Conical Tank Level Supervision using a Fractional Order Model Reference Adaptive Control Strategy

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Abstract: This paper proposes a fractional order model reference adaptive control (FO-MRAC) design in order to command the level of a conical tank system. The FO-MRAC is based on the choice of a fractional reference model which specifies the closed loop desired performances. Also, the control strategy adopted introduces fractional integration in the phase of corrector parameters updating. Model reference adaptive controller of integer order and of fractional order are applied to the non linear system and compared. From the simulation results, we concluded that FO-MRAC is the controller presenting the best performances, and especially in case of measurements noise, and parametric variations.

1 INTRODUCTION

Fractional order systems are attracting more and more researchers in different domains of science and engineering (Mathieu et al., 2003; Ma et al., 2009; El-sayed and Gaafar, 2003). Fractional order control is a generalization of the classic control theory of integer order, its major interest is to improve the control system performances using the concepts of non-integer derivation and fractional order systems. Dynamic Systems and Fractional Order Controllers, which are based on the fractional calculation, have gathered the attention of several researchers. The most known fractional order control structures are: CRONE controller (Outsaloup, 1991), Fractional $PI^{\lambda}D^{\mu}$ controller (Podlubny, 1999), and fractional adaptive control (Ladaci et al., 2008).

Adaptive control is one of the popular control techniques applied in industrial applications. This command consists in adapting the regulator on line with the variations of the regulated process to ensure a constant quality of performances. The main reason which have encouraged researchers to move toward fractional order adaptive control and essentially to the fractional order model reference adaptive control (FO-MRAC) is that The MRAC command is based on the choice of a reference model that specifies the desired performances in closed loop, and many research works proved the very good performances of fractional systems relatively to those of integer order (Outsaloup, 1995; Ladaci and Bensafia, 2016).

Many fractional order control structure based on MRAC have been developed in literature (Ladaci and Charef, 2012). An indirect model reference adaptive control is used to control a class of fractional order systems was introduced in (Chen et al., 2016). In (Vinagre et al., 2002; Ladaci and Charef, 2003), the authors investigated the use of fractional order parameter adjustment rule and the employment of fractional order reference model. The use of a fractional model and the introduction of fractional derivative filter at the plant output was introduced in (Ladaci and Charef, 2006).

In this paper, we propose to control a nonlinear dynamical plant with fractional order model reference adaptive control to improve the system performances. We will deal with the problem of conical tank level control, which has been widely studied and several control techniques have been employed, even with model predictive control (Warier and Venkatesh, 2012) and fractional order $PI^{\lambda}D^{\mu}$ controllers (Jauregui et al., 2016).

This work is organized as follows: Section 2 presents some theoretical concepts on fractional order systems.

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The proposed FMRAC strategy is presented in Section 3, and the conical tank modelization is given in Section 4. Section 5 shows the simulation results. Section 6 is dedicated to the main conclusions and future researches.

2 FRACTIONAL ORDER SYSTEMS

Fractional calculus is the domain of analytical mathematics which deals with the study and application of arbitrary order of integrals and derivatives.

2.1 Definitions

The commonly used definitions of the fractional order integrals and derivative are the Riemann-Liouville (R-L) and Grünwald-Letnikov (G-L) definitions (Oldham and Spanier, 1974; Bourouba et al., 2018).
The R-L fractional order integral of order $\lambda > 0$ is defined as:

$$\begin{aligned} I^{\lambda}_{RL} g(t) &= D^{-\lambda}_{RL} g(t) \\ &= \frac{1}{\Gamma(\lambda)} \int_0^t (t-\tau)^{\lambda-1} g(\tau) d(\tau) \end{aligned} \quad (1)$$

and the expression of the R-L fractional order derivative of order $\mu > 0$ is:

$$D^{\mu} g(t) = D^{m}\left[D^{-\nu} g(t)\right], \qquad \mu > 0, \quad (2)$$

where $\Gamma(.)$ is the Euler's gamma function and the integer m is such that $(m-1) < \mu < m$ and $\nu = m - \mu > 0$.
The GL fractional order integral of order $\lambda > 0$ is given by:

$$\begin{aligned} I^{\lambda}_{GL} g(t) &= D^{-\lambda}_{GL} g(t) \\ &= \lim_{h\to 0} \sum_{j=0}^{k} (-1)^j \binom{-\lambda}{j} g(kh - jh) \end{aligned} \quad (3)$$

