Dynamic Analysis of the Fractional PID Controller

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Abstract: This article presents as main objective the study and application of the fractional PID controller in a positioning system, a controller that has basis on the fractional calculus theory originated in 1695 and, despite having generated several paradoxes in the decade, nowadays there are important applications of this theory, as the one reported in this paper. Initially, the controller will be designed by means of computational simulation for the nominal model of a plant, using a program in Matlab and optimization algorithms and, then, applying in a real process using a data acquisition technique in order to analyse its dynamic behavior in the presence of real external disturbances. Given that the fractional PID is a generalization of the traditional PID, the goal is to obtain, in practice, the benefits of this one in relation to the another, mainly observing the requirements of robustness and stability that must be present in the system.

1 INTRODUCTION

The great industrial growth in recent years, along with the technological advance, produced constant changes in society as a whole. Increasingly the industrial processes become independent of human being and industrial automation reflects this process, for example, in the use of the robotic arm, which allows, when properly controlled, to perform welding, painting, displacement of objects, among other applications, automatically.

The PID controller (Proportional + Integrative + Derivative), widely used in industry, is one of the most traditional controllers in control theory and there are several methods for obtaining it (Dorf and Bishop, 2009). Result of the combination of three basic controllers, in other words, the combination of proportional, integrative and derivative controllers, its efficiency on making a response of a system display predetermined characteristics motivated studies that provided several methods for tuning this type of controller (Ogata, 1998).

The following transfer function (Dorf and Bishop, 2009) describes that controller which, in this paper, will be denominated as traditional PID (or classical PID, as it is also known in academic publications):

$$PID_{trad}(s) = K_p + \frac{K_i}{s} + K_d s, \qquad (1)$$

where K_p, K_i and K_d are, respectively, proportional,

integrative and derivative gains of the controller.

Over the years, a new use possibility for the traditional PID controller has been noticed, which is know as fractional PID. As it is a generalization of the traditional PID, it promises to be a model closer to reality, and provide a more refined control system.

Herewith, it can be thought about implementing the fractional PID in several industrial applications, being able to replace the so called traditional PID. As an example, in (Tepljakov et al., 2011) the authors comment on the focus given to fractional calculus in the last years, applied to control systems design due to more precise modeling and control enhancement possibilities. Already in (Tavazoei, 2012) the author highlights the use of fractional order dynamics to obtain more realistic models for real world phenomena and physical processes such as thermal systems and polarization phenomena. Other practical applications are also mentioned, such as suppression of chaotic oscillations in electrical circuits and compensating disturbances on the position and velocity servo systems.

In more recent studies, as in (Sandhya et al., 2016), examples of the best fractional controller that can be designed are given, arguing that it overcomes the best integer order controller even though it is verified that this latter works comparatively well. One of these examples considers a fractional controller for an integer order plant (DC motor with elastic shaft - a model from Mathworks, 2006), and the optimiza-

tion algorithms used are ITAE (Integral of Time Multiplied by Absolute Error) and ISE (Integral of the Square of the Error). The author emphasizes the use of software and hardware for the efficient implementation of these controllers in industrial and robotic applications. Aspects of the implementation of these systems are also evaluated in robotic arms, showing that the robustness - for variable loads of the object and small disturbances at the reference - is present in the system.

In the paper (Binazadeh and Yousefi, 2017) the authors consider a cascade control structure with fractional controllers slave (internal loop) and master (of external links) and computer simulations exhibits the good performance of the proposed project.

More recent articles as seen in (Morsali et al., 2017), (Khubalkar et al., 2016a) and (Khubalkar et al., 2016b), further reinforce the validity of the Fractional PID controller study.

Finally, the objective of this research is to evaluate if the fractional PID can be really more useful for a control system concerning to robustness, stability and limitation of the control effort. It is organized as follows: in section 2 a fractional calculus idea is provided; the methodology adopted in this work is presented in section 3; in section 4 the controllers are designed and applied in the mathematical model of the plant; same controllers applied in the real plant in presence of disturbances and the results are presented in section 5. Finally, the conclusions are presented in section 6.

