Recursive Identification of Continuous Time Variant Dynamical Systems with the Extended Kalman Filter and the Recursive Least Squares State-Variable Filter

Flávio Luiz Rossini[1], Guilherme Santos Martins[2], João Paulo Silva Gonçalves[2] and Mateus Giesbrecht[2]

[1]*Department of Electronic Engineering, Federal University of Technology - Paraná (UTFPR) Campo Mourão campus, Via Rosalina Maria dos Santos, 1233, Campo Mourão, PR, Brazil*

[2]*Department of Semiconductors, Instruments and Photonics, School of Electrical and Computer Engineering (FEEC), University of Campinas (UNICAMP), Av. Albert Einstein, 400, Campinas, SP, Brazil*

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Abstract: In this paper, a method for the continuous time varying dynamical systems identification is presented. The study is based on the integration of the State-Variable Filter (SVF), the Extended Kalman Filter (EKF) and the Recursive Least Squares State-Variable Filter (RLSSVF). The main contribution of the algorithm applied in this paper is that a state space continuous time model can be estimated based on the system sampled inputs and outputs. To validate the method, a continuous time varying benchmark system is simulated and the benchmark parameters are compared to the estimated model parameters. The benchmark outputs are also compared to the model outputs to verify the accuracy of the proposed method. The results obtained show that the model reproduces the benchmark behavior accurately.

1 INTRODUCTION

Real dynamical systems are subjected to uncertainties regarding their structure and the values of the parameters (Slotine et al., 1991). Thus, the analytical modelling of dynamical systems, also known as white box modelling, becomes inaccurate, since it is necessary to know the system completely, from the physical laws that describe it, to the physical properties of all the materials involved (Awrejcewicz, 2016).

To overcome those difficulties, system identification methods are proposed in the literature (Ljung, 1999) and (Katayama, 2006). The main advantage of those techniques is that they require little or no previous knowledge about the system.

The system identification consists in finding a model structure and a set of parameters in order to minimize the error between the model and the real system outputs, given that both are fed by the same input. In this article it is supposed that the model structure is known a priori in the state space continuous form, and the parameters are to be determined.

Once a model structure is known or proposed, the parameters can be estimated either by an offline approach or by an online approach. The offline approach can be executed by block or batch, that is, the entire data set must be available to execute the estimation algorithm. The offline estimation can also be applied recursively, according to the convenience and the algorithm suitability. Subspace methods for system identification are examples of offline algorithms capable of estimation perform, however there is no direct correlation of the estimated and true parameters (Katayama, 2006). Therefore, recursive algorithms, such as the optimal refined instrumental variable method (Padilla et al., 2016), can be executed offline to provide refined recursive estimates and block estimates for the model parameters (Young, 2011). The recursive identification algorithms are also the ones recommended for online applications. In this work the focus is on the recursive system identification application.

A large number of researches are presented regarding recursive time domain identification methods for discrete time systems. A frequently used recursive approach to estimate the discrete time varying system parameters is the Recursive Least Squares with Forgetting Factor Algorithm. The mentioned algorithm

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consists in finding the parameters that minimize the prediction error weighting the past data in order to reduce its influence on the present model parameters (Young, 2011). This technique can be applied to continuous systems which have been discretized, and the models outputs accurately represent the outputs sampled from the real system.

An alternative method that allows the recursive estimation of the continuous system parameters for a slowly time variant single-input single-output (SISO) system is the Recursive Least Squares State-Variable Filter (RLSSVF) method, introduced in (Padilla et al., 2016). In that paper, filtering and estimation approaches are proposed and applied in scalar form to SISO systems. In this paper, we propose an arrangement to perform the filtering and estimation in state space form, which allows the direct application of the method to multivariable systems.

When dealing with the identification of continuous time systems, the first difficulty encountered is the need to know a priori the temporal derivatives of the input and output plant signals. Several methods have been devised to circumvent this difficulty and reconstruct the signals temporal derivatives. In (Garnier et al., 2003) comparisons were made between the offline identification methods from experiments to study the sensitivity of each approach to design parameters, sample period, signal-to-noise ratio, noise power spectral density and the input signal type. The article (Young and Garnier, 2006) provides an introduction to the key aspects of existing time domain methods for identifying continuous time linear models from discrete time sampled data. Each method is characterized by specific advantages such as: mathematical convenience, simplicity in numerical implementation and computation, treatment of initial conditions, physical vision, precision, among others (Young, 2011). From those approaches, the State-Variable Filter (SVF) (Garnier et al., 2008) presents some advantages such as smoothing of the variables and, mainly, it deals satisfactorily with the system inputs and outputs derivatives. The method is originally developed for monovariable systems described by differential equations.

