A Fault-Tolerant Sensor Reconciliation Scheme based on Self-Tuning LPV Observers

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Abstract: This paper presents a fault-tolerant sensor reconciliation scheme for systems equipped with a redundant number of possibly faulty "physical" sensors. The reconciliator is in charge to discover on-line, at each time instant, the faulty physical sensors, if any, and exclude their measures from the generation of the "virtual" sensors, which, on the contrary, are supposed to be always healthy and suitably usable for control purposes without requiring the reconfiguration of the nominal control law. Amongst many, the solution proposed here is based on the use of a Linear Parameter Varying Luenberger Observers (LPV-LU) able to estimate both state, bias fault and loss of effectiveness fault. Such information is used to self adapting the parameters of the LPV representation. For simplicity, the sensor faults here considered are limited to variation of sensors' gain and offset values. The scheme is fully described and all of its properties investigated and proved. Finally, a simulation example is reported in details to show the effectiveness of the scheme.

1 INTRODUCTION

The capability of control systems to detect faulty sensors and recover in turn uncorrupted data has progressively gained more relevance in the last two decades. Traditional control schemes are usually designed by assuming perfect working conditions of the sensors to be used. However, in practice, sensors are subject to faults and, in that case, may provide wrong information about the system state, possibly degrading the system performance or even causing instability (Merrill et al., 1988; Samara et al., 2008). Therefore, Fault-Tolerant Control (FTC) is an important area of research in the safety critical systems domain.

One strategy to cope with this situation is to find a controller that assigns to the reconfigured closed-loop system a similar behaviour with respect to the nominal closed-loop system. A different strategy relies on the fault hiding approach that tries to hide the fault to the controller. In the latter approach the nominal controller remains in the loop while the reconfiguration block re-routes the output signals around the faulty component (Lunze and Richter, 2008). Such an approach has been dealt with in (Steffen, 2005) where a virtual sensor strategy has been proposed for fault accommodation purposes. The disadvantage of that method is that it is assumed that the sensor faults have already been detected and estimated correctly. The virtual sensors contribution to Active Fault Tolerant Control (AFTC) has been investigated in (Ponsart et al., 2010). There, the case of multiplicative sensor faults has not received any consideration. The virtual sensor approach has been investigated in (Behzad et al., 2016), for Sensor Reconciliation(SR) purposes.

This paper aims at presenting a general SR method for discrete-time linear systems with redundant physical sensors possibly subject to loss of effectiveness (gain) and offset (bias) faults. To this end, the proposed scheme consists of two interconnected modules: (*i*) a polytopic Luenberger Observer (LU) ((Bara et al., 2001)) in charge of estimating the current gain sensor faults, the state of the system and possible bias fault occurrences; (*ii*) a sensor reconciliation unit used to reconcile sensor measures. The key idea used in the proposed scheme is to consider the current gain and bias sensor faults with the system states, as an auxiliary state and consequently to design a polytopic LPV-UIO observer capable to estimate the faults via a specific Linear Matrix Inequality (LMI) procedure. Differently from (Behzad et al., 2016), where the con-

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vergence of the scheme was based on a *persistence of excitation* assumption, here we propose a Luenberger observer to jointly estimate the system state and fault parameters in a single step in order to guarantee the convergence of the whole scheme. Moreover, unlike the approach of (Steffen, 2005), our setup is capable of fault estimation along with fault accommodation. The key difference of our approach with respect to (Ponsart et al., 2010) is that both the multiplicative and additive faults may be taken into consideration. Properties of the proposed LPV-LU scheme are formally proved and discussed. A final numerical example is reported to show the effectiveness of the proposed strategy.

NOTATION

Let $\mathbb{R}$ denote the set of real numbers. A generic ball in an Euclidean n-space $\mathbb{R}^n$ is defined as $\mathcal{B}_\delta := \{x \in \mathbb{R}^n : |x|_2 \leq \delta\}$. Finally, let $\|x(\cdot)\|$ denote the ℓ_2-norm of a discrete-time signal $\{x(t)\}_{-\infty}^{\infty}$ (i.e. $\|x(\cdot)\| = \sqrt{\sum_{t=-\infty}^{\infty} |x(t)|_2^2}$).

Definition 1.1. ***(Cartesian Product)*** - For $\mathcal{P} \in \mathbb{R}^p$ and $\mathcal{Q} \in \mathbb{R}^q$ being two polytopes of dimension p and q respectively, their *Cartesian Product* is defined as

$$\mathcal{P} \times \mathcal{Q} = \{(x,y) : x \in \mathcal{P}, y \in \mathcal{Q}\}$$

Definition 1.2. ***(Unit Simplex)*** - The Polytope $\mathcal{S}_k := \{\xi \in \mathbb{R}^l | \xi_i \geq 0, i = 1,...,l, \sum_{i=1}^{l} \xi_i = 1\}$ is a k-dimensional *Unit Simplex*.