Where h is the sampling period and the coefficients and $\omega_0^{(-\lambda)} = \binom{-\lambda}{0}$ the coefficients of the following binomial:

$$(1-z)^{-\lambda} = \sum_{j=0}^{\infty} \omega_j^{(-\lambda)} z^j \quad (4)$$

The GL fractional order derivative of order $\mu > 0$ is also given by

$$\begin{aligned} D^{\mu}_{GL} g(t) &= \frac{d^{\mu}}{dt^{\mu}} g(t) \\ &= \lim_{h\to 0} h^{-\mu} \sum_{j=0}^{k} (-1)^j \binom{\mu}{j} g(kh - jh) \end{aligned} \quad (5)$$

where h is the sampling period and the coefficients $\omega_j^{(\mu)} = \binom{\mu}{j} = \frac{\Gamma(\mu+1)}{\Gamma(j+1)\Gamma(\mu-j+1)}$,
with $\omega_0^{(\mu)} = \binom{\mu}{0} = 1$,
are those of the polynomial:

$$(1-z)^{\mu} = \sum_{j=0}^{\infty} (-1)^j \binom{\mu}{j} z^j = \sum_{j=0}^{\infty} \omega_j^{(\mu)} z^j \quad (6)$$

2.2 Linear Approximation of Fractional Order Transfer Functions

For the purpose of our approach we need to use an integer order model approximation of the fractional order model reference in order to implement the adaptation algorithm. For this aim, we use the so-called singularity function method proposed by (Charef et al., 1992), and precisely for fractional first order system of the form:

$$H(s) = \frac{1}{(1 + s\, p_T)^{\beta}} \quad (7)$$

with β a positive real number such that $0 < \beta < 1$.
The approximation is given by:

$$H(s) \approx \frac{\prod_{\nu=0}^{L-1}\left(1 + \frac{s}{z_\nu}\right)}{\prod_{\nu=0}^{L}\left(1 + \frac{s}{p_\nu}\right)} \quad (8)$$

Where the singularities are given by:

$$\begin{cases} p_\nu = (ab)^{\nu} p_0 & \nu = 1,2,3,\ldots,L-1 \\ z_\nu = (ab)^{\nu} a p_0 & \nu = 1,2,3,\ldots,L-1 \end{cases} \quad (9)$$

with,

$$\begin{aligned} p_0 &= p_T\, 10^{\frac{\varepsilon}{20\beta}} \\ a &= 10^{\frac{\varepsilon}{10(1-\beta)}} \\ b &= 10^{\frac{\varepsilon}{10\beta}} \\ \beta &= \frac{\log(a)}{\log(ab)} \end{aligned} \quad (10)$$

ε is the tolerated error in $d\beta$. $L+1$ is the total number of singularities that can be determined by the frequency band of the system.

2.3 Numerical Approximation of Riemann Fractional Integral

In our work, we need to use a numerical approximation for the analytical formulas of the fractional order

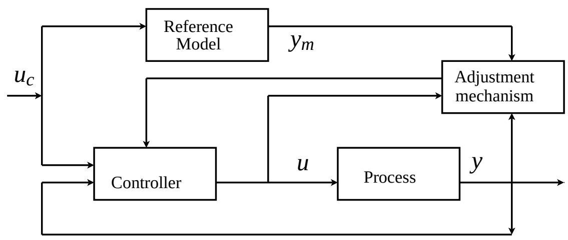

Figure 1: Block Diagram for MRAC approach.

