Path Tracking with Orthogonal Parametrization for a Satellite with Partial State Information

Wojciech Domski and Alicja Mazur
Wrocław University of Science and Technology, Wrocław, Poland

Keywords: Free-floating Satellite, Path Following, Object Capturing.

Abstract: This article presents an orthogonal parametrization for a space robot. A simulation scenario was presented where a satellite with a fixed 2R planar manipulator arm is chasing a moving object in the space. A path tracking algorithm with partial state information such as distance to chased object and difference in orientation was used. Due to the undemanding nature of proposed algorithm the satelite can successfully chase an object with limited state information. The chase manoeuvre is divided into two stages. The first stage is to enter an orbit around the object while following curvilinear path. In turn, the second stage is to settle in a point on the orbit to create favourable conditions for any further operations.

1 INTRODUCTION

Space robotics is widely researched topic were variety of problems are still being solved i.e. trajectory planning (Rybus and Seweryn, 2015). One of applications of space robots is debris removal (Kanazaki et al., 2017), (Li et al., 2016). During the space missions a number of debris are left on Earth orbit. Sometimes such debris are just leftovers and sometimes those are parts of space object which were involved in crash of various nature. The number of such space litter is increasing and removing it is an emerging problem for sustaining safety of future missions. In this article we propose an approach which allows a satellite to chase other space objects and makes possible further interaction with it. However, in space the information about state of chased object is limited and sometimes it is very difficult to obtain relevant information about the object's state (Diaz and Abderrahim, 2006), (C. M. Allen et al., 2008). We have presented an algorithm of path tracking problem with orthogonal parametrization using Serret–Frenet description. The Serret-Frenet frame was successfully used in path following problem with marine vessels (Do and Pan, 2003). Undoubtedly, the biggest advantage of proposed approach is the ability to work with limited information about the object. Two variables are required – a distance to chased object and its orientation error. As discussed before, in contrast to other methods which require a specific position and orientation of chased object in global frame, our approach only requires information about the distance to the object and the orientation error. The distance between our satellite and the object can be acquired with a laser scanner while the orientation can be determined with a camera system. In this work we consider a flat satellite with 2 DoF robotic arm attached to the base with offset.

In Section 2 mathematical model of satellite's dynamics is presented. In turn, in Section 3 orthogonal parametrization relative to Serret–Frenet frame is described. Control problem and additional assumptions are presented in Section 4. In Section 5 the control algorithm has been introduced. The Section 6 presents conducted simulations for flat satellite with manipulating arm while Section 7 offers summary and discussion on obtained simulations.

2 MATHEMATICAL MODEL

In this article we will consider flat satellite, so-called *base*, with manipulating arm. The satellite is made of a rectangular shape where a rigid manipulator with fixed two rotational joints is mounted. This object moves on a plain with affiliated global coordination system $X_0Y_0Z_0$. The position of local coordination system associated with centre of the mass of the satellite's base is $X_bY_bZ_b$.

Schematic of considered satellite and onboard manipulator is presented in Fig. 1.

Domski, W. and Mazur, A.
Path Tracking with Orthogonal Parametrization for a Satellite with Partial State Information.
DOI: 10.5220/0006835102520257
In *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2018) - Volume 2*, pages 252-257
ISBN: 978-989-758-321-6

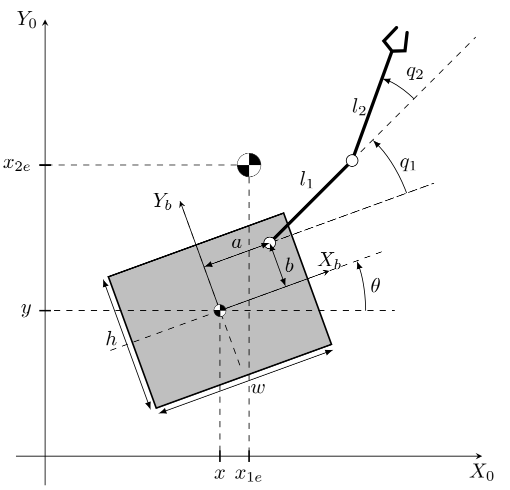

Figure 1: Satellite with marked coordination systems.