Definition 1.3. ***(Convex hull)*** - For l matrices $M_i \in \mathbb{R}^{n\times m}, i = 1,...,l$, their *Convex Hull*, denoted by $\mathrm{Co}\{M_i\}, i = 1,...,l$, is the polytope arising by all convex combinations of matrices M_i i.e $\{\sum_{i=1}^{l} \rho_i M_i, [\rho_1,...,\rho_l]^T \in \mathcal{S}_l\}$ with $\mathcal{S}_l$ being a l-dimensional unit simplex.

Definition 1.4. ***(Pontryagin Set Difference)*** - For given sets $\mathcal{A}, \mathcal{E} \subset \mathbb{R}^n$, the set determined as $\mathcal{A} \sim \mathcal{E} := \{a : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$ is the *Pontryagin Set Difference* of $\mathcal{A}$ with respect to $\mathcal{E}$.

2 PROBLEM FORMULATION

Let us consider a plant whose dynamics is described by the following discrete-time state-space representation

$$x_p(t+1) = Ax_p(t) + Bu(t) + Ev(t) \quad (1)$$

$$y(t) = \Delta\big(\gamma(t)\big)C_y x_p(t) + Fb(t) \quad (2)$$

$$z(t) = H_z C_y x_p(t) \quad (3)$$

where $A \in R^{n\times n}$, $B \in R^{n\times n_u}$, $E \in R^{n\times n_v}$, $C_y \in R^{m\times n}$ and $F \in R^{m\times q}$ are constant matrices. Moreover, $x_p(t) \in \mathbb{R}^n$ is the state vector assumed to be confined in the following set

$$\mathcal{X}_p := \{x_p : \underline{x}_p \leq x_p \leq \overline{x_p}\} \quad (4)$$

Notice that the upper and lower bounds are required to describe the polytopic representation of the output. This representation will be used to design a LPV observer.

Moreover $u(t) \in \mathbb{R}^{n_u}$ is a known input while $v(t) \in \mathbb{R}^{n_v}$ is an unknown input. $y(t) \in \mathbb{R}^m$ represents the *plant output* provided by physical redundant sensors possibly effected by both bias $b(t) \in \mathbb{R}^q$ and loss of effectiveness faults, the latter being modeled by the gain matrix $\Delta(\gamma) \in \mathbb{R}^{m\times m}$ that, for simplicity, we assume hereafter to have the following elementary structure:

$$\Delta(\gamma(t)) = \begin{bmatrix} \gamma_1(t) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \gamma_m(t) \end{bmatrix} \quad (5)$$

Finally, $z(t) \in \mathbb{R}^r$, with $r \leq m$, is defined as the *virtual output* of the system and represents the healthy information we need to get from the plant for control purposes regardless of any fault possibly occurring on the physical sensors.

It is clear that in the absence of faults one would have $\Delta(\gamma) = I_m$ and $b(t) = 0_q$. However, in the more general case $b(t) \neq 0_q$ and $\Delta(\gamma) \neq I_m$, with γ confined in the generic polytope

$$\Gamma \subseteq \{\gamma : 0_m \leq \gamma \leq 1_m\} \quad (6)$$

For this reason, it is not convenient to evaluate the signal $z(t)$ as $z(t) = H_z y(t)$ because it would be affected by possibly corrupted information brought by $y(t)$. However, because the state $x_p(t)$ is assumed not directly measurable, $z(t)$ cannot be evaluated as simply as in (3), but a more sophisticated machinery is required. This aspect motivates the design of the *Sensor Reconciliator* unit before mentioned which basically aims at addressing the following problem:

Sensor Reconciliaton Design Problem (SRDP-Problem):
Given the system (1)-(3)*, compute, at each time* $t \geq 0$ *on the basis of the real output* $y(t)$ *measures, a suitable estimate* $\hat{z}(t)$ *of the virtual output* $z(t) := H_z C_y x_p(t)$*, despite of the presence of both fault occurrences, corrupting the vector*$y(t)$*, and disturbances* $v(t)$.