operator and more precisely for the integral of Riemann introduced in (Ladaci and Charef, 2006):
Putting:

$$t = k\Delta$$

Where t is the current time, k an integer, and Δ sampling period. We obtain:

$$\begin{aligned} I^{\lambda} g(k\Delta) &= \frac{\Delta}{\Gamma(\lambda)} \sum_{\tau=0}^{k-1} (k\Delta - \tau\Delta)^{\lambda-1} g(\tau\Delta) \quad (11) \\ &= \frac{\Delta^{\lambda}}{\Gamma(\lambda)} \sum_{\tau=0}^{k-1} (k-\tau)^{\lambda-1} g(\tau\Delta) \end{aligned}$$

3 FRACTIONAL ORDER MODEL REFERENCE ADAPTIVE CONTROL

3.1 Model Reference Adaptive Control

It's one of the most used adaptive control approaches, in which the desired performances are specified by the choice of a reference model.
A block diagram representing the principle of this approach is given in the figure 1.

The reference model adaptive control system has an ordinary feedback loop composed of the process and the regulator and another feedback loop which allows the change of the regulator parameters.
We consider a single input single output system (SISO) described by the equation:

$$Ay(t) = Bu(t) \tag{12}$$

Where u is the control signal and y is the output signal. A and B represent polynomials functions of either the differential operator $p = d/dt$, or the shift operator in advance q.
The desired closed-loop response is specified by the reference model output y_m.

$$A_m y_m(t) = B_m u_r(t) \tag{13}$$

A general linear regulator can be described by:

$$Ru(t) = Tu_r(t) - Sy(t) \tag{14}$$

where R, S, and T are polynomials. And we get the closed-loop system:

$$y(t) = \frac{BT}{AR+BS} u_r(t) \tag{15}$$

The mechanism of adjustment of the regulator parameters can be obtained using the law of MIT, which is the original approach for the MRAC.
To represent the MIT law, we consider a closed-loop system in which the regulator has a vector θ of adjustable parameters. Let e be the error between the output y of the closed loop and that of the reference model y_m. The adjustment of the parameters is done in such a way as to minimize a cost function defined by

$$J(\theta) = \frac{1}{2} e^2 \tag{16}$$

To minimize J, we have to change the parameters in the direction of the negative gradient of J, and we have the famous MIT law:

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = \gamma \varphi e \tag{17}$$

$\varphi = -\frac{\delta e}{\delta \theta}$, and γ is the adaptation gain.
The following standardized algorithm is less sensitive to signal levels:

$$\frac{d\theta}{dt} = \gamma \frac{\varphi e}{\alpha + \varphi^T \varphi} \tag{18}$$

The control signal is computed using the following relation:

$$u = \varphi^T \theta \tag{19}$$

Where, φ is the regression vector containing the measured input and output signals u and y and the input reference signal u_r.

3.2 Fractional approach

In this work we will consider a fractional order reference model which will be implemented using the singularity approximation method proposed by (Charef et al., 1992). Also, in the phase of updating the parameters of the corrector, we will use the fractional order parameter adaptation law proposed in (Ladaci and Charef, 2006) instead of equation (18) given by:

$$\frac{d^m\theta}{dt^m} = \gamma \frac{\varphi e}{\alpha + \varphi^T \varphi} \tag{20}$$

And we will use the numerical Riemann approximation (11) to obtain fractional order integral of the relation (20).

4 CONICAL TANK SYSTEM

The layout of the conical tank system is shown in Figure 2. The water is pumped from the bottom of the recirculating tank to the upper part of the conical tank by means of a pump driven by an induction motor of variable speed driven by a variable frequency drive. From (Jauregui et al., 2016), the nonlinear model for

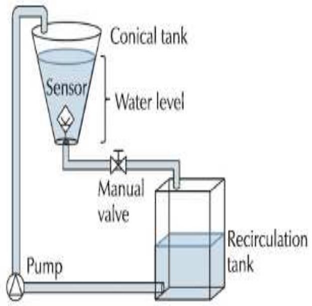

Figure 2: Conical tank configuration.

the conical tank is represented by the following equation:

$$\dot{h} = \frac{5.43f - 78.23 + \mu\sqrt{h}}{0.65h^2 + 11.4h + 17.1} = g(h, f) \tag{21}$$