2.1 Kinematics

Transformation from global coordination system to satellite's local coordination system can be expressed as

$$A_0^b = Trans(X,x)Trans(Y,y)Rot(Z,\theta). \tag{1}$$

Transformation from global coordination system to the end of the first link can be written as

$$A_0^1 = A_0^b Trans(X,a)Trans(Y,b)Rot(Z,q_1)Trans(X,l_1), \tag{2}$$

while the transformation from global coordinate system to the end of the second link, so-called end-effector, is described as

$$A_0^2 = A_0^1 Rot(Z,q_2)Trans(X,l_2). \tag{3}$$

For further consideration we need the kinematics of the end-effector including its position and orientation relative to global frame. Finally, we can define kinematics of the end–effector as

$$k_{ef} = \begin{pmatrix} x_{ef} \\ y_{ef} \\ \theta_{ef} \end{pmatrix} \tag{4}$$

where:

$$\begin{aligned} x_{ef} &= x + a\cos_\theta - b\sin_\theta - l_1(\sin_1\sin_\theta - \cos_1\cos_\theta) \\ &\quad - l_2(\cos_2(\sin_1\sin_\theta - \cos_1\cos_\theta) \\ &\quad + \sin_2(\cos_1\sin_\theta + \cos_\theta\sin_1)) \\ y_{ef} &= y + b\cos_\theta + a\sin_\theta + l_1(\cos_1\sin_\theta + \cos_\theta\sin_1) \\ &\quad + l_2(\cos_2(\cos_1\sin_\theta + \cos_\theta\sin_1) \\ &\quad - \sin_2(\sin_1\sin_\theta - \cos_1\cos_\theta)) \\ \theta_{ef} &= \theta + q_1 + q_2. \end{aligned}$$

Symbols have following meaning: $\sin_i = \sin q_i$ and $\cos_i = \cos q_i$, $i = 1,2$.

Calculating time derivative of k_{ef}, we obtain

$$\dot{k}_{ef} = \frac{\partial k_{ef}}{\partial q_b}\dot{q}_b + \frac{\partial k_{ef}}{q_r}\dot{q}_r = J_b\dot{q}_b + J_r\dot{q}_r \tag{5}$$

where state vector $q = (q_b, q_r)^T : q_b = (x,y,\theta)^T$ and $q_r = (q_1, q_2)^T$.

2.2 Dynamics

Dynamics of the whole system was expressed in generalized coordinates. Satellite's dynamics can be expressed in following equation (Siciliano and Khatib, 2007)

$$M\ddot{q} + C = Bu, \tag{6}$$

or more in detail as

$$\begin{bmatrix} M_b & M_{bm} \\ M_{bm}^T & M_m \end{bmatrix} \begin{pmatrix} \ddot{q}_b \\ \ddot{q}_r \end{pmatrix} + \begin{pmatrix} c_b \\ c_r \end{pmatrix} = \begin{bmatrix} B_b & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} u \\ 0 \end{pmatrix}, \tag{7}$$

where M is inertia matrix, and c_b and c_r are other elements of dynamics related to Coriolis and centrifugal forces of the base and of the manipulator arm. u is input vector for controlling the thrusters which in turn actuate the flat satellite and the B_b is the input matrix.

It is assumed that the joint position of the manipulator are fixed and do not change, thus the dynamics regarding the acceleration of manipulator's joint can be omitted. As written above the manipulator joints are fixed at initial position and do not move. This condition is true during the whole phase of chasing an object, as explained latter.

Another assumption was made for gravitation forces, since the research object is in space it is influenced by the micro-gravity and therefore the gravitation force can be neglected. Also, as mentioned earlier, the satellite's base is actuated with three thrusters.

2.3 Thrusters Modelling

Moments generated by thrusters are present in local coordination system of the satellite. However, to model those actuator the moments should be transformed into global coordinate system. First vector of force for i^{th} thruster has to be decomposed into three elements: an element which direction is aligned with X_b axis of the flat satellite coordinate system and with Y_b axis. The third element has to be perpendicular to the line connecting the thruster mount point and the centre of the mass of the satellite. For each thruster a vector of three elements is calculated and then transformed to the global coordination system. Those elements can be expressed with following formulas:

$$v_x = \alpha\cos(\beta) \tag{8}$$

$$v_y = \alpha\sin(\beta) \tag{9}$$

$$v_\theta = \sqrt{(1-\alpha^2)(x^2+y^2)}\operatorname{sgn}(\sin(\phi-\beta)) \tag{10}$$