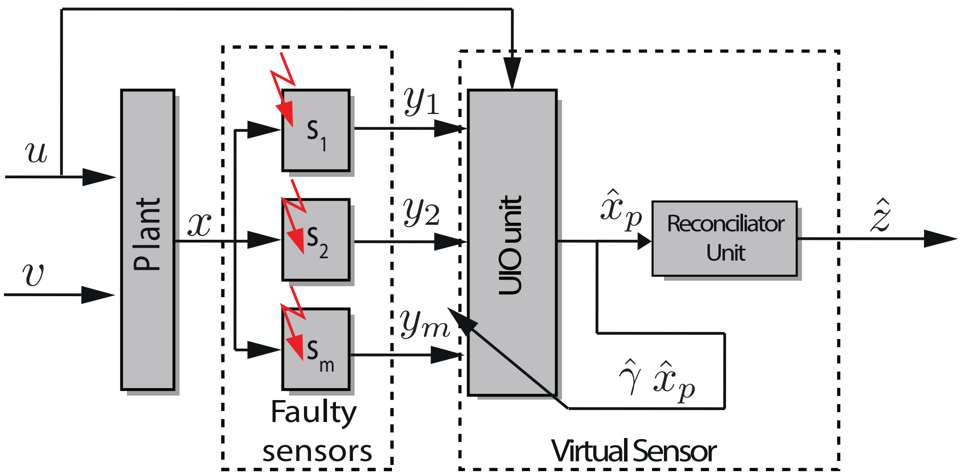

Figure 1: Virtual Sensor Architecture.

3 VIRTUAL SENSOR ARCHITECTURE

The **SRDP-Problem** solution here proposed is based on the evaluation of an estimate $\hat{x}_p(t)$ of the state $x_p(t)$ that is exploited to compute the corresponding approximation $\hat{z}(t)$ of $z(t)$ through the following equation

$$\hat{z}(t) = H_z C_y \hat{x}_p(t) \tag{7}$$

Such an approach requires to face two crucial issues: 1) How to estimate the fault occurrences corrupting $y(t)$? 2) How to get a good estimation $\hat{x}_p(t)$ in the presence of an unknown input $v(t)$ and time-varying sensor gains and bias?

The above mentioned questions are dealt with by introducing the *virtual sensor* architecture depicted in Fig. 1 that consists of two modules: a *Luenberger Observer* (LO) unit which is the core of this scheme and is designed to jointly compute current estimates of the state $x_p(t)$, the bias term $b(t)$ and the effectiveness matrix (5) parameters; a *Reconciliator Unit* that simply performs the computation indicated in (7).

4 SENSOR FAULT AUGMENTED MODEL

In order to design the Luenberger observer, the following augmented state is considered that includes the bias fault $b(t)$ and multiplicative fault $\gamma(t)$ vector among its components

$$x(t) = \begin{bmatrix} x_p(t) \\ b(t) \\ \gamma_1(t) \\ \vdots \\ \gamma_m(t) \end{bmatrix} \tag{8}$$

Notice that in order to describe the augmented model, one has to assume that the multiplicative sensor fault term $\gamma(t)$, the bias term $b(t)$ and the parameters $\Delta\gamma(t)$ and $\Delta b(t)$ are bounded in the l_2 norm sense, where

$$\Delta b(t) := b(t+1) - b(t)$$
$$\Delta\gamma(t) := \gamma(t+1) - \gamma(t)$$

In this way, the related augmented model can be described as

$$\begin{aligned} x(t+1) &= \bar{A}x(t) + \bar{B}u(t) + \bar{E}v(t) \\ &\quad + \bar{F}\Delta b(t) + \bar{D}\Delta\gamma(t) \\ \bar{y}(t) &= \bar{C}_{\gamma(t),x_p(t)} x(t) \end{aligned} \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix}, \bar{F} = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}$$
$$\bar{C}_{\{\gamma(t),x_p(t)\}} = \begin{bmatrix} \triangle(\gamma(t))C_y & F & 0 \\ 0 & F & diag(C_y x_p(t)) \end{bmatrix}, \bar{D} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \tag{10}$$

5 LUENBERGER OBSERVER

In this section we describe the basic ingredients of the proposed observer. Let us assume to be provided with estimates $\hat{\gamma}(t)$ of $\gamma(t)$ and $\hat{x}_p(t)$ of $x_p(t)$ at each time t. Then, a possible structure for an observer for the model (9) is given by

$$\begin{aligned} \hat{x}(t+1) &= \bar{A}\hat{x}(t) + \bar{B}u(t) \quad + L\Big(\bar{y}(t) - \hat{y}(t)\Big) \\ \hat{y}(t) &= \bar{C}_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}\hat{x}(t) \end{aligned} \tag{11}$$

As a consequence, the one-step ahead evolution of the state estimation error

$$e(t) := x(t) - \hat{x}(t) \tag{12}$$

would take the following form

$$\begin{aligned} e(t+1) = &\Big(\bar{A} - L_{\hat{\gamma}(t),\hat{x}_p(t)}\bar{C}_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}\Big)e(t) + \bar{E}v(t) \\ &+ \bar{F}\Delta b(t) + \bar{D}\Delta\gamma(t) - L_{\hat{\gamma}(t),\hat{x}_p(t)}d(t) \end{aligned} \tag{13}$$

with

$$d(t) := \big(\bar{C}_{\{\gamma(t),x_p(t))\}} - \bar{C}_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}\big)x(t)$$

If $\Big(\bar{A} - L_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}\bar{C}_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}\Big)$ were chosen as a stable matrix $\forall\gamma \in \Gamma$ and $\forall x_p \in \mathcal{X}_p$, the state estimation error would remain bounded whenever $\Delta b(t)$ and $\Delta\gamma(t)$ remain bounded as well. Hence, the state $x(t)$ of the system can be estimated.