2 AN IDEA OF FRACTIONAL CALCULUS

The fractional PID has its fundamental basis embedded in the theory of Fractional Calculus (Camargo, 2009). This theory arose in 1695, and from there numerous studies were done to contribute to the development of the fractional calculus, highlighting, Abel and Liouville, who were the first to find an application for this theory. However, among the various definitions for the fractional order differential and the fractional order integral, the following definitions stand out:

- Grünwald-Letnikov definition (GL):

$$\mathcal{D}_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[\frac{t-\alpha}{h}]} (-1)^j \binom{\alpha}{j} f(t - jh). \quad (2)$$

- Riemann-Liouville definition (RL):

$$\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt} \right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} \, d\tau, \quad (3)$$

for $m - 1 < \alpha < m$, $m \in \mathbb{N}$, where $\Gamma(\cdot)$ is Euler's gamma function.

Therefore, based on the fractional calculus theory, the denominated fractional PID controller arises, in which the derivative and integrative terms have fractional orders. It has the following transfer function (Tavazoei, 2012):

$$PID_{frac}(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\beta, \quad (4)$$

where λ and β are arbitrary constants, positive and less than 1.

As can be seen, while in traditional PID the aim is to find the optimal K_p, K_i and K_d gains, in fractional PID there are five parameters to be adjusted: K_p, K_i, K_d, λ and β. This, obviously, might allow a more refined tuning of the PID so that the system produces a dynamic response as expected. However, in relation to the traditional PID, it is worth noting that if $\lambda = \beta = 1$, the fractional PID becomes equal to the traditional, and, therefore, the traditional PID is a particular case of the fractional PID.

However, among all the existing studies about fractional PID, some questions are still present: can the fractional PID produce robustness to the system, concerning to the rejection of external disturbances?; the tuning of all fractional PID parameters based on a single optimization method is more efficient than merging optimization methods?; the fractional PID applied on a real plant, with all its nonlinearities, will maintain the same performance presented for a nominal plant? These questions will be answered during this work.

3 METHODOLOGY

The purpose here is to control a position of a servo system using the traditional PID controller and then using a fractional PID.

Initially, both controllers were tuned using the ITAE performance index. ITAE is the most employed index available as the minimum value of the integral is easily discernible when the system parameters are varied. As the absolute error is time weighted, this criteria has good applicability when a reduction of the contribution of large errors are necessary and a greater emphasis is placed on errors that occur later in the transient response of the system.

The expression that describes this index is (Dorf and Bishop, 2009):

$$ITAE = \int_0^T t \, |e(t)| \, dt, \quad (5)$$

where, typically, the upper limit T of the integral is defined as the settling time T_S, which is the time required for the response reach and remain within specified limits (usually 2% to 5% of the final value). Therefore, it is a sufficiently large time span that includes transient and steady state. Thus, a system is considered optimal when the chosen performance index reaches an extreme minimum value. In other words, the optimal system to be developed is one that minimizes this index (Dorf and Bishop, 2009).

In a second case, specifically for the fractional PID, the gains were tuned using the Linear Quadratic Regulator method (LQR) (MUKHOPADHYAY, 1978) and the parameters that define the order of the derivative and integrative terms were defined by ITAE. Which means the tuning via Linear Quadratic Regulator will be used for optimizing the parameters K_P, K_i, K_d of the controller to be designed. A comparison between this method and ITAE becomes relevant since a more suitable tuning can be obtained and, consequently, achieving optimal systems.

The method presents precepts of the Modern Control theory as concepts of state space modeling, with detriment of the transfer function model complexities, to develop an equivalent solution to find the gains K_P, K_i, K_d of a system, with some advantages that are properly discussed in (MUKHOPADHYAY, 1978).