where $\alpha = \cos(\phi - \beta)$, $\beta = \mathrm{atan}(\frac{y}{x})$. x, y, ϕ are thrusters position in X_b axis, Y_B axis and angle of mounting thruster on the base in its local coordinate system respectively. After calculation of all three elements (v_x, v_y, v_θ) for each thruster a matrix can be composed where i^{th} column holds three values calculated for i^{th} thruster. This matrix is then transformed to the global coordinate system with rotation matrix around Z axis. The general form of control matrix B_b is following:

$$B_b = \begin{bmatrix} v_{x,1} & v_{x,2} & & v_{x,n} \\ v_{y,1} & v_{y,2} & \cdots & v_{y,n} \\ v_{\theta,1} & v_{\theta,2} & & v_{\theta,n} \end{bmatrix} \quad (11)$$

By choosing the placement, the orientation and the number of thrusters on the satellite we can ensure that the matrix is invertible. In the Section 6 placement of the thrusters and its orientation was provided and also the value of B matrix determinant.

3 ORTHOGONAL PATH PARAMETRIZATION

In this paragraph we want to present specific description of the satellites's motion – not relative to fixed frame but relative to moving space debris.

The path P is characterized by a curvature $\kappa(s)$ which is an inversion of the radius of the circle tangent to the path at a point characterized by the parameter s, see Fig. 2. We consider a moving point M and the associated Serret-Frenet frame defined on the curve P by the normal and tangent unit vectors x_n and $\frac{dr}{ds}$. The point M is any point, which should be tracked, e.g. position of the end-effector in our case. In turn M' is the orthogonal projection of the point M on the path $P(s)$.

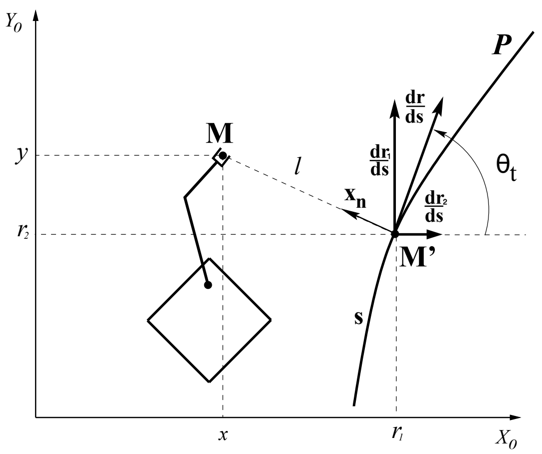

Figure 2: Path P and projection of point M on the path P.

To describe the satellite's motion relative to given path we can use certain geometric relationships (Mazur, 2004)

$$\dot{l} = -\sin\theta_t \dot{x}_{ef} + \cos\theta_t \dot{y}_{ef}, \quad (12)$$

$$\dot{\tilde{\theta}} = \frac{\kappa(s)\cos\theta_t}{\kappa(s)l - 1}\dot{x}_{ef} + \frac{\kappa(s)\sin\theta_t}{\kappa(s)l - 1}\dot{y}_{ef} + \dot{\theta}_{ef}, \quad (13)$$

$$\dot{s} = -\frac{\cos\theta_t}{\kappa(s)l - 1}\dot{x}_{ef} - \frac{\sin\theta_t}{\kappa(s)l - 1}\dot{y}_{ef}. \quad (14)$$

l is the distance between point on a path and the end–effector, $\tilde{\theta}$ is the orientation error – it is a difference between the orientation of the end–effector and the orientation of the Serret-Frenet frame on the path. Curvilinear distance s is a term which defines where on the contour of the path the Serret-Frenet frame is located.

Since the orthogonal parametrization is local transformation the following conditions has to be fulfilled during control (Fradkov et al., 1999)

$$r_{min} > 0, \quad (15)$$

$$\kappa(s) \leq \frac{1}{r_{min}}, \quad (16)$$

$$l < r_{min}. \quad (17)$$

It means that the radius of any circle tangent to path $P(s)$ in two or more points has to be limited from below with some positive number r_{min} while interior of such a circle can not contain any point which belongs to the curve $P(s)$. What is more, the distance l has to be smaller then the minimal radius which in turn yields that this parametrization is indeed local.