Once the inputs and outputs of a dynamical system are known, it is possible to estimate with the Recursive Least Squares Method a set of parameters, which minimizes the error between the model and the system outputs. However, for the state space form, the system inputs, outputs and states must be known to estimate the model parameters. This makes their physical implementation more complex, since generally the states of the real system are not measurable. Thus, to circumvent such a difficulty, the Kalman Filter (KF) algorithm uses the system inputs and outputs to estimate the states given that a model is known (Rayyam et al., 2015). Therefore, it allows the implementation of algorithms for state space system identification.

This algorithm is based on linearizing the model around the current state. With this method it is also possible to estimate linear system parameters by rewriting their state space model, so that the parameters to be estimated are computed as part of the state vector, making the model non-linear. Examples of that can be found on the references (Rayyam et al., 2015).

In this paper a hybrid algorithm composed of three stages that perform the sampling, the filtering and the estimation of a continuous time variant dynamical state space model, is shown. In the first stage of the algorithm, sampling is performed together with the filtering of the system input and output signals by the SVF method. In the second stage, the Extended Kalman Filter (EKF) is applied based on the current estimated model to estimate the states from the filtered signals. And in the third stage, from the filtered input and output signals and the estimated states, the RLSSVF method is executed for recursive estimation of the system parameters in state space form. The motivation for the hybrid algorithm development is the possibility to estimate recursively continuous time system parameters in the state space form only from the system sampled inputs and outputs. This has strong appeal to real applications where the multivariable continuous systems parameters are required.

The article is organized as follows: in the section 2 the fundamentals of sampling, filtering and identification of the continuous time systems are presented. In the section 3 the methodology and the case study are illustrated. In the section 4 a case study is presented, in the section 5, the numerical results of the method application are presented and in the section 6 the conclusions found in this work are reported.

2 FILTERING AND DIRECT IDENTIFICATION OF CONTINUOUS TIME MODEL

In this section, the main algorithms that are used in the study proposed in this paper are presented. In the subsection 2.1, the formulation and arrangements for sampling and filtering the system input and output signals are presented. In the subsection 2.2, the fundamentals of state variable filtering are reported. In the subsection 2.3 the filtering and identification method for linear continuous time slowly time variant

(LCTSTV) systems are shown. Finally, in the subsection 2.4 the EKF algorithm is presented.

2.1 Differential Equations Models

A linear continuous monovariable time invariant model can take the form of a differential equation of constant coefficients, expressed by (Garnier et al., 2008):

$$\begin{aligned} &\tfrac{d^n y(t)}{dt^n} + a_1 \tfrac{d^{n-1}y(t)}{dt^{n-1}} + \ldots + a_n y(t) = \ldots \\ &\ldots b_0 \tfrac{d^m u(t)}{dt^m} + b_1 \tfrac{d^{m-1}u(t)}{dt^{m-1}} + \ldots + b_m u(t) + v(t) \end{aligned} \tag{1}$$

where $\frac{d^*(.)}{dt^*}$ is the differential operator of order $*$, $u(t)$ the input signal, $y(t)$ the output signal, $v(t)$ an additive white noise and a_i and b_j, with $i = 1,\ldots n$ and $j = 0,\ldots m$, are the model parameters, which are constant, since the system is supposed to be time invariant.

The equation (1) can be rewritten using the differential operators $A(p)$ and $B(p)$ as:

$$A(p)y(t) = B(p)u(t) + v(t) \tag{2}$$

where $A(p) = p^n + a_1 p^{n-1} + \ldots + a_n$ and $B(p) = b_0 p^m + b_1 p^{m-1} + \ldots + b_m$, with $n \geq m$ and $p = \frac{d}{dt}$ the differential operator.

For any instant of time $t = t_k$, the equation (1) can be rewritten in the regression form as:

$$\frac{d^n y(t_k)}{dt_k^n} = \varphi^T(t_k)\theta + v(t_k) \tag{3}$$

where $\varphi(t_k)$ is the regressor vector and θ is the parameters vector, which are defined by:

$$\begin{aligned} &\varphi^T(t_k) = \ldots \\ &\ldots \left[-\tfrac{d^{n-1}y(t_k)}{dt_k^{n-1}} \quad \ldots \quad -y(t_k) \quad \tfrac{d^m u(t_k)}{dt_k^m} \quad \ldots \quad u(t_k) \right] \end{aligned} \tag{4}$$

and

$$\theta^T = \begin{bmatrix} a_1 & \ldots & a_n & b_0 & \ldots & b_m \end{bmatrix}. \tag{5}$$

Supposing that $p^n y(t_k)$ and the regressor vector $\varphi(t_k)$ are sampled in a number of time instants greater than the dimension of the parameter vector θ, this vector can be estimated from the equation (3) using the least squares method. However, the regressor vector $\varphi(t_k)$ contains input and output time derivatives that are not available as measurement data in most practical cases. A method to estimate those derivatives from sampled input and output data is presented in the following subsection.