The FOMCON toolbox (Tepljakov et al., 2011) has an optimization tool (which uses an optimize function) to tune a controller by minimizing the function given by a predetermined performance index. The FOMCON was idealized as a reflection of the growth in research and development of the fractional controller due to the new possibilities generated through the modeling of this type of system. The tool is simple to manipulate, provides a graphical user interface and several resources for system analysis, which allows fast practical results to be generated (Tepljakov et al., 2011).

In all these cases simulations were made for the mathematical model of the servo system. Finally, these same controllers were applied to the real system through the data acquisition technique, which allows the communication between Matlab and a real plant.

Comparisons with the traditional PID will be made and a robustness analysis of the fractional PID will be performed.

The real system used here is a servo system (a servo positioner) shown in Figure 1.

The nominal transfer function of the servo positioner, obtained by experimental analysis is:

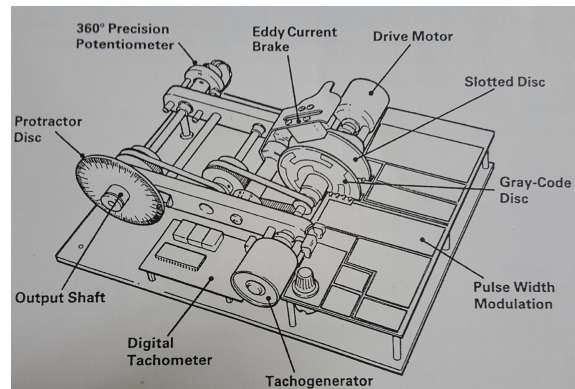$$H(s)_{servo\ system} = \frac{40}{s^2 + 4s}. \qquad (6)$$

Figure 1: Real plant - Servo positioner.

Some components of the real plant are presented as follows:

- DC Motor: It is a direct current motor that can reach up to 2500 rpm;

- Tachogenerator (or Tachometer): It is an electromagnetic device that, when rotated, generates an output voltage proportional to the speed of its axis. This property will be used to obtain the speed feedback of the control system (LJ Technical Publications Department, 2016);

- Magnetic brake: It consists of a permanent magnet attached to a pivoted rod that allows the introduction of a magnetic brake when placed in front of the aluminum disc. The magnetic brake insertion rod has three positions (0,1 and 2), and the load intensity can be changed according to this position selection (LJ Technical Publications Department, 2016).

Some nonlinearities are inherent to this plant, such as dead zone of the motor and backlash on gears, which can cause the so-called backlash effect, besides noise. Therefore, it can be noticed that the servo system used here is a set that meets the intentions of this work and can enable the achievement of relevant practical results.

4 DESIGN AND SIMULATION

The five parameters of the fractional PID controller, as well as the three parameters of the traditional PID controller, were tuned using the FOMCON toolbox.

First, K_P, K_i and K_d will be defined by identifying an entire order model for the plant, and a suitable tuning method.

Then, the parameters λ and β are tuned using the optimization tool. A performance index was defined

(in this case, the ITAE, which considers the absolute error over time).

A block diagram was developed in Simulink for the traditional and fractional PID, both applied to the same plant. These diagrams are given in Figure 2.
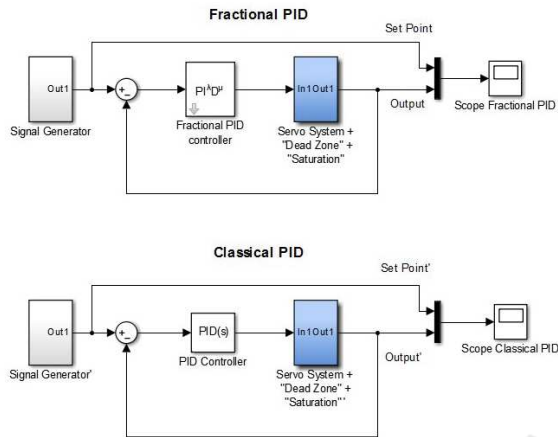

Figure 2: Block diagrams in Simulink.