Moving from these considerations, in order to design the Luenberger observer (11), it is is sufficient to determine a parameter varying gain L_{γ,x_p} that robustly stabilizes the system (13) against all possible occurrences of $\Delta b(t)$, $\Delta\gamma(t)$ and $d(t)$. Such a problem has been addressed in a significant amount of works in different contexts by exploiting well-known results on robust control theory and LMI formalism. Specifically, among many observer design approaches, it is interesting to mention here (Heemels et al., 2010), where the LPV gain is determined in the case of constant output matrix. A similar approach has been considered in (Zhou et al., 2013), where a LMI based procedure has been proposed to compute a constant gain.

Here we design a discrete-time self-tuning LPV observer where the time-varying parameter γ and the state x_p are not perfectly known. In particular, the approach considers systems (13) characterized by a structured uncertainty related to γ , x_p and attempts to determine a LPV gain that can be tuned on-line by exploiting an estimate $\hat{\gamma}(t)$ and $\hat{x}(t)$ of the true $\gamma(t)$ and $x(t)$. In this respect, it is worth pointing out that we assume hereafter to be provided by a polytopic embedding approximation for matrix $\bar{C}_{\{\hat{\gamma}(t),\hat{x}_p(t)\}}$ given by

$$\bar{C}_\rho = \sum_{i=1}^{l} \rho_i(\gamma, x_p)\tilde{C}_i, \tag{14}$$

for a certain continuous functions $\rho_i : \Gamma \times \mathcal{X}_p \sim \mathcal{B}_\delta \to \mathbb{R}$ of γ and x_p and matrices $\tilde{C}_i$, $i = 1,...,l$.

Now, we have all the ingredients to design a LPV gain $L_{\hat{\rho}}$ defined as follows

$$L_{\hat{\rho}} = \sum_{i=1}^{l} \hat{\rho}_i(\gamma, x)L_i \tag{15}$$

where the gains L_i, $i = 1,...,l$ are properly chosen to stabilize the observer with the estimation error subject to

$$e(t+1) = N_{\hat{\rho}(t)}e(t) + F_{\hat{\rho}(t)}w(t) \tag{16}$$

with

$$N_\rho := \big(\bar{A} - L_\rho\bar{C}_\rho\big)$$

$$F_\rho := \begin{bmatrix}\bar{E} & \bar{F} & T_\rho\bar{D} & -L_\rho\end{bmatrix}, \quad w(t) := \begin{bmatrix} v(t) \\ \Delta b(t) \\ \Delta\gamma(t) \\ d(t)\end{bmatrix} \tag{17}$$

In addition, those gains have to guarantee that for each estimate $\hat{\gamma}, \hat{x}_p$, $\tilde{C}_\rho$ lies in the convex hull $\mathrm{Co}\{\tilde{C}_i\}, i = 1,...,l$. Moreover it is assumed that the signal $w(t)$ belong to ℓ_2 space with a known bound $\bar{w}$ on its ℓ_2-norm, i.e.

$$\|w(\cdot)\| \leq \bar{w} \tag{18}$$

Then the problem just stated translates into finding a parameter-dependent gain such that difference equation (16) is stable for any arbitrary time variation of the parameters $\hat{\rho}(t) \in \mathcal{S}_l$ and such that, for any input $w(t)$ satisfying (18), the error $e(t)$ is bounded as

$$||e(\cdot)|| < \sigma||w(\cdot)||, \tag{19}$$

A convex optimization methodology to solve the above stated design problem is provided in the next Theorem 1.

Theorem 1. *Assume a symmetric strictly positive definite matrices P_i and matrices G_i and Y_i, $i = 1,...,l$ exist such that the optimization problem*

$$\min_{P_i,G_i,Y_i,\mu} \mu$$

subject to

$$\begin{gathered} \Xi_{ij} := \begin{bmatrix} G_i + G_i^T - P_j & Q_{12} & G_iF_i \\ \star & P_i - I & 0 \\ \star & \star & \mu I \end{bmatrix} > 0, \mu < \tfrac{\delta^2}{\bar{w}^2} \\ i = 1,...,l,\ j = 1,...,l \\ Q_{12} := G_i\tilde{T}_i\tilde{A} - Y_i\tilde{C}_i \end{gathered} \tag{20}$$