2.2 State-variable Filtering Method

First, consider the model of the differential equation (2) without the additive white noise $v(t)$:

$$A(p)x(t) = B(p)u(t) \tag{6}$$

where $x(t)$ represents noise-free output variables.

Let a filter $F_0(p)$ be applied to the signals $x(t)$ and $u(t)$. Then the equation (6) becomes:

$$A(p)F_0(p)x(t) = B(p)F_0(p)u(t). \tag{7}$$

The filter $F_0(p)$ is defined as state variable filter and typically is chosen as presented in the equation (8):

$$F_0(p) = \frac{1}{(p+\lambda)^n} \tag{8}$$

where λ is the parameter used to define the bandwidth of the filter and n is the order of the system to identify.

Expanding the equation (7), the following equation is obtained:

$$\begin{aligned} &\left(p^n + a_1 p^{n-1} + \ldots + a_n\right) F_0(p)x(t) = \ldots \\ &\ldots \left(b_0 p^m + b_1 p^{m-1} + \ldots + b_m\right) F_0(p)u(t). \end{aligned} \tag{9}$$

Substituted the equation (8) in (9) the result is:

$$\begin{aligned} &\left(\tfrac{p^n}{(p+\lambda)^n} + a_1 \tfrac{p^{n-1}}{(p+\lambda)^n} + \ldots + a_n \tfrac{1}{(p+\lambda)^n}\right) x(t) = \\ &\ldots = \left(b_0 \tfrac{p^m}{(p+\lambda)^n} + b_1 \tfrac{p^{m-1}}{(p+\lambda)^n} + b_m \tfrac{1}{(p+\lambda)^n}\right) u(t) \end{aligned} \tag{10}$$

or yet

$$\begin{aligned} &(F_n(p) + a_1 F_{n-1}(p) + \ldots + a_n F_0(p)) \ldots \\ &\ldots x(t) = (b_0 F_m(p) + \ldots + b_m F_0(p))\, u(t) \end{aligned} \tag{11}$$

where $F_i(p)$ to $i = 0,1,\ldots,n$ is a set of filters defined as:

$$F_i(p) = \frac{p^i}{(p+\lambda)^n}. \tag{12}$$

The equation (11) can be expressed by:

$$\begin{aligned} &x_f^{(n)}(t) + a_1 x_f^{(n-1)}(t) + \ldots + a_n x_f^{(0)}(t) = \ldots \\ &\ldots b_0 u_f^{(m)}(t) + b_1 u_f^{(m-1)}(t) + \ldots + b_m u_f^{(0)}(t) \end{aligned} \tag{13}$$

with

$$x_f^{(i)}(t) = f_i(t) * x(t) \tag{14}$$

and

$$u_f^{(i)}(t) = f_i(t) * u(t) \tag{15}$$

where $f_i(t)$, for $i = 0,1,\ldots,n$, represents the impulse response of the filters defined in the equation (12) and the operator $*$ indicates the convolution.

The outputs of the filters $x_f^{(i)}(t)$ and $u_f^{(i)}(t)$ provide the pre-filtered time derivatives of the inputs and outputs in the bandwidth of interest, which can be used in the equation (3) to estimate the time invariant model parameters with the least squares method in a batch procedure.

If the system is time variant, the parameters vector θ depends on t. In those cases, the recursive approach, such as the one discussed in the next section, allows the determination of the time variant parameters.