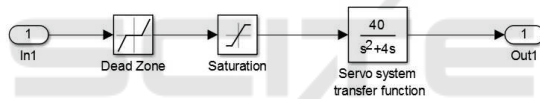Figure 3 shows the contents of the servo system block.

Figure 3: Contents of the block which represents the servo system shown in Figure 2.

Firstly, the traditional PID was tuned via ITAE performance index. Then, the fractional PID parameters were obtained via toolbox FOMCON, fixing K_p, K_i and K_d, previously found in the traditional PID tuning, and optimizing λ and β via ITAE. The parameters were shown in Table 1.

Table 1: Controller parameters for ITAE tuning.

	Fractional PID	Traditional PID
K_p	5,4115	5,4115
K_i	2,0841	2,0841
K_d	5,2487	5,2487
λ	0,15918	-
β	0,67146	-

The system responses using the fractional PID tuned and also the traditional PID are presented in figures 4 and 5 respectively.

As can be seen, the tuning of the fractional PID provided a faster response without overshoot.

One more tuning of coefficients was performed in which K_P, K_i and K_d are adjusted by the Linear Qua-
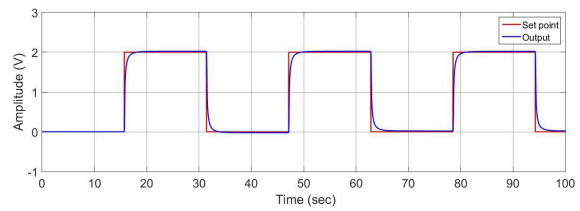
Figure 4: Response for the fractional PID when K_p=5,4115; K_i=2,0841; K_d=5,2487; λ=0,15918; β=0,67146.
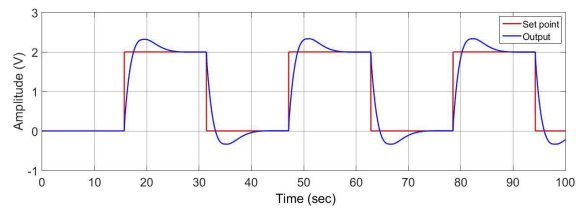

Figure 5: Response for the traditional PID when K_p=5,4115; K_i=2,0841; K_d=5,2487.

dratic Regulator (obtained through a program developed in Matlab) and λ and β are, then, optimized through the toolbox FOMCON based on the ITAE performance index (fixing K_P, K_i and K_d found previously in the LQR tuning via Matlab and allowing the toolbox to optimize λ and β). This procedure has as objective to analyze if the composition of two adjustment methods is efficient for tuning the fractional PID controller. The parameters obtained are shown in Table 2.

Table 2: Controller parameters for Linear Quadratic Regulator tuning (traditional PID) and for Linear Quadratic Regulator and ITAE tuning (fractional PID).

	Fractional PID	Traditional PID
K_p	1,2551	1,2551
K_i	1	1
K_d	0,1877	0,1877
λ	0,36832	-
β	0,89995	-

The system responses using the fractional PID tuning and also the traditional PID are shown in figures 6 and 7 respectively.
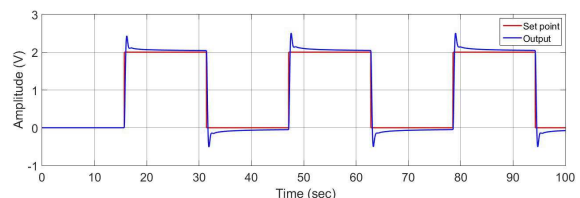

Figure 6: Response for the fractional PID when K_p=1,2551; K_i=1; K_d=0,1877; λ=0,36832; β=0,89995.

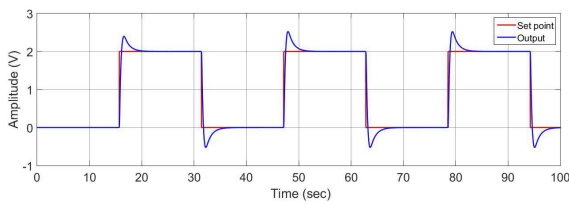Figure 7: Response for the traditional PID when K_p=1,2551; K_i=1; K_d=0,1877.