4 CONTROL PROBLEM STATEMENT

The goal of the paper is to develop the control algorithm for the flat satellite with onboard manipulating arm, which allows to catch space debris with only partial information about state of the object. The control scheme will be divided into two steps:

1. Object tracking – motion of the base with immobile manipulator realized with thrusters. After this phase of motion satellite should be in the neighbourhood of object, in such distance that it would be possible to catch object only with manipulator's motion.
2. Object catching – near object the satellite should stop, turn-off thrusters and try to catch the object. This motion phase should be realized only with manipulating arm.

The information, which we have during the first phase, is only partial – we don't get information about position relative to the global coordinate system $X_0Y_0Z_0$ but we have only local information obtained from sensors:

– shortest distance between satellite's point M and its orthogonal projection on object's path M',
– orientation error $\tilde{\theta} = \theta - \theta_t$ – difference between object's and chaser satellite's angle.

4.1 Assumptions

In this article we have taken the following assumptions:

- the motion of the tracked object and the chasing satellite (chaser) is realized on the same plane
- the path, the object is moving along, is regular – it can be approximated by the smooth flat curve
- dynamics of the chaser is fully known.

5 CONTROL ALGORITHM

As it has been mentioned in the previous section, the control strategy contains two phases: tracking the path of the object and reduction of the distance between the chaser and the object and, next, catching the object with manipulating arm, if satellite is near the object in reachable distance. However, in this paper we have reduced our interest to design the control method for the first phase.

5.1 Chasing the Object – Path Following Problem

In the text we have written that chasing the object can be treated as path following problem. For this reason lets take satellite's equations expressed relative to the moving object (12)-(14) and rewrite them in the matrix form

$$\dot{\xi} = \Lambda \dot{k}_{ef}, \tag{18}$$

where path tracking variables are defined as follows

$$\xi = \begin{pmatrix} l \\ \tilde{\theta} \\ s \end{pmatrix}, \tag{19}$$

and matrix Λ equals to

$$\Lambda = \begin{bmatrix} -\sin\theta_t & \cos\theta_t & 0 \\ \frac{\kappa(s)\cos\theta_t}{\kappa(s)l-1} & \frac{\kappa(s)\sin\theta_t}{\kappa(s)l-1} & 1 \\ -\frac{\cos\theta_t}{\kappa(s)l-1} & -\frac{\sin\theta_t}{\kappa(s)l-1} & 0 \end{bmatrix}. \tag{20}$$

Matrix Λ is nonsingular as long as local parametrization is valid, i.e.

$$\det \Lambda = \frac{-1}{\kappa(s)l-1} \neq 0 \tag{21}$$

where fraction denominator is not equal to 0.

5.2 Decoupling and Linearization

To design a control algorithm based on orthogonal parametrization, we need to differentiate (18) with time

$$\ddot{\xi} = \dot{\Lambda}\dot{k}_{ef} + \Lambda\ddot{k}_{ef}. \tag{22}$$

On the other hand we have assumed, that the joint positions of the satellite's manipulator are fixed and do not move during path following process. It means that the dynamics (7) regarding the acceleration of manipulator's joint can be omitted, giving the simplified form

$$M_b\ddot{q}_b + c_b = B_b u. \tag{23}$$

The condition treating about fixed joint is true during the whole phase of chasing an object, therefore (5) can be recalculated as follows

$$\dot{k}_{ef} = \frac{\partial k_{ef}}{\partial q_b}\dot{q}_b = J_b(q_b)\dot{q}_b \tag{24}$$

with Jacobi matrix given below

$$J_b = \begin{bmatrix} 1 & 0 & -b\cos_\theta - a\sin_\theta - l_2\sin_{12\theta} - l_1\sin_{1\theta} \\ 0 & 1 & a\cos_\theta - b\sin_\theta + l_2\cos_{12\theta} + l_1\cos_{1\theta} \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix J_b is always invertible, because it holds

$$\forall\, q_b \in R^3 \qquad \det J_b(q_b) = 1.$$

The term $\ddot{k}_{ef}$ can be obtained from equation (24)

$$\ddot{k}_{ef} = \dot{J}_b\dot{q}_b + J_b\ddot{q}_b. \tag{25}$$

Now, we put (25) into (22) and get

$$\ddot{\xi} = \dot{\Lambda}\dot{k}_{ef} + \Lambda\left(\dot{J}_b\dot{q}_b + J_b\ddot{q}_b\right). \tag{26}$$

The final step is to take into consideration dynamics of the satellite.

