

Anomaly Detection using System Identification Techniques

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Abstract: As cyber-physical systems are becoming more human independent any anomaly or system failure should be detected and solved in an autonomous way. In the last decade significant research was performed to find more intelligent and accurate anomaly detection methods. Most of these methods are analyzing only the output(s) of a system hoping to find some inconsistencies in the data stream. Our attempt is to consider the system's model as well and develop an anomaly detection methodology that tries to identify slight changes in the behavior of the system, detectable through model changes. The key part of our detection method is the system identification step through which we compute the system's model considered as a differential equation between input and output signals or as an autoregression formula. We demonstrate the feasibility of the proposed method through a simulated and a real-life example.

1 INTRODUCTION

In cyber-physical systems anomalies may have various causes like natural ones as environmental noise, communication errors or device faults as well as artificial ones caused by malicious attacks (e.g. virus attacks). In each case an automated method should discriminate between correct and abnormal data. In case of computer controlled systems, incorrect parameter values will produce wrong control decisions which may bring the physical system in an unstable, even dangerous state. Therefore, the measured values should be analyzed, and outlier values should be eliminated before a control decision is made.

There are different kinds of anomalies that should be detected:

- statistically detectable anomalies
- single value outliers (in time series and spatially distributed data)
- abnormal signal shapes or patterns
- system behavior changes

This article is focused mainly on the detection of anomalies from the last category where a slight change in the behavior of a system may be interpreted as an anomaly. As it will be shown in the next paragraphs, the problem of anomaly detection can be solved using system identification techniques. We measure continuously the input and output of a

given system and compute the coefficients of the system's model described as a differential equation or as an auto-regression. Any significant change in these coefficients is considered an anomaly. This detection technique is based on the supposition that a given physical system is not changing its mathematical model in time.

This method can detect anomalies that are not so evident for a human observer. Also, some other kind of anomaly detection methods based only on the continuity or statistical parameters of the output signal may fail to detect changes in the system model. This method also eliminates some false anomalies (that may be detected by other methods) that may occur because of some significant change in the input signals.

The rest of the paper is organized as follows: the next section presents some related work in the area of anomaly detection; section 3 explains the basic idea behind our method, it gives the mathematical background and demonstrates the feasibility of the method through some experiments; the conclusions of this research are summarized in the last chapter.

2 RELATED WORK

In the last decade, a lot of research was performed (Barnett et al., 1994) (Cateni et al, 2008) (Gupta et al, 2014) in the direction of developing new anomaly

detection methods that assure higher accuracy and precision in different domains on interest: economy and business, industry, social sciences, weather and environmental prediction, etc.

One tendency (Agrawal et al, 2015) is to find general methods that work well on a wide range of datasets. These methods exploit some inherent correlations and redundancies present in the collected data, without analyzing the technical significance of the different attributes present in the datasets. For instance, the same clustering method used for anomaly detection may work well on financial data as well as on environmental data or data collected from an industrial process.

The other tendency (Rassam et al, 2013) (Zhang et al, 2010) is toward a more specialized approach where the method and its configuration depend on the applications' domain, the type of data (e.g. statistical data, time series, sensorial data, etc.) and the kind of anomaly taken into consideration. In (Estevez-Tapiador et al, 2014) the authors present a taxonomy of anomaly detection methods based on the above-mentioned criteria.

In principle, any anomaly detection method (Chandola et al, 2009) is built upon some kind of correlation between the samples of some measured parameters (or signals); the correlation may be found in the continuity of a signal (e.g. for time series) or as a functional correlation between different parameters (e.g. given by a physical law). An anomaly is breaking these correlation rules, offering the ground for the automated detection process.

Our proposed method tries to go "behind the scene" in the sense that it does not look only on the collected data, it tries to build the model of the system that generates the data. In our approach any change in the system model (e.g. behavior) is an indicator of a possible anomaly. In this sense we consider that this kind of anomaly detection technique is a different approach and it may solve some cases when more traditional methods fail to detect the anomaly.