$$\Xi_{ijk} := \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ * & P_i + P_k - I & 0 \\ * & * & \mu I \end{bmatrix} > 0$$
$$i = 1,\dots,l-1,\, j = 1,\dots,l, k = i+1,\dots,l$$
$$R_{11} := G_i + G_i^T + G_k + G_k^T + P_j \tag{21}$$
$$R_{12} := G_i \tilde{T}_k \bar{A} + G_k \tilde{T}_i \bar{A} - Y_i \tilde{C}_k - Y_k \tilde{C}_i$$
$$R_{13} := G_i F_k + G_k F_i$$

has a solution. Then, the convergence of the observer estimation error dynamically characterized by equation (16) is ensured, the guaranteed $\mathcal{H}_\infty$ performance gain (19) *given by*

$$\sigma = \sqrt{\mu^\star} < \frac{\delta}{\bar{w}}, \mu^\star = \min \mu \tag{22}$$

is achieved and $\hat{\rho}(t)$ is ensured to vary into $\mathcal{S}_l$. Moreover, the observer gain vertices defined in (15) *are given by*

$$L_i = G_i^{-1} Y_i, i = 1,\dots,l \tag{23}$$

and stabilize the observer for any arbitrary time variation of the parameter $\hat{\rho}(t)$ in the polytope $\mathcal{S}_l$.

Proof: Consider the parameter-dependent Lyapunov function

$$V\big(e(t)\big) = e^T(t) P_{\hat{\rho}(t)} e(t) \tag{24}$$

with

$$P_{\hat{\rho}(t)} = \sum_{i=1}^{l} \hat{\rho}_i(t) P_i, P_i = P_i^T, i = 1,\dots,l \tag{25}$$

The related one-step ahead evolution of the Lyapunov function on the observer error trajectory is given by

$$V\big(e(t+1)\big) = e^T(t+1) P_{\hat{\rho}(t+1)} e(t+1) \tag{26}$$

where $P_{\hat{\rho}(t+1)}$ can be written as

$$P_{\rho(t)} = \sum_{j=1}^{l} \rho_j(t) P_j, P_j = P_j^T, j = 1,\dots,l \tag{27}$$

Using (27), one can recast (26) into

$$V\big(e(t+1)\big) = \Big(N_{\hat{\rho}(t)} e(t) + F_{\hat{\rho}(t)} w(t)\Big)^T P_{\rho(t)} \Big(N_{\hat{\rho}(t)} e(t) + F_{\hat{\rho}(t)} w(t)\Big) \tag{28}$$

Then, the Lyapunov function increment derived by (24) and (28) results to be given by

$$\begin{aligned} \Delta V(e(t)) &= V(e(t+1)) - V(e(t)) \\ &= e^T(t) \Big(N_{\hat{\rho}(t)}^T P_{\rho(t)} N_{\hat{\rho}(t)} - P_{\hat{\rho}(t)} \Big) e(t) \\ &\quad + 2 e^T(t) N_{\hat{\rho}(t)}^T P_{\rho(t)} F_{\hat{\rho}(t)} w(t) \\ &\quad + w^T(t) F_{\hat{\rho}(t)}^T P_{\rho(t)} F_{\hat{\rho}(t)} w(t) \end{aligned} \tag{29}$$

It is well-known that the stability of system with $\mathcal{H}_\infty$ guaranteed performance (19) is ensured if

$$\Delta V(e(t)) < -e^T(t) e(t) + \mu w^T(t) w(t),\ \forall t \tag{30}$$

By replacing $\Delta V(e(t))$ with the expression (29), one is able to rewrite (30) as $\Gamma^T(t) U \Gamma(t) < 0$ with $\Gamma(t) := \begin{bmatrix} e^T(t) & w^T(t) \end{bmatrix}^T$ and

$$U := \begin{bmatrix} U_{11} & U_{12} \\ \star & U_{22} \end{bmatrix} \tag{31}$$
$$U_{11} := N_{\hat{\rho}(t)}^T P_{\rho(t)} N_{\hat{\rho}(t)} - P(\rho(t)) + I$$
$$U_{12} := N_{\hat{\rho}(t)}^T P_{\rho(t)} F_{\hat{\rho}(t)},\ U_{22} := F_{\hat{\rho}(t)}^T P_{\rho(t)} F_{\hat{\rho}(t)} - \mu I$$

Clearly, by imposing $U < 0$ it is possible to guarantee (30) for all $e(t) \neq 0$ and $w(t) \neq 0$ by satisfying the following inequality

$$\begin{bmatrix} N_{\hat{\rho}(t)}^T P_{\rho(t)} \\ F_{\hat{\rho}(t)}^T(\rho(t)) P_{\rho(t)} \end{bmatrix} P_{\rho(t)}^{-1} \begin{bmatrix} P_{\rho(t)} N_{\hat{\rho}(t)} & P_{\rho(t)} F_{\hat{\rho}(t)} \end{bmatrix} - \begin{bmatrix} P_{\hat{\rho}(t)} - I & 0 \\ 0 & \mu I \end{bmatrix} < 0 \tag{32}$$