With $\mu = -20.63$, f is the input and represent the electrical network frequency expressed in % as a percentage of the nominal frequency ($50Hz$) and ranges in the interval $0\% - 100\%$, h is the output and represent the water level inside the conical tank expressed in cm, $g(h, f)$ is a non linear function of these two variables showing clearly the nonlinearities of the conical tank system.
The system is linearized around 3 operating points (h_{op}, f_{op}). These approximations are given by:

$$\dot{h} = W(h - h_{op}) + Z(f - f_{op}) \tag{22}$$

With $W = \frac{\delta g(h,f)}{\delta h}\Big|_{(h_{op}, f_{op})}$ and $Z = \frac{\delta g(h,f)}{\delta f}\Big|_{(h_{op}, f_{op})}$.
To determine the operating points the whole operation range $15\,cm - 60\,cm$ is divided into three segments, as illustrated in Table 1.

Table 1: Operating points and parameters of linearized models.

	Low level	Medium level	High level
Interval (cm)	15-30	30-45	45-60
$h_{op}(cm)$	22.5	37.5	52.5
$f_{op}(\%)$	32.42	37.66	41.92
W	-0.0036	-0.0012	-0.0006
Z	0.009	0.004	0.0022

In our work, we try to control the nonlinear system around only one operation point (Low level). So, the linearized system is represented by equation (22), with $W = -0.0036$, and $Z = 0.009$, By posing: $y(t) = h - hop$, and $u(t) = f - fop$, we obtain the following transfer function (Deghboudj and Ladaci, 2017):

$$G(s) = \frac{0.009}{s + 0.0036} \tag{23}$$

5 SIMULATION RESULTS

In this Section, we will apply the proposed fractional order adaptive control technique to the tank level control, with the transfer function given in equation (23).Let us chose reference model transfer functions of integer order and fractional order respectively:

$$G_m(s) = \frac{1}{1 + 20s} \tag{24}$$

and

$$G_{mf}(s) = \frac{1}{(1 + 20s)^{0.6}} \tag{25}$$

G_{mf} is approximated to an integer order model using the singularity function method (Charef et al., 1992). The sampling period is $\Delta = 0.1\,sec$.

The step response of the fractional order reference model is compared with that of the integer order model and with the open loop system step response in Figure 3.
Figure 4 illustrates the bode plot of the open loop system, of the integer order reference model, and the fractional order reference model.

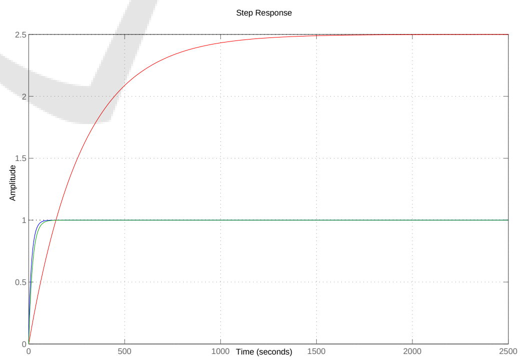

Figure 3: Open loop step response of the tank system (red), step response of the integer order reference model (green), step response of the fractional order reference model (blue).

From Figure 3, we clearly see that the fractional order model is faster than the integer order model, which is confirmed also in Figure 4, where we see that the fractional reference model has the largest pass band.
In order to test the effectiveness and the performances

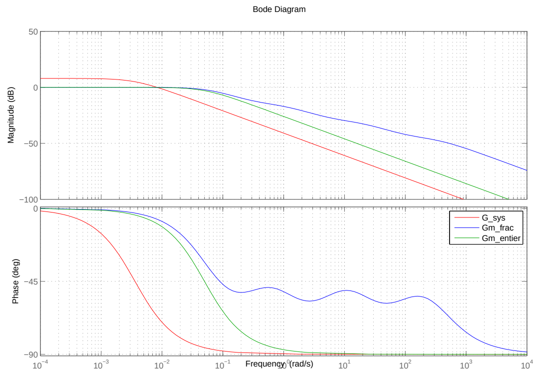

Figure 4: Bode plot of G (red), of G_m (red), and of G_{mf} (blue).

of each controller (MRAC and FO-MRAC), a standard reference signal $u_r(t)$ is applied, consisting of steps with different amplitudes described in table 2.

