T = 8$, where $T_i$ is air temperature at a measurement number $i$ ("at a time moment $i$"), $d \in \{28, 30, 31\}$ is a number of days in a month.

First input parameter of a stochastic model is one-dimensional distribution of each component $T_i$. To construct a stochastic model, the use of sample one-dimensional distributions is not advisable, since the sample distributions don't have any tails, and therefore do not allow to estimate the probability of occurrence of extreme values of a meteorological element. In this connection, it is necessary to approximate the sample distributions by some analytic densities, which, on the one hand, do not greatly alter the form of the distribution and its moments, and on the other, possess tails. In (Derenok and Ogorodnikov, 2008) a Gaussian density was used for such approximation. Analysis of real data shows that at some weather stations sample distribution of air temperature is bimodal, and it can't be approximated well with a Gaussian distribution. To define the best approximation (in sense of the Pearson's criterion and closeness of approximating distributions moments to empirical ones) several types of approximating densities and different methods of densities parameters estimation were compared. Numerical experiments show that mixtures

$$g_k(x) = \theta_k \frac{1}{b_{k1}\sqrt{2\pi}} \exp\left(-\frac{(x-a_{k1})^2}{2b_{k1}^2}\right) +$$
$$+(1-\theta_k)\frac{1}{b_{k2}\sqrt{2\pi}} \exp\left(-\frac{(x-a_{k2})^2}{2b_{k2}^2}\right),$$
$$0 < \theta_k < 1, \ k = \overline{1,8}.$$

of two Gaussian distributions approximate closely sample histograms of air temperature for all measurements $k = \overline{1,8}$ (and, therefore, for all moments of time $i = 1, 2, \ldots, 8d$ ) at all considered weather stations. Fig. 2 shows examples of air temperature sample histograms $s_k(x)$, $x\,^oC$ and corresponding approximating densities. Parameters $\theta_k, a_{k1}, b_{k1}^2, a_{k2}, b_{k2}^2$ were chosen using an algorithm, proposed in (Marchenko and Minakova, 1980). This algorithm let to choose such parameters of a mixture $g_k(x)$ that mathematical expectation, variance and skewness of a random variable with a density $g_k(x)$ are equal to corresponding sample characteristics and function $g_k(x)$ minimizes the Pearson's functional, that describes difference between $s_k(x)$ and $g_k(x)$. For each $k$ sample mathematical expectation, variance and skewness were estimated on a basis of $23d$ -element sample.

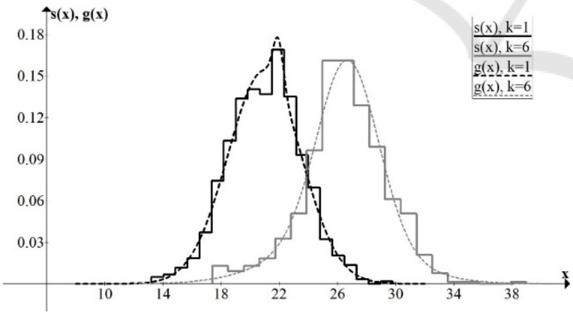


Figure 2: Sample and approximation distribution densities of air temperature. Sochi, July.

Another input parameter of a model is correlation matrix of the weather process. In this paper a sample correlation matrix $R$ is used to describe correlation structure of air temperature time-series (approximation of the sample correlation function of the process with some analytic parametric function is a work in progress). It should be noted, that the matrix

$$R = \begin{pmatrix} corr(T_1, T_1) & \ldots & corr(T_1, T_{8d}) \\ \vdots & & \vdots \\ corr(T_{8d}, T_1) & \ldots & corr(T_{8d}, T_{8d}) \end{pmatrix} =$$
$$= \begin{pmatrix} corr(1,1) & \ldots & corr(1,8d) \\ \vdots & & \vdots \\ corr(8d,1) & \ldots & corr(8d,8d) \end{pmatrix},$$

estimated under assumption that the process $\vec{T}$ is periodically correlated, is a block-Toeplitz matrix. Analysis of real data shows that for all meteorological stations and months considered, the amplitudes of diurnal oscillations of the corresponding autocorrelation functions $corr(i, i+h)$ of air temperature are significant. Fig. 3 shows examples of sample correlation coefficients $corr(i, i+h)$, as functions of time $i$ for a fixed shift $h$ (presented in Fig. 3 functions are periodic because estimations of correlation coefficients were done under the assumption that the process $\vec{T}$ is periodically correlated). As a function of the shift $h$, the correlation coefficients decrease rapidly, as illustrated in Fig. 4.