The latter, thanks to the use of a Schur's complement argument, is equivalent to

$$U' := \begin{bmatrix} P_{\rho(t)} & P_{\rho(t)} N_{\hat{\rho}(t)} & P_{\rho(t)} F_{\hat{\rho}(t)} \\ \star & P_{\hat{\rho}(t)} - I & 0 \\ \star & \star & \mu I \end{bmatrix} > 0 \tag{33}$$

that can be recast into

$$M U' M^T > 0 \text{ with } M := \begin{bmatrix} G_{\hat{\rho}(t)} P_{\rho(t)}^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{34}$$

and, in turn, into

$$\begin{bmatrix} G_{\hat{\rho}(t)} P_{\rho(t)}^{-1} G_{\hat{\rho}(t)}^T & G_{\hat{\rho}(t)} N_{\hat{\rho}(t)} & G_{\hat{\rho}(t)} F_{\hat{\rho}(t)} \\ \star & P_{\hat{\rho}(t)} - I & 0 \\ \star & \star & \mu I \end{bmatrix} > 0 \tag{35}$$

Using previously defined matrices and considering that $\rho \in \mathcal{S}_l$ and $\hat{\rho} \in \mathcal{S}_l$, inequality (35) can be written as

$$\sum_{i=1}^{l} \hat{\rho}_i^2(t) \sum_{j=1}^{l} \rho_j(t) \Xi_{ij} + \sum_{i=1}^{l-1} \sum_{k=i+1}^{l} \hat{\rho}_i(t) \hat{\rho}_k(t) \sum_{j=1}^{l} \rho_j(t) \Xi_{ijk} > 0 \tag{36}$$

with Ξ_{ij} defined in (20) and Ξ_{ijk} defined in (21). For more details please refer to (Heemels et al., 2010). □

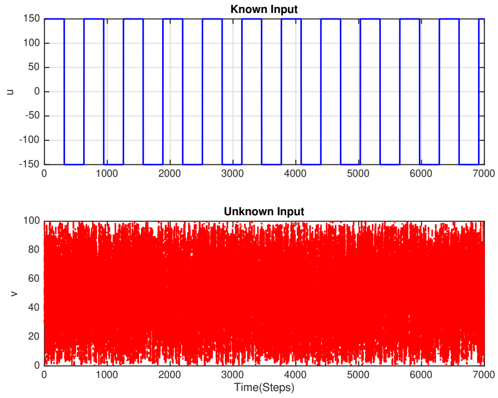

Figure 2: Known Input (up) and Unknown Input (down).

6 ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of the proposed **UIO-SR** scheme is investigated by considering the following linear stable model

$$\begin{aligned} x_p(t+1) &= Ax_p(t) + Bu(t) + Ev(t) \\ y(t) &= \Delta(\gamma(t))C_y x_p(t) + Fb(t) \end{aligned} \tag{37}$$

where

$$A = \begin{bmatrix} 0.98806 & 0.0096049 \\ -0.32754 & 0.93033 \end{bmatrix}, B = \begin{bmatrix} -0.0001 \\ -0.0921 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, E = 0.01 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

with γ assumed to be confined within the polytope $\Gamma := \left\{\gamma : [\underline{\gamma}_1, \underline{\gamma}_2, \underline{\gamma}_3]^T \leq \gamma \leq [\bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3]^T\right\}, \underline{\gamma}_1 = \underline{\gamma}_2 = 0, \underline{\gamma}_3 = 0.1, \bar{\gamma}_i = 1, i = 1,2,3.$

The goal of this simulation is to verify the capability of the proposed method to correctly estimate the first component of $x_p(t)$ in (37) under faults that is used as *virtual output* $z(t) = H_z C_y x_p(t)$ for the system. Correspondingly, the following sensor reconciliation matrix

$$H_z = \begin{bmatrix} 0.5 & 0.5 & 0 \end{bmatrix} \tag{38}$$

results with the known input $u(t)$ depicted in Figure 2 (up) and the unknown input $v(t)$ supposed to be a white noise with standard deviations equal to 10 (depicted in Figure 2 (down)). Moreover, the bias profile on the three available physical sensors is assumed to change along the simulation time according to the following profile

$$b(t) = 1 + 100 sind(0.3t) \tag{39}$$

and faults on the matrix effectiveness gain will affect the first two sensors as depicted in Figure 3.

In this scenario, without any sensor reconciliator block the *virtual output* would result falsified, as depicted in Figure 5 (blue dashed line), because of faults occurrences on the physical sensors.