2.3 Recursive Method of Filtering and Identification

An alternative model to that presented in the equation (2) was proposed in the reference (Padilla et al., 2016), to deal with the time variant case. It is expressed as:

$$A(p,t,\theta)x(t) = B(p,t,\theta)u(t) \tag{16a}$$

$$y(t_k) = x(t_k) + e(t_k) \tag{16b}$$

where p is the differential operator, t is the continuous time, $x(t)$ is the noise-free continuous output signal, $u(t)$ is the continuous input signal, t_k is the time instant in which the signals are sampled, $x(t_k)$ is the sample of $x(t)$ at the time instant t_k and $e(t_k)$ is a zero mean white noise at the time instant t_k. Also, $A(p,t,\theta)$ and $B(p,t,\theta)$ polynomials in p and with time variant parameters a_i and b_j, with $i = 1,...,n$ e $j = 0,...,m$, given by:

$$A(p,t,\theta) = p^n + a_1(t)p^{n-1} + ... + a_n(t) \tag{17a}$$

$$B(p,t,\theta) = b_0 p^m + b_1(t)p^{m-1} + ... + b_m(t) \tag{17b}$$

where $n \geq m$ and the time-varying parameters can be grouped into the parameter vector of the form:

$$\theta(t) = \begin{bmatrix} a_1(t) & ... & a_n(t) & b_0(t) & ... & b_m(t) \end{bmatrix}. \tag{18}$$

For the system (16), the following assumptions must be satisfied:

(i) the degrees n and m of the polynomials $A_0(p,t)$ and $B_0(p,t)$, respectively, are a priori known; and

(ii) The systems parameters variations are slow.

The identification problem is to estimate recursively the parameter vector (18) from the system (16) considering the assumptions listed above.

As pointed out in the section 2.2, direct estimation of continuous time models requires values for the signals time derivatives that are often not available. One approach to solve this problem is to use the SVF filter presented in the equation (8).

When applying the filter described by the equation (12), in a manner analogous to the one that was done to obtain the equation (13), the following equation is obtained:

$$\begin{aligned} &y_f^{(n)}(t_k) + a_1(t_k)y_f^{(n-1)}(t_k) + ... + \\ &...a_n(t_k)y_f^0(t_k) = b_0(t_k)u_f^{(m)}(t_k) + ... + \\ &...b_m(t_k)u_f^0(t_k) + v_f(t_k) \end{aligned} \tag{19}$$

where

$$v_f(t_k) = F_0(p)v(t_k) = F_0(p)A(p)e(t_k). \tag{20}$$

The equation (19) can be rewritten as a linear regression similar to the equation (3) of the form:

$$y_f^{(n)}(t_k) = \varphi_f^T(t_k)\theta(t_k) + v_f(t_k) \tag{21}$$

with filtered regressor vector given by:

$$\begin{aligned} \varphi_f^T(t_k) = \Big[&-y_f^{(n-1)}(t_k) \quad ... \quad -y_f(t_k) \quad ... \\ &... \quad u_f^{(m)}(t_k) \quad ... \quad u_f(t_k) \Big] \end{aligned} \tag{22}$$

and $\theta_f(t_k)$ are the estimates, which can be obtained recursively with an algorithm based on the recursive least squares method below, as described (Padilla et al., 2016):

Prediction stage:

$$\hat{\theta}(t_k|t_{k-1}) = \hat{\theta}(t_{k-1}) \tag{23a}$$

$$P(t_k|t_{k-1}) = P(t_{k-1}). \tag{23b}$$

Correction stage:

$$\hat{\theta}(t_k) = \hat{\theta}(t_k|t_{k-1}) + L(t_k)\varepsilon(t_k) \tag{24a}$$

$$\varepsilon(t_k) = x_f^{(n)}(t_k) - \varphi_f^T(t_k)\hat{\theta}(t_k|t_{k-1}) \tag{24b}$$

$$L(t_k) = \frac{P(t_k|t_{k-1})\varphi_f(t_k)}{1 + \varphi_f^T(t_k)P(t_k|t_{k-1})\varphi_f(t_k)} \tag{24c}$$

$$P(t_k) = P(t_k|t_{k-1}) - L(t_k)\varphi_f^T(t_k)P(t_k|t_{k-1}). \tag{24d}$$

The algorithm (23)-(24) is called the RLSSVF random search method and allows the determination of a time variant continuous model based on differential equations. The method presented in this article is an extension of that method to the determination of time variant continuous state space models. For that, samples from the state and its first order derivative must be available. The states are estimated by the EKF, presented in the next section and its derivatives are calculated with the SVF. Further details are presented in the section 3.

2.4 Extended Kalman Filter

The Extended Kalman Filter (EKF) is a popular algorithm to estimate states of nonlinear state space models from input and output measures. It also can be used to estimate physical parameters simultaneously with state variables and has been used to solve real-world application problems (Kowalski and Wierzbicki, 2007), (Meziane et al., 2008) and (Rayyam et al., 2015). To apply the EKF, an analytical model of the system is needed. This model is linearized around the current state. In the sequence, the prediction of the future states is performed, as well as the execution of the correction step with the gain calculations and the update of state and error covariance matrix.