Table 2: Reference signal.

$t\,(sec)$	$[0\ 500]$	$[500\ 1000]$	$[1000\ 1500]$
$r(t)$	20	29	15

Also, to evaluate and compare the performances of each control method, we choose the performance index given by the sum of the absolute error (SAE) defined in the expression (26), and the sum of the square input (SSI) defined in (27):

$$J_e = \sum_{k=0}^{N} |e(k\Delta)| \tag{26}$$

$$J_u = \sum_{k=0}^{N} u(k\Delta)^2 \tag{27}$$

where N is the samples number, $e(k\Delta) = y(k\Delta) - r(k\Delta)$.

5.1 Study in the Ideal Case (Without Disturbances)

Respectively, the real output of the system and the control signal of the classical MRAC and the FO-MRAC are shown in Figure 5 and Figure 6.

From obtained results, we remark that the output of the system follows the referential signal, even the sudden change of the set point signal.
Better results are obtained when using the FO-MRAC method, where we have the lowest cost function of error, whereas the MRAC strategy presents the lowest energy consumption by the control signal. The system is faster and more precise with the FO-MRAC strategy.
In order to study the influence of the fractional order

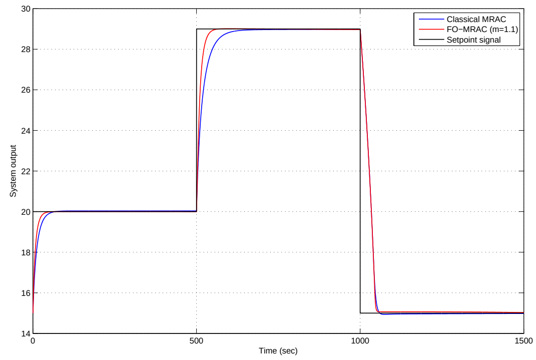

Figure 5: The system response with classical MRAC and with FO-MRAC ($m = 1.1$).

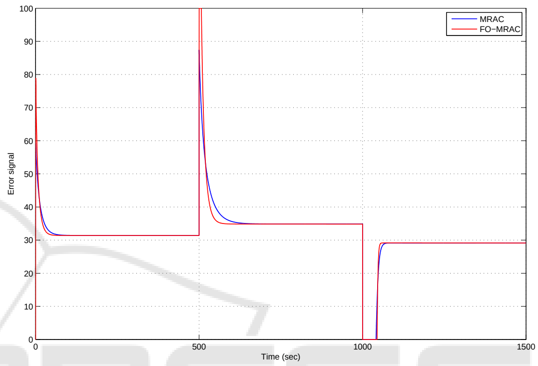

Figure 6: The control signal of the classical MRAC and the FO-MRAC ($m = 1.1$).

m value on the performance of the control system, we vary its value from 0.1 to 1.5; the results are grouped in the following Table 3.

Table 3: Cost functions Versus the fractional order m.

m	J_e	J_u
0.1	$5.6025\ 10^3$	$1.6313\ 10^7$
0.2	$5.6011\ 10^3$	$1.6313\ 10^7$
0.3	$5.5988\ 10^3$	$1.6313\ 10^7$
0.4	$5.5948\ 10^3$	$1.6314\ 10^7$
0.5	$5.5879\ 10^3$	$1.6315\ 10^7$
0.6	$5.5758\ 10^3$	$1.6318\ 10^7$
0.7	$5.5550\ 10^3$	$1.6322\ 10^7$
0.8	$5.5207\ 10^3$	$1.6331\ 10^7$
0.9	$5.4667\ 10^3$	$1.6346\ 10^7$
1	$5.6484\ 10^3$	$1.6096\ 10^7$
1.1	$5.3649\ 10^3$	$1.6424\ 10^7$
1.2	$5.5223\ 10^3$	$1.6332\ 10^7$
1.3	$5.4757\ 10^3$	$1.6348\ 10^7$
1.4	$5.4264\ 10^3$	$1.6377\ 10^7$
1.5	$5.4376\ 10^3$	$1.6369\ 10^7$