3 ANOMALY DETECTION AS A SYSTEM IDENTIFICATION PROBLEM

Most of the anomaly detection techniques used for datasets containing time series try to analyze the output of a given system in order to identify an abnormal value or sequence of values. In these cases, the anomaly detection is based on the

supposition that there is a time correlation between consecutive samples of the same (process) parameter. The graphic of the time series should be a continuous function over time. Any sample that breaks this correlation is considered an anomaly of some kind. Sometimes also spatial correlation (e.g. present in data collected through a distributed sensor network) or any kind of functional correlation between multiple parameters may be used for this purpose.

Unfortunately, in all these cases a significant change in the evolution of the output signal(s) is considered an anomaly. But there are justified cases when the output of a system changes significantly without it being an anomaly.

For instance, in a temperature regulated room a door/window is opened, and the temperature is dropping fast, or in the case of an electric motor the speed varies significantly if the input voltage of the load changes. Therefore, in such systems also the input signal(s) that may produce a change in the output signal(s) has to be considered, eliminating those cases when the drastic change of the output was caused by a justified change in the input.

There are also cases when a system is changing its behavior without a significant change in the output signal. As it will be shown in figures 2 and 3 it is hard even for the human eye to determine the moment when such a change happened. These cases should also be considered as anomalies in the behavior of a system.

The goal of our research was to find an anomaly detection approach that covers both cases: to eliminate false alarms if a change is caused by natural causes and to detect slight changes in the behavior of a system as an indicator for an anomaly.

Figure 1 shows the basic setup of our experiment. We consider that there is a given system (a black box) who's input and output parameters are measurable. A component, called the estimator, tries to build the mathematical model of the given system, based on the measured input and output signals. Any significant change in the system's model detected during the time may be considered as an abnormal behavior or an anomaly.

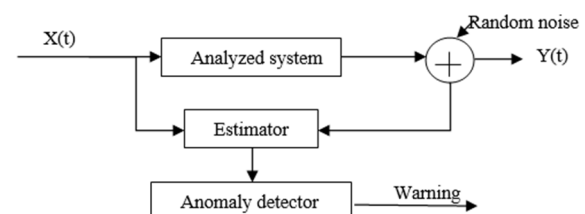


Figure 1: The setup scheme.

3.1 The Mathematical Approach

In most cases a real (physical) system may be approximated with a first order or second order differential equation. This assumption is based on the fact that most real systems have one or maybe two dominant time constants that correspond to some inertial (or energy accumulating) components. For instance, in electronics an RC filter (resistor condenser) is described with a first order differential equation and a circuit with a resistor a condenser and an inductor as a second order equation (the condenser and the inductor are the energy conserving components). In a similar way in mechanics a suspension system composed of a spring and a dumper may be modelled with a second order equation.

A first order differential equation has the following form:

$$dy/dt = A * y(t) + B * x(t) \tag{1}$$

Where: $y(t)$ is the output signal and $x(t)$ is the input signal; A, B are the constants that define the the system model.

In digital domain, the equation becomes:

$$y(n) = a * y(n - 1) + b * x(n) \tag{2}$$

Where: $y(i), x(i)$ are the n^{th} sample of the output and input signals; a, b are constants that describe the first order system.

In a similar way, a second order system is described in digital domain as:

$$y(n) = a * y(n - 1) + b * y(n - 2) + c * x(n) \tag{3}$$

In order to model a given physical system with an acceptable precision a first or a second order differential equation is adopted and then the constant parameters of the equation should be determined. This process is called system identification. There are many methods that can be used for this purpose. For instance, one method works as follows: a step signal is applied at the input of the system; the corresponding values measured at the output of the system will give the integral of the system's transformation function. By differentiating the output signal, we obtain the transformation function of the system.

In our case the above method cannot be applied because the identification process should be continuous, and the input signal is not controlled. Another issue is the noise which affects the signals and consequently the correctness of the equation (2). In the presence of noise equation (2) becomes:

$$y(n) = a * y(n - 1) + b * x(n) + \epsilon(n) \tag{4}$$

Where: $\epsilon(n)$ is the error experienced because of the noise

Therefore, we compute the constants of the first or second order equations using the measured values of the output (y) and input (x) signals. In the first stage we also ignore the error generated by noise. Its effect will be reduced later using a low-pass-filter. For instance, in the case of the first order equation we generate a system of 2 linear algebraic equations with „a” and „b” as unknown variables.

$$\begin{cases} y(n) = a * y(n - 1) + b * x(n) \\ y(n - 1) = a * y(n - 2) + b * x(n - 1) \end{cases} \tag{5}$$