Both simulations are very similar when observing the overshoot, but the fractional PID give a more faster accommodation time response. On the other hand, the fractional PID controller presents an stationary error as can be seen in Figure 6, not observed in the traditional PID simulation (Figure 7).

In the next session, the same controllers will be applied in the real plant in presence of nonlinearities and disturbances.

5 APPLICATION OF FRACTIONAL AND TRADITIONAL PID IN REAL PLANT MODEL

After the controllers are tuned and simulated by means of computational analysis, it is necessary to verify their functionalities in a real system, the servo positioner, which has several nonlinearities that can compromise the desired final behavior, such as the dry friction of the plant of the system, gap in the gears, dead zone of the motor and saturation of the power amplifier.

The servo system was powered and connected to the National Instruments data acquisition interface for obtaining data.

To establish the transmission and reception of data between the servo system and Matlab/Simulink, the National Instruments PCI 6221-37 pin data acquisition board was used. This board has 16 channels of analog inputs and 2 channels of analog outputs and through the CB-37FH connector, it is possible to use the inputs and outputs of the board to perform the communication between the PID contained in Matlab and the real plant.

A block diagram has been developed in Simulink to integrate virtual and real components. The analog input block represents the control output (the signal which comes from the servo positioner), connected with an adjustable transfer function filter to eliminate high frequency noise produced by the derivative action. The analog output block contains the signal that is inserted into the motor. Besides these compo-

nents, the traditional and fractional PID blocks were used to simulate the responses.

Therefore, the same procedure performed for the nominal model of the system was performed in the real model of the plant. Figure 8 shows the block diagram of the control system with the performance of the traditional PID tuned with ITAE.
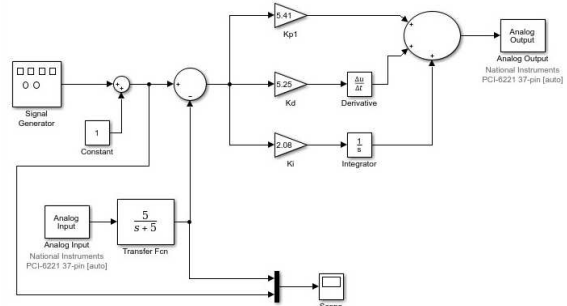

Figure 8: Block diagram in Simulink for the traditional PID.

Figures 9 and 11 presents the system response, using the traditional PID, without external disturbance and with external disturbance, respectively. Figures 10 and 12 presents, respectively, the control efforts of the controller without external disturbance and with disturbance. The intent is to verify the efficiency that the controller manages to maintain this control effort within an acceptable limit.
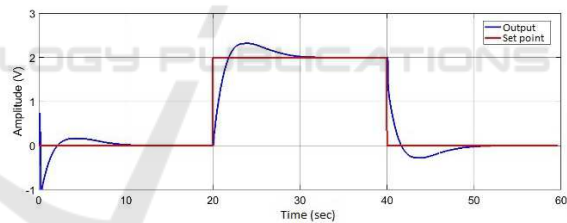

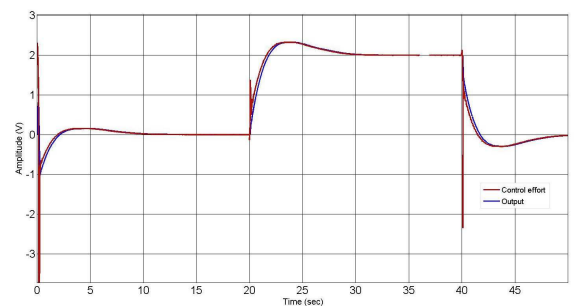Figure 9: Response for the traditional PID when K_p=5,4115; K_i=2,0841; K_d=5,2487.

Figure 10: Control effort for the traditional PID when K_p=5,4115; K_i=2,0841; K_d=5,2487.