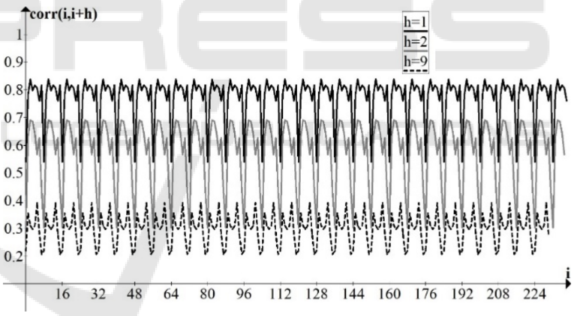


Figure 3: Sample correlation coefficients $corr(i, i+h)$ of air temperature. Tomsk, June.

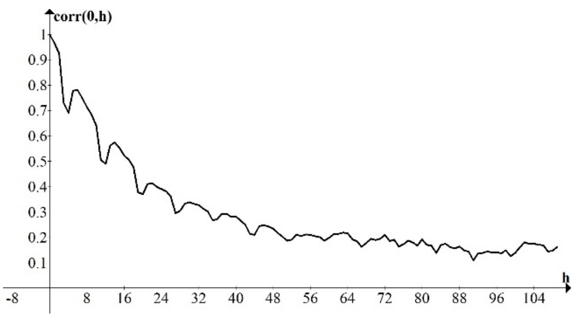


Figure 4: Sample correlation coefficients $corr(0, h)$ of air temperature. Sochi, December.

For simulation of $\vec{T}$ with given one-dimensional distributions $g_k(x),\ k=\overline{1,8}$ and given correlation matrix $R$ a method of inverse distribution function may be used (Piranashvili 1966; Ogorodnikov and Prigarin, 1996). In the framework of this method, simulation of $\vec{T}$ comes down to an algorithm with 3 steps:

1. Calculation of a matrix $R'$ that is a correlation matrix of an intermediate standard Gaussian process $\vec{T'}=\left(T_1',T_2',\ldots,T_{8d}'\right)$. Element $r'(\mathrm{i},\mathrm{j}),\ i,j=\overline{1,8d}$ of the matrix $R'$ is a solution of an equation

$$corr(\mathrm{i},\mathrm{j}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i^{-1}\left(\Phi(x)\right)F_j^{-1}\left(\Phi(y)\right)\varphi\left(x,y,r'(\mathrm{i},\mathrm{j})\right)dxdy,$$

where

$$\varphi\left(x,y,r'(\mathrm{i},\mathrm{j})\right) = \left[2\pi\sqrt{1-\left(r'(\mathrm{i},\mathrm{j})\right)^2}\exp\left(\frac{2r'(\mathrm{i},\mathrm{j})xy-x^2-y^2}{2\left(1-\left(r'(\mathrm{i},\mathrm{j})\right)^2\right)}\right)\right]$$

is a distribution density of a bivariate Gaussian vector with zero mean, variance equal to 1 and correlation coefficient $r'(\mathrm{i},\mathrm{j})$ between components number $i$ and $j$, $\Phi(\cdot)$ is a CDF of a standard normal distribution, $F_i$, $F_j$ are CDFs corresponding to densities $g_i(x)$, $g_j(x)$.

2. Simulation of a standard Gaussian sequence $\vec{T'}$ with correlation matrix $R'$.
3. Transformation of $\vec{T'}$ into $\vec{T}$:

$$T_i = F_i^{-1}\left(\Phi\left(T_i'\right)\right),\ i=\overline{1,8d}.$$

If matrix $R'$, obtained in the first step, is not positively defined, it must be regularized. Several methods of regularization are described in (Ogorodnikov and Prigarin, 1996). In this paper a method of regularization based on substitution of negative eigenvalues of the matrix $R'$ with small positive numbers was used. Simulation of a standard Gaussian sequence $\vec{T'}$ with correlation matrix $R'$ in the second step could be done using Cholesky or spectral decomposition of the matrix $R'$. However, due to a special structure of the matrix $R'$, there are methods to reduce time required for simulation of $\vec{T'}$. As it was mentioned above, matrix $R$ is a block-Toeplitz matrix. Therefore, matrix $R'$ is also block-Toeplitz. This means that the sequence $\vec{T'}$ may be interpreted as a vector stationary sequence, that could be simulated with efficient algorithms presented in (Ogorodnikov, 1990; Robinson, 1983). In this paper an algorithm of Levinson was used. It should be noted that, due to the block-Toeplitz structure of the matrix $R'$, on the first step of the simulation algorithm, it is enough to solve only $8\times 8d$ equations for $r'(\mathrm{i},\mathrm{j}),\ i=\overline{1,8},\ j=\overline{1,8d}$.