After inserting dynamics (23) into the above equation we get affine control system

$$\ddot{\xi} = F + Gu \tag{27}$$

where $F = \dot{\Lambda}\dot{k}_{ef} + \Lambda\dot{J}_b\dot{q}_b - \Lambda J_b M_b^{-1}c_b$ and $G = \Lambda J_b M_b^{-1}B_b$.

It is easy to check that G matrix is invertible i.e. it fulfils regularity condition $detG \neq 0$, because

$$G^{-1} = B_b^{-1}M_b J_b^{-1}\Lambda^{-1}$$

and every element in G^{-1} is well defined and invertible.

Next we define control law for the system (27)

$$u = G^{-1}(v - F), \tag{28}$$

where v is a new input to the system. The closed-loop system (27)-(28) is fully decoupled and linearised

$$\ddot{\xi} = v, \tag{29}$$

i.e. it is equivalent to the linear system of “double integrator” type.

To control such system we propose PD-control with compensation signal as follows

$$v = \ddot{\xi}_d - K_d \dot{e}_\xi - K_p e_\xi \tag{30}$$

where $e_\xi = \xi - \xi_d$ and $\dot{e}_\xi = \dot{\xi} - \dot{\xi}_d$.

As a desired reference signals ξ_d we take

$$\xi_d = \begin{pmatrix} l_d \\ \tilde{\theta}_d \\ s_d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ k_s t \end{pmatrix} \tag{31}$$

where k_s is a desired velocity of linear reducing distance between the chaser satellite and the object. Parametrization of curvilinear distance $s_d(t)$ is free to choose. We want it to be $s_d(t) = k_s t$. $s_d(t)$ describes desired way of moving along the path.

6 SIMULATION STUDY

We have placed three thrusters. Their positions and orientations relative to the satellite base frame are following:

$$\begin{aligned} (x_1, y_1, \phi_1) &= \left(\frac{w}{2}, 0, 0\right), \\ (x_2, y_2, \phi_2) &= \left(0, \frac{h}{2}, \frac{\pi}{2}\right), \\ (x_3, y_3, \phi_3) &= \left(\frac{w}{2}, \frac{h}{2}, \frac{3\pi}{4}\right) \end{aligned}$$

where w and h are width and height of the satellite base respectively equal to $0.5m$ both. Thus the determinant of matrix B_b is equal 0.3536.

During the simulation studies we have examined a scenario which is conducted in two stages:

1. Start tracking a path parametrized with time.
2. Settle in a point on the curve $P(s)$ with desired orientation.

We assume that the research object does not move, thus it has zero momentum at the initial stage. What is more, each joint of the attached manipulator also does not and is fixed in constant position; $(\dot{q}_1, \dot{q}_2) = (0, 0)$ and $(q_1, q_2) = (0, 0)$. We track a path which has a contour of a circle with $R = 1$ which yields $\kappa(s) = 1$. The desired path is described with following equations

$$\begin{aligned} l_d &= 0, \\ \tilde{\theta}_d &= 0, \\ s_d &= 0.02t. \end{aligned}$$

after time $t_s = const = 600$ seconds we settle in a point on the path with following criteria

$$\begin{aligned} l_d &= 0, \\ \tilde{\theta}_d &= -0.4, \\ s_d &= 0.02t_s. \end{aligned}$$

In the Fig 2 we present overall process of following a path and in the Fig. 4 we can see errors in orthogonal space. The errors are converging to 0, thus the path is followed. In 600 second we change the desired path

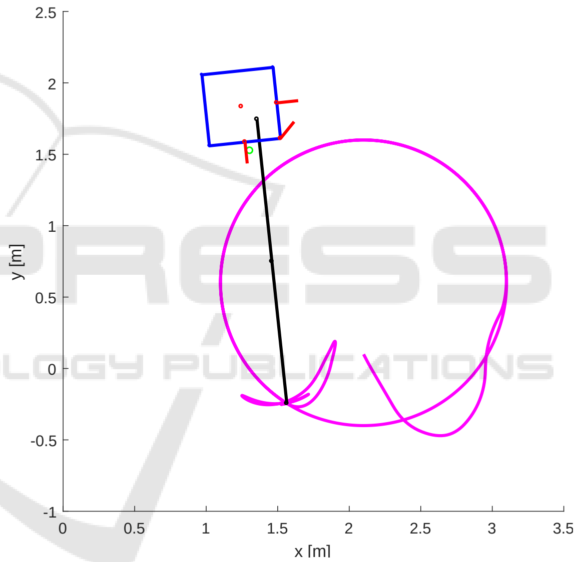

Figure 3: Full trajectory of the satellite.