The traditional PID tuned via ITAE resulted in a similar response with the one viewed previously in computational simulation (Figure 5). The controller
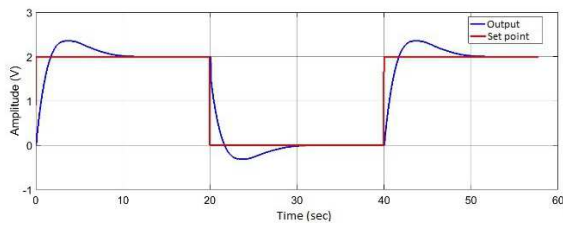
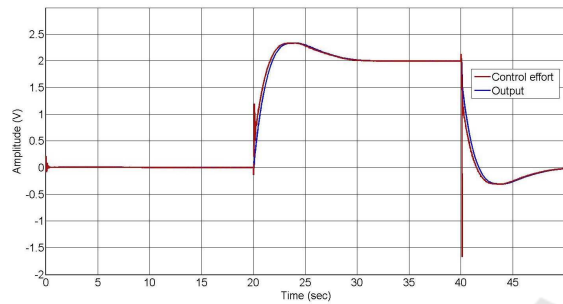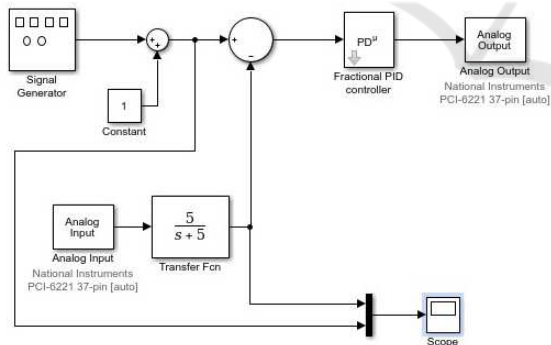Figure 11: Response for the traditional PID with disturbance when K_p=5,4115; K_i=2,0841; K_d=5,2487.

Figure 12: Control effort for the traditional PID with disturbance when K_p=5,4115; K_i=2,0841; K_d=5,2487.

managed very well in front of the disturbances, but when observing the control effort, the peaks may indicate a loss for the system as a great effort needs to be spent in a very small time.

Figure 13 shows the block diagram of the control system with the performance of the fractional PID tuned with ITAE.

Figure 13: Block diagram in Simulink for the fractional PID.

As a result, figure 14 show the system response for the fractional PID with K_P, K_i, K_d fixed by ITAE and λ and β optimized singly, also with ITAE.

In this case, it can be noted that the system, different from the computational results, has an unstable behavior.

As an alternative to this practical case, the fractional PID controller was tuned using the Linear Qua-

Figure 14: Response for the fractional PID with K_P, K_i, K_d fixed by ITAE and λ and β optimized singly, also with ITAE. The obtained parameters: K_P=5,4115; K_i=2,0841; K_d=5,2487;λ=0,15918; β=0,67146.

Figure 15: Response for the traditional PID when K_p=1,2551; K_i=1; K_d=0,1877.

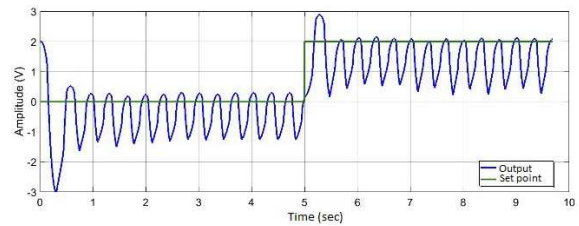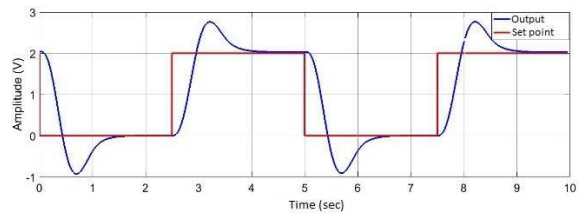dratic Regulator method (MUKHOPADHYAY, 1978) and the integrator and derivative orders tuned with ITAE performance index, as was done in section 4. But before, the traditional PID was also tuned by the Linear Quadratic Regulator method. Figures 15 and 17 presents, respectively, the responses using the traditional PID without and with disturbances and Figures 16 and 18 highlight the control effort present in the system.
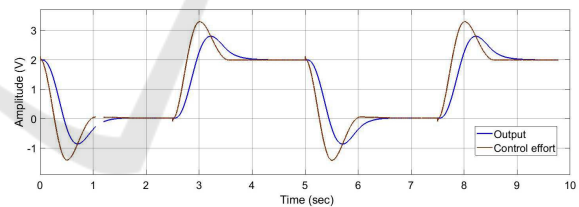