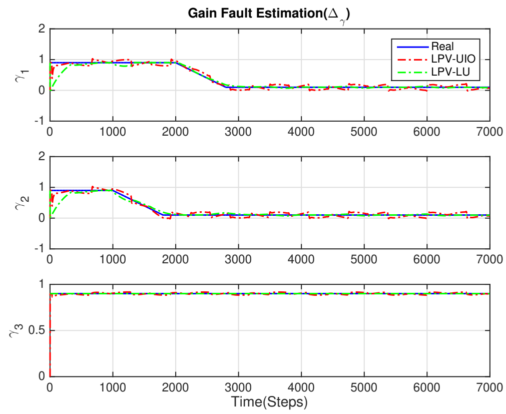

Figure 3: Loss of effectiveness parameters.

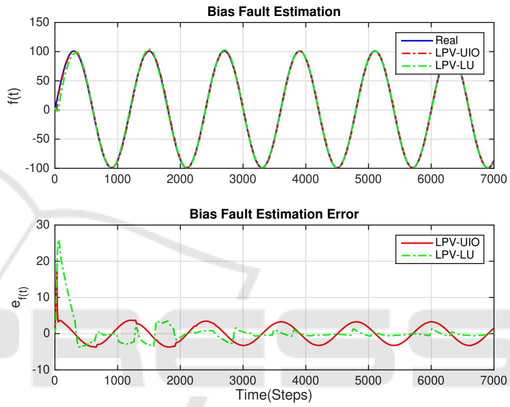

Figure 4: Bias fault and its estimation.

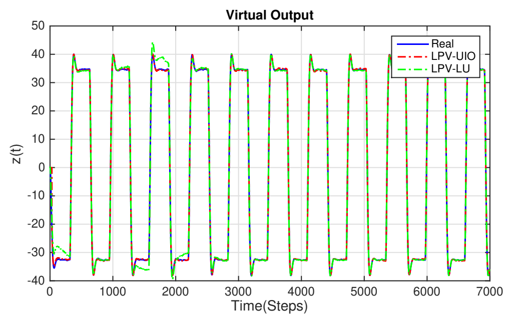

Figure 5: Virtual Output and its estimation.

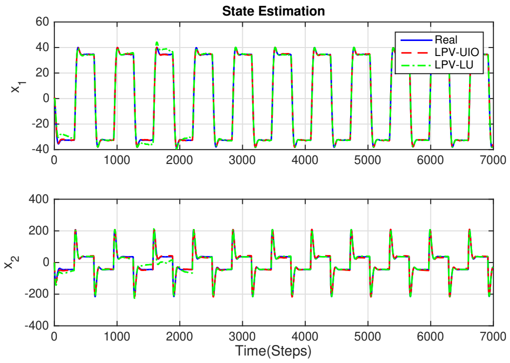

Figure 6: Loss of effectiveness parameters.

By defining the auxiliary state

$$x(t) := \begin{bmatrix} x_p(t) & b(t) & \gamma_1(t) & \gamma_2(t) & \gamma_3(t) \end{bmatrix}^T$$

the augmented model (9) is derived where

$$\bar{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_{3\times3} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0_{1\times1} \\ 0_{3\times1} \end{bmatrix}, \bar{E} = \begin{bmatrix} E \\ 0_{1\times1} \\ 0_{3\times1} \end{bmatrix},$$

$$\bar{F} = \begin{bmatrix} 0_{2\times1} \\ 1 \\ 0_{3\times1} \end{bmatrix}$$

$$\bar{C}_{\{\gamma(t),x_p(t)\}} = \tag{40}$$

$$\begin{bmatrix} \gamma_1(t) & 0 & 1 & 0 & 0 & 0 \\ \gamma_2(t) & 0 & 1 & 0 & 0 & 0 \\ \gamma_3(t) & \gamma_3(t) & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & x_{p_1}(t) & 0 & 0 \\ 0 & 0 & 1 & 0 & x_{p_2}(t) & 0 \\ 0 & 0 & 1 & 0 & 0 & x_{p_1}(t)+x_{p_2}(t) \end{bmatrix}$$

$$\bar{D} = \begin{bmatrix} 0_{2\times3} \\ 0_{1\times3} \\ I_{3\times3} \end{bmatrix} \tag{41}$$

In order to exploit a polytopic Luenberger observer for (9), an embedding polytopic representation of the form (14) has been derived. To this end, the matrix $\bar{C}_{\gamma,x}$ can be embedded in $P_C := Co\{C_1,\ldots,C_{32}\}$ with related vertices computed by evaluating matrix $\bar{C}_{\gamma,x}$ on the extremum points of Γ and x_p. Then, a suitable polytopic representation (14) can be achieved by first deriving the LPV scheduling parameter $\rho(p)$, that in this example is composed by 32 components, each one having the following structure

$$\rho_1 := \prod_{i,j}(1-p_i^1)(1-p_j^2)$$

$$\vdots$$

$$\rho_{32} := \prod_{i,j} p_i^1 p_j^2 \tag{42}$$

where the measurable parameters p_i^1 and p_j^2 obtained by normalizing and centering the physical signals $\gamma(t)$ and $x_p(t)$

$$p_i^1(t) = \frac{\gamma_i(t) - \underline{\gamma}_i}{\bar{\gamma}_i - \underline{\gamma}_i}, \quad i = 1,..,3$$

$$p_j^2(t) = \frac{x_{p_j}(t) - \underline{x}_{p_j}}{\bar{x}_{p_j} - \underline{x}_{p_j}}, \quad j = 1,2 \tag{43}$$