For the description of the EKF algorithm, consider a nonlinear system of the form (Rayyam et al., 2015):

$$x(t_{k+1}) = f(x(t_k), u(t_k), w(t_k)) \tag{25a}$$

$$y(t_{k+1}) = h(x(t_{k+1})) + v(t_{k+1}) \tag{25b}$$

where $f(x(t_k), u(t_k), w(t_k))$ and $h(x(t_{k+1}))$ are nonlinear functions supposedly known, $x(t_k)$ is the state vector, $u(t_k)$ is the input signal, $w(t_k)$ is the state update noise, $h(x(t_{k+1}))$ is the noise free output signal and $v(t_{k+1})$ is the measurement noise.

The nonlinear system is then linearized around the most recent estimate, from the partial derivatives of the nonlinear functions from the equations 25, as follows:

$$F(t_k) = \left.\frac{\partial f(x(t_k), u(t_k), w(t_k))}{\partial x^T(t_k)}\right|_{x(t_k)=\hat{x}(t_k|t_k)} \tag{26a}$$

$$H(t_k) = \left.\frac{\partial h(x(t_k))}{\partial x^T(t_k)}\right|_{x(t_k)=\hat{x}(t_{k+1}|t_k)} \tag{26b}$$

where $k+1|k$ represents the prediction in time $k+1$ based on the time data k.

The implementation of the EKF algorithm is performed in two steps: the prediction step and the correction step. These steps are summarized below:

(i) Prediction step:

$$\hat{x}(t_{k+1}|t_k) = f(x(t_k), u(t_k), w(t_k)) \tag{27a}$$

$$F(t_k) = \left.\frac{\partial f(x(t_k), u(t_k), w(t_k))}{\partial x^T(t_k)}\right|_{x(t_k)=\hat{x}(t_k|t_k)} \tag{27b}$$

$$\hat{P}(t_{k+1}|t_k) = F(t_k)\hat{P}(t_k|t_k)F^T(t_k) + Q \tag{27c}$$

$$H(t_k) = \left.\frac{\partial h(x(t_k))}{\partial x^T(t_k)}\right|_{x(t_k)=\hat{x}(t_{k+1}|t_k)} \tag{27d}$$

(ii) Correction step:

$$S(t_{k+1}) = H(t_k)\hat{P}(t_{k+1}|t_k)H^T(t_k) + R \tag{28a}$$

$$G(t_{k+1}) = \hat{P}(t_{k+1}|t_k)H^T(t_k) + R \tag{28b}$$

$$\hat{x}(t_{k+1}|t_{k+1}) = \hat{x}(t_{k+1}|t_k) + G(t_{k+1})\ldots$$

$$\ldots(y(t_{k+1}) - H(t_k)\hat{x}(t_{k+1}|t_k)) \tag{28c}$$

$$\hat{P}(t_{k+1}|t_{k+1}) = (I - G(t_{k+1})H(t_k))\,\hat{P}(t_{k+1}|t_k) \tag{28d}$$

where $G(t_k)$ is the Kalman gain matrix. To execute the algorithm, the initial conditions $x(t_0)$ and $P(t_0|t_0)$ are required.

3 METHODOLOGY

In this section, it is discussed the hybrid algorithm application proposal in this article. In the figure 1, a block diagram is illustrated to better clarify the flow of information and processing blocks.

Consider a time-varying linear multivariate continuous system, given by:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{29a}$$

$$y(t) = C(t)x(t) \tag{29b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the system input signal, $y(t) \in \mathbb{R}^p$ is the output signal, $A(t) \in \mathbb{R}^{n\times n}$ is the system state transition matrix, $B(t) \in \mathbb{R}^{n\times m}$ is the input matrix, $C(t) \in \mathbb{R}^{p\times n}$ is the output matrix and t is the continuous time.

In the figure 1, the system to be identified is inside the dashed rectangle and in the external part of that rectangle the applied hybrid algorithm is shown.

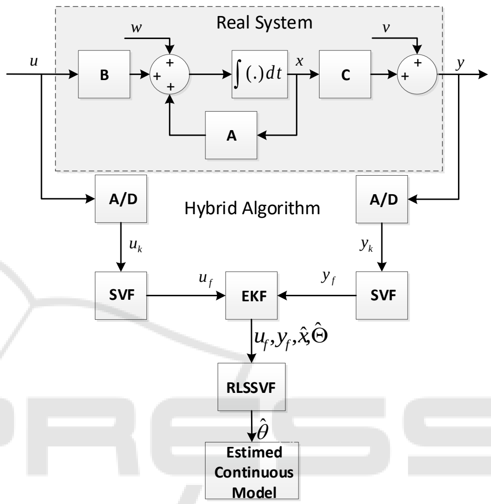

Figure 1: Diagram of Blocks of the real system and hybrid algorithm.