From the results, the minimum cost of the error is obtained for $m = 1.1$, where the control energy con-

sumption is the greatest. The error cost function is better for FO-MRAC whereas the criterion J_u is better for integer order MRAC, which means that the fractional order MRAC improves the reference tracking by mean of a greater input energy effort.
From the Table 3, there exists a trade off between the control energy consumption and the tracking error cost. The FO-MRAC strategy allows the resolution of this trade off by choosing the proper m value.
It is also worthy to notice that the adaptation gain γ is very small for all fractional orders m (less than 10^{-6}) comparatively to the integer order MRAC ($\gamma = 0.01$) which improves the relative stability of the adaptive control system.

5.2 Study in the Presence of Measurement Noises

In order to test the robustness of these controllers in the case where the measurement is tainted with noise, we have injected a noise to the output (random signal) of zero average and standard deviation equal to 0.03. We study the influence of the fractional order m value on the performance of the FO-MRAC system in presence of measurement noise by varying its value from 0.1 to 1.5, the results are grouped in Table 4.

Table 4: Cost functions Versus the fractional order m in presence of noises.

m	J_e	J_u
0.1	$7.0855\ 10^3$	$1.6168\ 10^7$
0.2	$7.0839\ 10^3$	$1.6168\ 10^7$
0.3	$7.0812\ 10^3$	$1.6168\ 10^7$
0.4	$7.0763\ 10^3$	$1.6169\ 10^7$
0.5	$7.0673\ 10^3$	$1.6170\ 10^7$
0.6	$7.0509\ 10^3$	$1.6173\ 10^7$
0.7	$7.0215\ 10^3$	$1.6177\ 10^7$
0.8	$6.9698\ 10^3$	$1.6185\ 10^7$
0.9	$6.8811\ 10^3$	$1.6201\ 10^7$
1	$8.1872\ 10^3$	$1.5961\ 10^7$
1.1	$6.5166\ 10^3$	$1.6305\ 10^7$
1.2	$6.9527\ 10^3$	$1.6186\ 10^7$
1.3	$6.8560\ 10^3$	$1.6202\ 10^7$
1.4	$6.6940\ 10^3$	$1.6235\ 10^7$
1.5	$6.7269\ 10^3$	$1.6226\ 10^7$

From Table 4, we see that the FO-MRAC is more robust against noise than the classical MRAC strategy, for the reason that with FO-MRAC, we obtain always the lowest error cost and that for any value of the fractional order of integration m. The lowest error cost is obtained for $m = 1.1$.
Figure 7 and Figure 8 illustrate respectively the system output and the control signal for the two applied strategies.

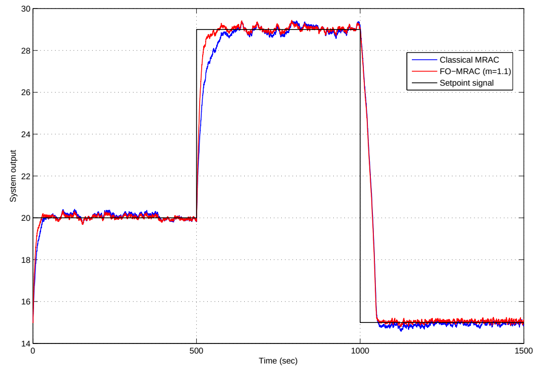

Figure 7: The system response with classical MRAC and with FO-MRAC ($m = 1.1$) in presence of measurement noises.

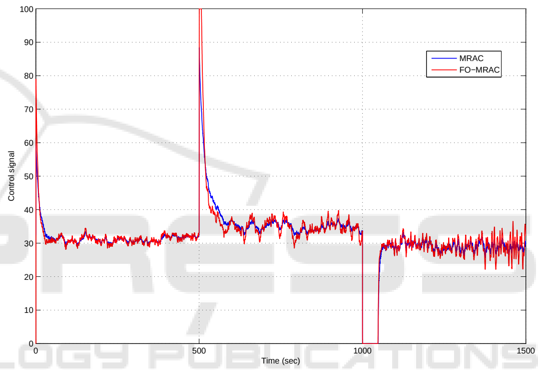

Figure 8: The control signal of the classical MRAC and the FO-MRAC ($m = 1.1$) in presence of measurement noises.