Finally, it is possible to get a polytopic embedding approximation for $\bar{C}_{\gamma,x}$ as follows

$$\bar{C}_\rho = \sum_{i=1}^{l} \rho_i(p) C_i \tag{44}$$

A simulative comparison has been attempted between the presented **LPV-LU-SR** scheme and the Sensor Reconciliating approach of (Behzad et al., 2016), here referred to as **LPV-UIO-SR** and endowed with a LPV polytopic observer. In order to compute the observer's gain, the same embedding polytopic representation used in (Behzad et al., 2016) has been here considered for the matrix $\bar{C}_\rho$, that consists in a polytope characterized by 64 vertices.

In Figures 3-6 these schemes have been compared. Although both observers achieve good performance in getting an estimate of $x_p^{(1)}$, as expected, the **LPV-LU-SR** scheme exhibits a slight better behavior with respect to **LPV-UIO-SR**, both in estimating the state and the bias. This is mostly due to the fact the observer exploits the knowledge on the input $v(t)$ supposed unknown in the case of the **LPV-UIO-SR** scheme. Such an aspect translates in a better effectiveness parameter (gain matrix) estimation (Figure 4) and in a more accurate *virtual output* generation (Figure 6).

However, beyond the numerical results, it is worth discussing some practical aspects of the considered strategies. Unlikely the **LPV-UIO-SR**, the **LPV-LU-SR** does not require any persistent excitation of the state estimate that is needed to ensure parameter estimation convergence. Unfortunately, such an expedient is not enough in general to guarantee convergence on the state estimation. On the other hand, the **LPV-UIO-SR** presents two advantages with respect to **LPV-LU-SR**: it does not require neither the input knowledge nor any pre-specified bounds on the state x_p.

7 CONCLUSIONS

In this paper, LPV Luenberger observers have been proposed to solve fault-tolerant sensor reconciliation design problems for linear discrete-time systems subject to possible faults on sensor gain and bias. The role of the observer relies on the estimation of both the state of the system and the current fault of the physical sensors. The resulting design procedure is quite simple and guarantees bounded errors on the estimation of both the plant state and fault parameters. The scheme has been fully described, its properties rigorously proved and, in the final simulation example, it has been shown to achieve good performance in recovering useful data from the pool of redundant sensors.

REFERENCES

Bara, G. I., Daafouz, J., Kratz, F., and Ragot, J. (2001). "Parameter-dependent state observer design for affine LPV systems". *International Journal of Control*, 74(16):1601–1611.

Behzad, H., Casavola, A., Tedesco, F., and Sadrnia, M. A. (2016). "A fault-tolerant sensor reconciliation scheme based on LPV Unknown Input Observers". In *IEEE 55th Conference on Decision and Control (CDC), 2016*, Las Vegas, NV. IEEE.

Heemels, W. M. H., Daafouz, J., and Millerioux, G. (2010). "Observer-based control of discrete-time LPV systems with uncertain parameters". *IEEE Trans. on Aut. Contr.*, 55(9):2130–2135.

Lunze, J. and Richter, J. H. (2008). "Reconfigurable fault-tolerant control: a tutorial introduction". *European Journal of Control*, 14(5):359–386.

Merrill, W. C., DeLaat, J. C., and Bruton, W. M. (1988). "Advanced detection, isolation, and accommodation of sensor failures-real-time evaluation". *Journal of Guidance, Control, and Dynamics*, 11(6):517–526.

Ponsart, J.-C., Theilliol, D., and Aubrun, C. (2010). "Virtual sensors design for active fault tolerant control system applied to a winding machine". *Control Engineering Practice*, 18(9):1037–1044.

Samara, P. A., Fouskitakis, G. N., Sakellariou, J. S., and Fassois, S. D. (2008). "A statistical method for the detection of sensor abrupt faults in aircraft control systems". *IEEE Transactions on Control Systems Technology*, 16(4):789–798.

Steffen, T. (2005). "Reconfiguration using a virtual sensor". In *Control Reconfiguration of Dynamical Systems*, pages 69–79. Springer.

Zhou, M., Shen, Y., and Wang, Q. (2013). "Robust UIO-based fault estimation for sampled-data systems: An LMI approach". In *IEEE International Conference on Information and Automation (ICIA), 2013*, pages 1308–1313, Yinchuan, China.