In the algorithm, illustrated in figure 1, the input $u(t)$ and output $y(t)$ of the real system, which are sampled, $u(t_k)$ and $y(t_k)$, in the respective blocks nominated by A/D are used. Then, the signals $u(t_k)$ and $y(t_k)$ are filtered in the blocks SVF from the equations (14) and (15), being made available u_f e y_f to the EKF block. The EKF block estimates the samples of the system states $x(t_k)$ and certain parameters $\hat{\Theta}$, selected by the designer to be part of the state vector. With this, the linear system (29) becomes nonlinear, as represented by the equation (25). Finally, with the input and output signals filtered u_f and y_f, respectively, together with the estimated state vector $\hat{x}$, block $RLSSVF$ is executed for recursive estimation of all the remaining model parameters, according to equations (23)-(24). Where its main advantage the direct estimation of model parameters, i.e., there is correspondence from one to one the estimated parameters with the parameters of continue time real system.

In the next section, an application example of the algorithm elaborate in this paper is shown. Then, in the section 5, the results obtained with the algorithm application are presented and discussed.

4 CASE STUDY

To demonstrate the use of the algorithm, a continuous time variant linear monovariable benchmark with state space representation taken from the reference (Ohsumi and Kawano, 2002) is used. This benchmark is expressed by:

$$\dot{x}(t) = A(t)x(t) + Bu(t) \tag{30a}$$

$$y(t) = Cx(t) \tag{30b}$$

where $A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The parameters $a_{11}(t)$ and $a_{12}(t)$ are subjected to abrupt variations, as follows:

$$a_{11}(t) = \begin{cases} 0 & \text{if } 0 \leq t < 500 \\ 0.2 & \text{if } t \geq 500 \end{cases} \tag{31}$$

and

$$a_{12}(t) = \begin{cases} -0.5 & \text{if } 0 \leq t < 1000 \\ -0.8 & \text{if } t \geq 1000. \end{cases} \tag{32}$$

The simulation was performed from $t = 0s$ to $t = 2000s$. The input signal $u(t)$ was chosen as a random signal with a Gaussian distribution of zero mean and variance of 0.1. Then, the input and output signals were sampled to perform the filtering and parameter estimation.

The system (30) was discretized, which takes the form (25), given by:

$$x(t_{k+1}) = \begin{bmatrix} 1 + a_{11}\tau & a_{12}\tau \\ \tau & 1 - \tau \end{bmatrix} x(t_k) + \dots$$

$$\dots \begin{bmatrix} \tau \\ 0 \end{bmatrix} u(t_k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w(t_k) \tag{33a}$$

$$y(t_k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t_k) + v(t_k) \tag{33b}$$

where τ the sampling time, the cutoff frequency equation (12) of the filter $\lambda = 0.1$ and the noises, $w(t_k)$ and $v(t_k)$ are gaussians with zero mean and variance 0.05.

It should be noticed that the benchmark is a time continuous variant system, according to equations (31) and (32), while the matrices B and C are constant over time. Such parameters $a_{11}(t)$ and $a_{12}(t)$ are included in the system state vector, equation 30, as follows:

$$X(t) = \begin{bmatrix} x_1(t) & x_2(t) & a_{11}(t) & a_{12}(t) \end{bmatrix}^T \tag{34}$$

where $X(t)$ is the extended state vector, which will be estimated with the EKF algorithm.

When executing the hybrid algorithm, we considered the real system (30) and consequently its discrete form expressed by the equation (33), whose estimates obtained in each recursion are presented in the following numerical form.

5 SIMULATED RESULTS

In the figures 2 and 3, the real and the estimated values for the matrix A elements are presented. From the figures it is possible to notice that the estimated parameters converge to the real ones, even a_{11} and a_{12}, which are time variant.