## 4 NONSTATIONARY MODEL

It is possible to consider air temperature time-series as a non-stationary sequence without any periodic characteristics. At first thought, this assumption looks especially plausible for off-season months, when difference between average daily temperature in the beginning and in the end of a month is essential. Fig. 5 shows an example of fluctuation of average temperature in such month.

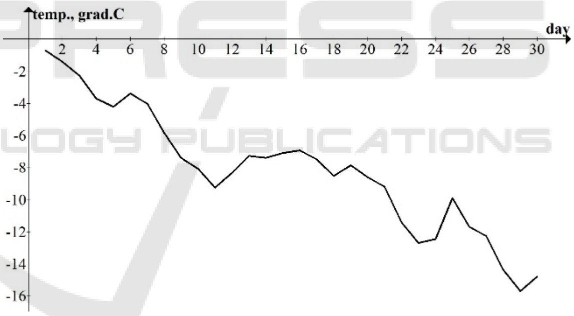


Figure 5: Average daily air temperature. Tomsk, November.

For simulation of a non-stationary sequence $\vec{S}=\left(S_1,S_2,\ldots,S_{8d}\right)$ of air temperature it is necessary to define $8d$ distribution densities (instead of $8$ densities in the periodically correlated model). As in the described above model, in the non-stationary model for approximation of sample histograms mixtures of Gaussian distributions are used. It should be noted that in this case size of a sample used for histogramming and estimation of sample moments is equal to $23$. Since sample is so small, the statistical uncertainty of distribution parameters estimation is relatively great. To decrease this uncertainty, a moving average procedure with a five-day averaging window was used for distribution parameters and correlation coefficients estimation.

Sample correlation matrix $C=\left(c\left(\mathrm{i},\mathrm{j}\right)\right)$ of a non-stationary sequence $\vec{S}$, used as a second input parameter of a model, doesn't have any specific features. It means that in the framework of inverse distribution function method it is necessary to solve $8d\times 8d$ equations to define a correlation matrix $C'=\left(c'\left(\mathrm{i},\mathrm{j}\right)\right)$ of an auxiliary standard Gaussian process $\overrightarrow{S'}$. In this paper simulation of $\overrightarrow{S'}$ with the correlation matrix $C'$ was done using Cholesky decomposition of the matrix $C'$.

## 5 NUMERICAL EXPERIMENTS

It is obvious, that any stochastic model must be verified before one starts to use simulated trajectories to study properties of a simulated process. For a model verification, it is necessary to compare simulated and real data based estimations of such characteristics, which, on the one hand, are reliably estimated by real data, and on the other hand are not input parameters of the model. Here are several examples of such characteristics.

Tab. 1 shows the probabilities of the event "air temperature is below a given level $l\,^oC$ during at least 3 hours (equivalently – during at least 2 consequent measurements)". For models verification, levels $l$ close to the mean values of temperature were chosen. Since both models accurately reproduce this characteristic, simulated trajectories were used to estimate the probabilities of the specified event for extreme low levels ( $l=-38,-40^oC$ ) for which an estimate from a small sample of real data yields a zero result, although the event is possible. Here and below $10^5$ simulated trajectories were used for estimations. To denote estimations based on real data, an abbreviation RD is used, and for estimations based on the periodically correlated model and on the nonstationary one abbreviations PCM and NSM are used respectively.

Another characteristic that was used both for models verification and study of air temperature time-series properties was a "probability of a rapid change of air temperature". As a rapid change of air temperature, a change for more than $\Delta\,^oC$ in less than 24 hours was considered. Tab. 2 shows corresponding estimations. Rapid temperature changes (both temperature drops and rises) are unpleasant weather events, that negatively influence on a human well-being and on open-ground planted crop species. This characteristic is reproduced well by both models for all considered weather stations.