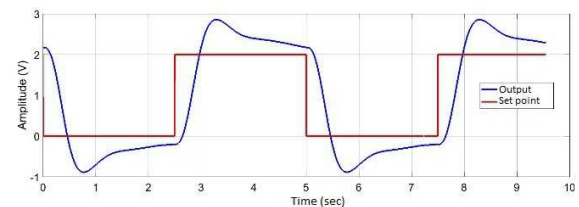Figure 16: Control effort for the traditional PID when K_p=1,2551; K_i=1; K_d=0,1877.

Figure 17: Response for the traditional PID with disturbance when K_p=1,2551; K_i=1; K_d=0,1877.

The Linear Quadratic Regulator method increased the overshoot for the traditional PID and, in face of the disturbance, also increased the settling time. It is required more from the controller when comparisons
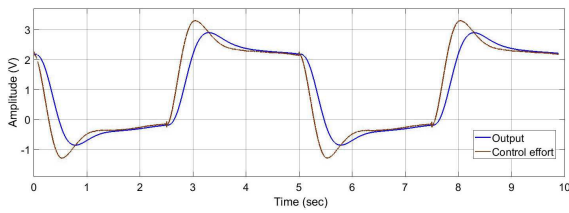
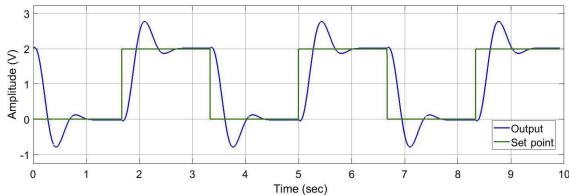Figure 18: Control effort for the traditional PID with disturbance when K_p=1,2551; K_i=1; K_d=0,1877.

Figure 19: Response for the fractional PID when K_p=1,2551; K_i=1; K_d=0,1877; λ=0,36832; β=0,89995.

of control effort are made with the tuning via ITAE.

Figures 19 and 21, respectively, presents the responses using fractional PID without and with disturbances, and Figures 20 and 22 presents the respective control efforts.
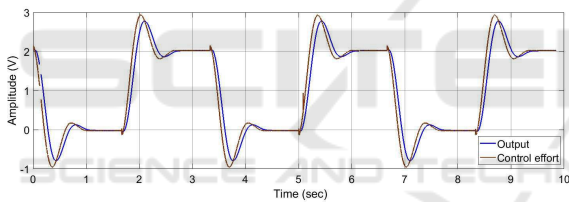

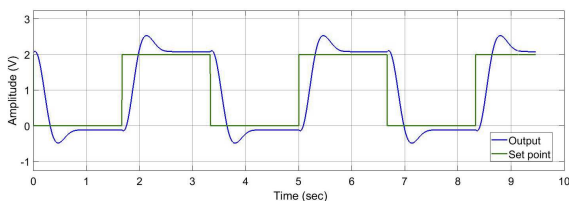Figure 20: Control effort for the fractional PID when K_p=1,2551; K_i=1; K_d=0,1877; λ=0,36832; β=0,89995.