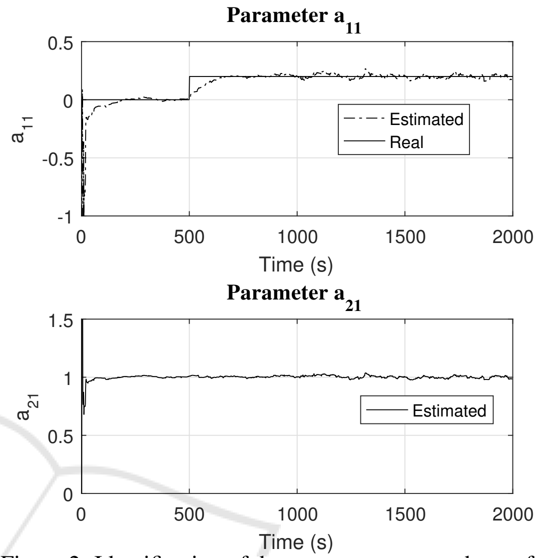

Figure 2: Identification of the parameters a_{11} and a_{21} of the system transfer matrix, $A(t)$, through the proposed Hybrid Algorithm.

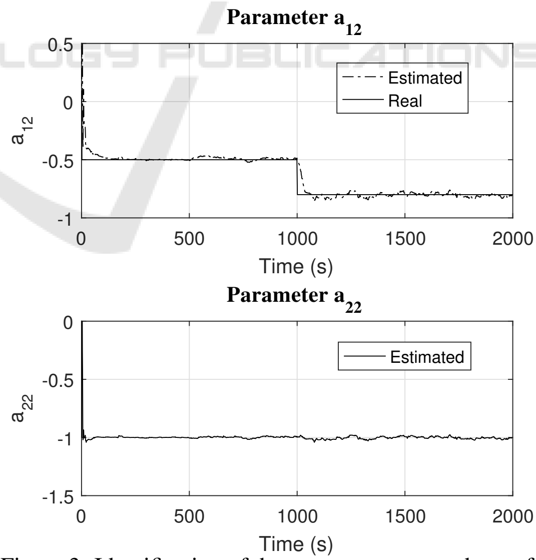

Figure 3: Identification of the parameters a_{12} and a_{22} of the system transfer matrix, $A(t)$, through the proposed Hybrid Algorithm.

In the figure 4 the results of the estimation of the matrix elements B are shown. Those parameters also converge to the real system parameters.

In the figure 5 the numerical results of the estimation of the elements of the matrix C are shown. As

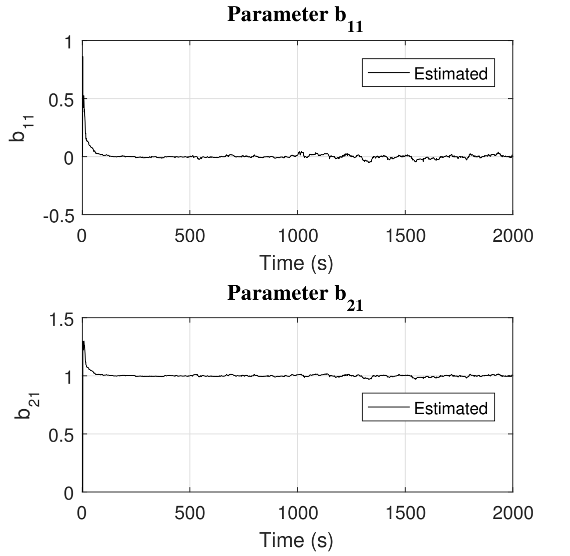

Figure 4: Identification of the parameters b_{11} and b_{21} of the system input matrix, B, through the proposed Hybrid Algorithm.

the other parameters, those values, converge in few recursions to the real system values.

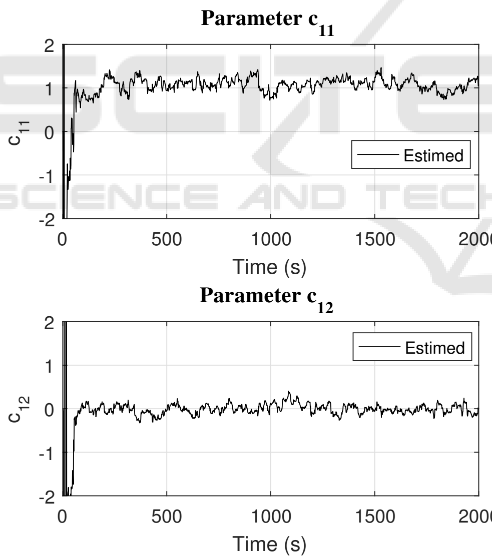

Figure 5: Identification of the parameters c_{11} and c_{21} of the system output matrix, C, through the proposed Hybrid Algorithm.

In the figure 6, a clipping of the discrete system states in the regions of the parameters change, a_{11} from $500s$ and a_{12} from $1000s$, are presented. In the first graph of figure 6, the discrete state $x_1(t)$ and its estimated value by the EKF algorithm is shown, this is output signal of the dynamical system. The second graph it refers to the state $x_2(t)$. From the figures it is noticed that the estimated states are closed to the real ones.