Table 1: Probabilities of the event "air temperature is below a given level $l\,^oC$ during at least 3 hours". Tomsk, December.

| $l^oC$ | RD | PCM | NSM |
|---|---|---|---|
| -10 | 0.63 | 0.61 | 0.62 |
| -14 | 0.46 | 0.44 | 0.42 |
| -16 | 0.37 | 0.38 | 0.40 |
| -18 | 0.31 | 0.20 | 0.31 |
| -32 | 0.04 | 0.03 | 0.04 |
| -38 | 0.00 | 0.01 | 0.01 |
| -40 | 0.00 | 0.01 | 0.01 |

Table 2: Probabilities of the air temperature rapid change. Tomsk, March.

| $\Delta^oC$ | RD | PCM | NSM |
|---|---|---|---|
| 5 | 0.85 | 0.84 | 0.86 |
| 9 | 0.52 | 0.54 | 0.51 |
| 13 | 0.23 | 0.23 | 0.22 |
| 17 | 0.06 | 0.05 | 0.07 |
| 21 | 0.01 | 0.02 | 0.02 |
| 25 | 0.00 | 0.01 | 0.01 |

One more characteristic that was studied on a basis of real and simulated data was "average number of days in a month with a minimum daily temperature above given level $l\,^oC$". Tab. 3 shows corresponding estimations. That last column of the Tab. 3 contains estimations of the characteristic under consideration, obtained with simulated trajectories of a well-known model WGEN (see, for example, Richardson, 1981; Richardson and Wright, 1984; Semenov et al., 1998). WGEN is a stochastic model of a weather complex "daily precipitation, daily maximum and minimum temperature, solar radiation". All three models (PCM, NSM and WGEN) give comparable results. It's worth noting that for extreme high levels $l=26,28^oC$ there is a big difference between estimations on real and simulated data. The most probable explanation of this fact is that size of a real data sample is too small for reliable estimation of rare (but physically possible) weather events.

The last characteristic presented in this paper is "average daily temperature in a day number $i$". Estimations of these probabilities for two different months (first of which is an in-season month and second is an off-season) are shown in Tab. 4 and

Tab.5. It’s easy to see, that NSM accurately reproduces this characteristic both for in-season and off-season months, while PCM gives plausible results only for the in-season month.

Table 3: Average number of days in a month with a minimum daily temperature above given level $l\,^oC$. Sochi, July.

| $l^oC$ | RD | PCM | NSM | WGEN |
|---|---|---|---|---|
| 14 | 30.78 | 30.81 | 30.76 | 30.80 |
| 16 | 29.83 | 29.63 | 29.64 | 29.60 |
| 18 | 25.70 | 25.31 | 25.98 | 25.30 |
| 20 | 16.74 | 16.70 | 16.79 | 16.70 |
| 22 | 7.09 | 6.96 | 7.03 | 7.07 |
| 24 | 1.70 | 1.63 | 1.62 | 1.60 |
| 26 | 0.04 | 0.21 | 0.20 | 0.19 |
| 28 | 0.00 | 0.11 | 0.11 | 0.10 |

Table 4: Average daily temperature in a day number $i$. Sochi, January.

| $i$ | RD | PCM | NSM |
|---|---|---|---|
| 1 | 7.41 | 7.42 | 7.39 |
| 11 | 6.17 | 6.16 | 6.19 |
| 21 | 5.69 | 5.66 | 5.63 |
| 31 | 5.84 | 5.83 | 5.81 |

Table 5: Average daily temperature in a day number $i$. Sochi, May.

| $i$ | RD | PCM | NSM |
|---|---|---|---|
| 1 | 13.87 | 16.01 | 13.96 |
| 11 | 15.99 | 16.06 | 15.83 |
| 21 | 18.47 | 15.99 | 18.01 |
| 31 | 19.45 | 16.04 | 19.52 |

## 6 CONCLUSIONS

Results of numerical experiments show that, in general, both considered models reproduce quite well the properties of a real air temperature time-series in an in-season month and can be used for study of the properties of extreme / rare meteorological events. But, since simulation of a periodically correlation sequence $\vec{T}\,'$ as a vector stationary sequence requires less time than simulation of a non-stationary sequence $\vec{S}\,'$, usage of the first model is preferable. For off-season months the periodically correlated model does not always give satisfactory results, so it is better to use the non-stationary model.

In future, the both models will be expanded – instead of air temperature time-series, three-component weather complexes “air temperature, relative humidity, atmospheric pressure” and “air temperature, relative humidity, wind speed modulus” will be simulated as joint time-series. Simulation of first weather complex is of interest, because on a basis of simulated trajectories it is possible to study the properties of humid air enthalpy time-series. Simulated trajectories of the second complex could be used as input data for models of forest / grassland fires spread.

Both models also could be easily transformed into conditional models that may be used for probabilistic forecasting of air temperature. Quality of such forecasts will be studied later.

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