5.3 Study in Case of Model Parametric Variations

No let us test the robustness of the proposed control law with respect to model parametric variations. We consider a set of variation on the parameter values in equation (23) from the instant 1500 *sec*. Results obtained with the classical MRAC and with the FO-MRAC strategy are exposed in Table 5.

Table 5: Cost functions Versus the parameter variation rate.

Param var. %	Classical MRAC		FO-MRAC $m = 1.1$	
	J_e	J_u	J_e	J_u
05	$7.07\,10^3$	$1.65\ 10^7$	$5.13\,10^3$	$1.67\,10^7$
10	$7.55\,10^3$	$1.68\ 10^7$	$5.28\,10^3$	$1.72\,10^7$
20	$8.61\,10^3$	$1.76\ 10^7$	$5.67\,10^3$	$1.79\ 10^7$
40	$1.07\,10^4$	$1.92\ 10^7$	$8.40\,10^3$	$1.94\ 10^7$
60	$1.23\,10^4$	$2.10\ 10^7$	$1.00\,10^4$	$2.12\ 10^7$

The system responses for the two control strategies applied in case of 20% parameter variation and the control signal of the classical MRAC and the FO-MRAC controllers are exposed in Figure 9 and Figure 10.

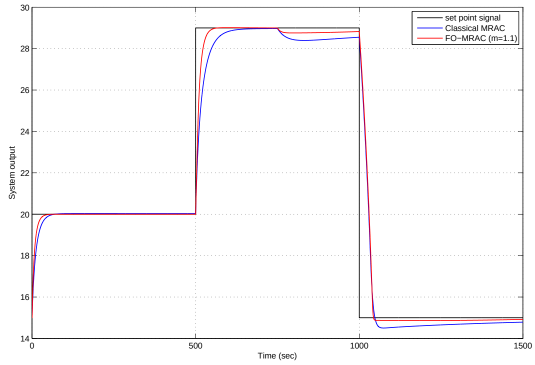

Figure 9: The system response with classical MRAC and with FO-MRAC ($m = 1.1$) in presence of measurement noises.

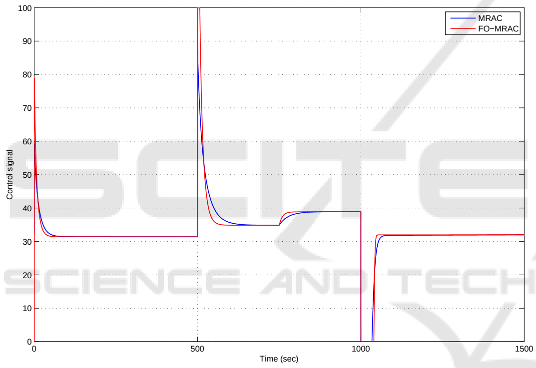

Figure 10: The control signal of the classical MRAC and the FO-MRAC ($m = 1.1$) in presence of measurement noises.

From the simulation results, we see that w the system is affected by the change of parameter value at the instant of 1500 *sec*, after that we observe that the system tries to follow the consign signal, and at the end maintains the desired performance. However, it is also remarkable that with an FO-MRAC strategy, the error signal has the lower amplitude.
From the Table 5, we see that the FO-MRAC has always the lowest error cost, even for sudden parameter variations. So, we conclude that the FO-MRAC is much more robust than the classical MRAC.

6 CONCLUSIONS

A fractional order model reference adaptive control has been designed in order to command a conical tank level with nonlinear dynamics. The reference model is set to be a fractional model system approximated by rational transfer function using the singularity function approach; also a fractional order parameter adaptation law is used to update the controller parameters. Simulation results illustrate the effectiveness of the proposed control scheme and confirm its robustness and especially in the presence of measurement noises and parametric variations.

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