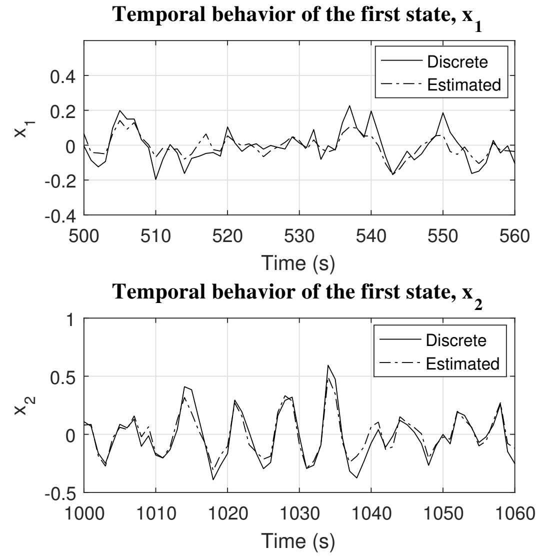

Figure 6: Behavior of the x_1 and x_2 states of the discrete time, along with their estimated states through the EKF.

In this section, were presented several graphs to illustrate the numerical results of the simulation, whose average computational cost per iteration was less than $0.9ms$.

6 CONCLUSIONS

In this article, a hybrid algorithm composed of three stages for sampling, filtering and estimating the states and the parameters of a state space model for a linear continuous dynamical system, was presented. The method allows the recursive estimation of the continuous system parameters, which may represent physical parameters from dynamical systems with a known structure.

From the case study, a good accuracy was observed in the estimation of the parameters of the benchmark studied. Future work is expected to: (*i*) identify MIMO systems; (*ii*) apply the algorithm to the identification of parameters of teleoperated systems such as the one presented in (Roshandel et al., 2011); and (*iii*) to develop indirect adaptive control algorithms.

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REFERENCES

Awrejcewicz, J. (2016). *Dynamical Systems: Modelling: Łódź, Poland, December 7-10, 2015*, volume 181. Springer.

Garnier, H., Mensler, M., and Richard, A. (2003). Continuous-time model identification from sampled data: implementation issues and performance evaluation. *International journal of Control*, 76(13):1337–1357.

Garnier, H., Wang, L., and Young, P. C. (2008). Direct identification of continuous-time models from sampled data: Issues, basic solutions and relevance. In *Identification of continuous-time models from sampled data*, pages 1–29. Springer.

Katayama, T. (2006). *Subspace methods for system identification*. Springer Science & Business Media.

Kowalski, C. and Wierzbicki, R. (2007). Application of the extended kalman filter for rotor and stator fault detection of the induction motor. *Poznan University of Technology Academic Journals. Electrical Engineering*, (55):145–155.

Ljung, L. (1999). *System identification: theory for the user*. Prentice-Hall, Upper Saddle River, NJ, USA.

Meziane, S., Toufouti, R., and Benalla, H. (2008). Nonlinear control of induction machines using an extended kalman filter. *Acta Polytechnica Hungarica*, 5(4):41–58.

Ohsumi, A. and Kawano, T. (2002). Subspace identification for a class of time-varying continuous-time stochastic systems via distribution-based approach. *IFAC Proceedings Volumes*, 35(1):241–246.

Padilla, A., Garnier, H., Young, P. C., and Yuz, J. (2016). Real-time identification of linear continuous-time systems with slowly time-varying parameters. In *2016 IEEE 55th Conference on Decision and Control (CDC)*, pages 3769–3774.

Rayyam, M., Zazi, M., and Hajji, Y. (2015). Detection of broken bars in induction motor using the extended kalman filter (ekf). In *Complex Systems (WCCS), 2015 Third World Conference on*, pages 1–5. IEEE.

Roshandel, A., Khosravi, A., and Alfi, A. (2011). An lmi based delay-dependent robust controller for transparent bilateral teleoperation system. In *The 2nd International Conference on Control, Instrumentation and Automation*, pages 457–462.

Slotine, J.-J. E., Li, W., et al. (1991). *Applied nonlinear control*, volume 199. Prentice hall Englewood Cliffs, NJ.

Young, P. C. (2011). *Recursive estimation and time-series analysis: An introduction for the student and practitioner*. Springer Science & Business Media.

Young, P. C. and Garnier, H. (2006). Identification and estimation of continuous-time, data-based mechanistic (dbm) models for environmental systems. *Environmental modelling & software*, 21(8):1055–1072.