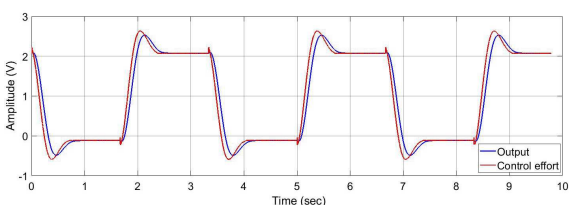Figure 21: Response for the fractional PID with disturbance when K_p=1,2551; K_i=1; K_d=0,1877; λ=0,36832; β=0,89995.

Figure 22: Control effort for the fractional PID with disturbance when K_p=1,2551; K_i=1; K_d=0,1877; λ=0,36832; β=0,89995.

The fractional PID clearly has obtained an efficient tuning with the Linear Quadratic Regulator Method in detriment of the ITAE tuning with gives an unstable behavior (Figure 14). The controller also manages well to maintain the control effort.

An fine adjustment for the fractional PID has performed in order to achieve the best controller that can be projected. The idea was to reduce K_i and to increment the K_d parameter (to reduce the excessive overshoot).The result is given in figures 23 and 24 which presents, respectively, the fine adjustment and the control effort associated.
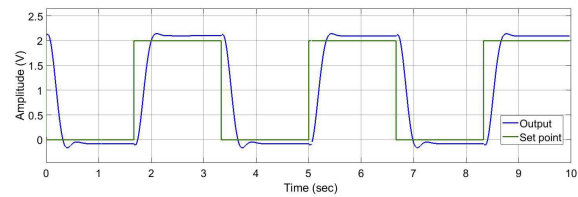

Figure 23: Fractional PID fine adjustment when K_p=1,2551; K_i=0,85; K_d=0,3377; λ=0,36832; β=0,89995.
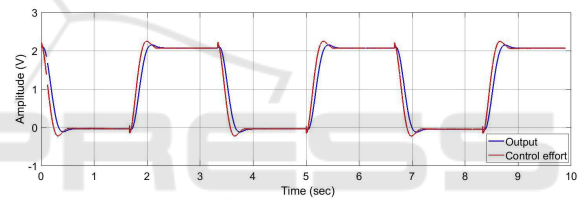

Figure 24: Control effort for the fractional PID fine adjustment when K_p=1,2551; K_i=0,85; K_d=0,3377; λ=0,36832; β=0,89995.

The potentialities of the fractional PID were shown as the fine adjustment generated a response without overshoot and a satisfactory control effort.

6 CONCLUSIONS

In this work we intended to tune the traditional and fractional PID in a positioning system. For the PID tuning, two methods were used: i) the ITAE performance index, ii) the Linear Quadratic Regulator. The first phase consisted in applying the PID in the nominal model of the plant and, in a second phase, the same controllers obtained were applied in the real plant with all its nonlinearities.

By means of the designed controllers, it is observed that the ITAE index is efficient to tune the traditional PID and also the fractional PID when applied to the mathematical model of the plant. In this sense, the superiority of the fractional PID efficiency is remarkable, since the system response has a shorter settling time and without overshoot. However, when

these controllers are applied to a real plant with its nonlinearities, the system controlled by the fractional PID starts to present an unstable response.

One solution for this problem was to tune fractional PID gains using Linear Quadratic Regulator method and adjust the derivative and integrative orders using the ITAE performance index. In the mathematical model of the plant, the responses of the system controlled by the fractional and traditional PID were very similar. However, when applied in the real plant, the fractional PID makes the system produce a stable and faster response when compared to the traditional PID, even in the presence of disturbances.

Therefore, when applying the controllers in the mathematical model and in the real plant, it could be observed that the fractional PID tends to have, in fact, a greater efficiency when compared with the traditional PID, with respect to the robustness of performance in the presence of disturbances, speed of response and reduction of overshoot. However, in a real model, it is necessary to take care in choosing the optimization method. The fractional PID proved to be more efficient in a real case when its gains were tuned using the Linear Quadratic Regulator and the integrator and derivative orders adjusted by the ITAE. Thus, the combination of two optimization methods for fractional PID tuning has proven to be a promising way to apply the controller to a real plant.

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