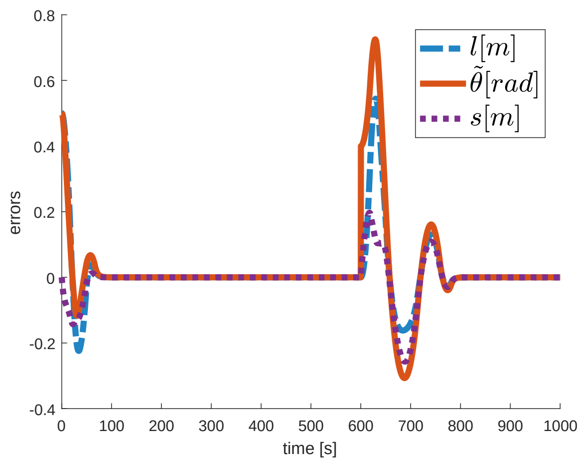

Figure 4: Errors in orthogonal parametrization space.

to a fixed point with orientation where the errors also converge to zero and the satellite in not moving any more.

7 CONCLUSIONS

The orthogonal path parametrization was presented for a space object – a flat satellite with 2 DoF planar manipulator arm. This approach was utilized for a certain scenario where a flat satellite was used to chase a object, debris or an other satellite, and during this manoeuvre the manipulator arm is fixed in a certain position. The orthogonal path parametrization was used to track a path parametrized with time where end effector of the manipulator was moving on the orbit around the chased object. The most important advantage of this approach is very scatter amount of information needed to successfully follow the path. The presented method only requires the distance to the object and orientation error between the followed path and the current orientation of the satellite. These type of data, in cosmic conditions, can be relatively easily collected. Presented manoeuvre in simulation study section shows two phases of the movement. Firstly, the satellite is following the orbit and then in the second stage it settles in a point on this path with desired orientation. This is good starting point to intercept the chased object with only the manipulator and without the thrusters as presented in our other research paper. Further research would involve merging those to stages into a single manoeuvre.

ACKNOWLEDGEMENTS

The works of Wojciech Domski was supported by National Science Centre grant no. 2015/17/B/ST7/03995. The works of Alicja Mazur was supported by the Wrocław University of Science and Technology statutory grant 0401/0142/17.

REFERENCES

C. M. Allen, A., Langley, C., Mukherji, R., B. Taylor, A., Umasuthan, M., and D. Barfoot, T. (2008). Rendezvous lidar sensor system for terminal rendezvous, capture, and berthing to the international space station.

Diaz, J. and Abderrahim, M. (2006). Visual inspection system for autonomous robotic on-orbit satellite servicing. *in: Proc. 9th ESA Symposium on Advanced Space Technologies in Robotics and Automation (ASTRA 2006).*

Do, K. D. and Pan, J. (2003). Robust path following of underactuated ships using Serret-Frenet frame. In *Proceedings of the 2003 American Control Conference, 2003.*, volume 3, pages 2000–2005 vol.3.

Fradkov, A., Miroshnik, I., and Nikiforov, V. (1999). *Nonlinear and Adaptive Control of Complex Systems*. Kluwer Academic Publishers, Dordrecht.

Kanazaki, M., Yamada, Y., and Nakamiya, M. (2017). Trajectory optimization of a satellite for multiple active space debris removal based on a method for the traveling serviceman problem. In *2017 21st Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES)*, pages 61–66.

Li, X., Yao, Y., Yang, B., and Wang, L. (2016). Guidance strategy design for space debris removal using fractionated spacecraft. In *2016 IEEE Chinese Guidance, Navigation and Control Conference (CGNCC)*, pages 264–270.

Mazur, A. (2004). Hybrid adaptive control laws solving a path following problem for nonholonomic mobile manipulators. *International Journal of Control*, 77(15):1297–1306.

Rybus, T. and Seweryn, K. (2015). Application of rapidly-exploring random trees (rrt) algorithm for trajectory planning of free-floating space manipulator. In *2015 10th International Workshop on Robot Motion and Control (RoMoCo)*, pages 91–96.

Siciliano, B. and Khatib, O. (2007). *Springer Handbook of Robotics*. Springer-Verlag New York, Inc.