In this way in every sampling moment we obtain a pair of values $a(n)$ and $b(n)$. In the case of a stable physical system and without noise the $a(n)$ and the $b(n)$ values are constant and represent the theoretical model of the physical system. But because of the noise the $a(n)$ and $b(n)$ are strongly affected by noise. Therefore, in order to obtain a “close to constant” value for “a” and “b” we have to apply a low pass filter. The length of the median low pass filter should be adjusted in accordance with the magnitude of the noise. Another, more accurate method of determining the „a” and „b” constants would be to apply a „least square method” that tries to minimize the sum: $\sum \epsilon^2(n)$. But in the case of differential equations this procedure is not a trivial one. As it will be showed through a practical example the simpler method proposed here generate satisfactory results for the anomaly detection purpose.

The next step in the anomaly detection process is to follow the evolution of the filtered „a” and „b” values and determine the moment when the two parameters change their values in a significant (detectable) way. The change in the values signifies a change in the model and consequently a change in the behavior of the system. This change may be interpreted as an anomaly.

3.2 Experimental Demonstration

To show the effectiveness of the proposed anomaly detection method we present an experimental simulated case study of a first order physical system. The experiment considers a RC (resistor condenser) low pass filter as an example of a first order physical system. For a more realistic case we consider that the output signal (the voltage on the condenser) is affected by a white noise. For a unit step input signal, the output is given by the equation:

$$y(t) = Amp * (1 - e^{-(t/RC)}) + noise \quad (6)$$

Where: Amp=5V is the amplitude of the input signal; 1/RC=150s is the time constant of the system

In digital domain the equation of the system (derived from the analog formula) is:

$$y(n) = a * y(n - 1) + b * x(n) + noise \quad (7)$$

Where: a=0.860707976

b=0.139292024

noise=0.05*(0.5-Random(0÷1))

sample period T=0.002s

Figures 2 and 3 show the chart of the input and output signal for a sinus input and a square digital input.

What is not so obvious in figures 2 and 3 is the fact that in both charts the system changed its behavior two times: once at sample time 90 and again at sample time 146. First the 1/RC changed from 150 to 200 and the second time to 250. These changes are reflected in parameters „a” and „b”.

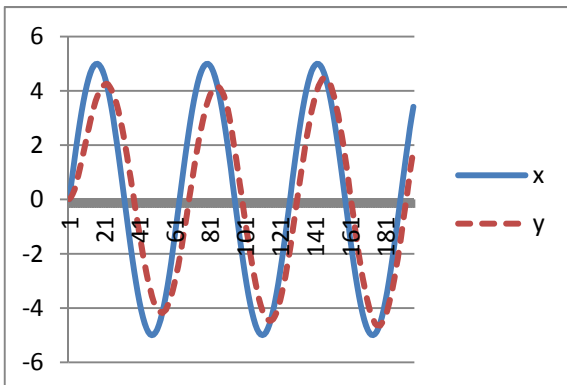


Figure 2: The input and output signals for an RC circuit with sinus input.

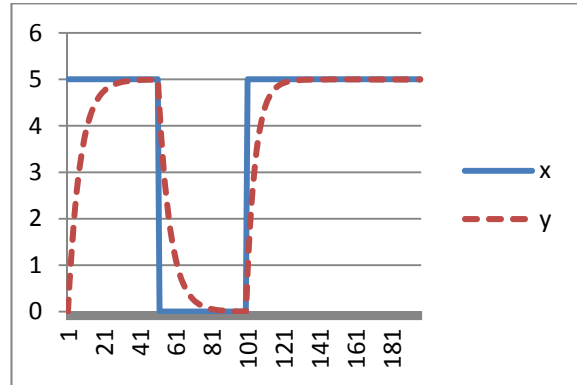


Figure 3: The input and output signals for an RC circuit with digital input.

Figure 4 represents the raw values of „a” and „b” computed using the equation system (5), for each sampling point. It can be seen that the „constant” values are strongly affected by the noise. Therefore, in order to obtain an average value for a and b a median low-pass-filter is applied. After a number of experiments the best results were obtained for a median filter computed on 15 consecutive samples. Figure 4 shows the filtered values of a, b, (a_LPF and b_LPF). It can be seen that the filtered values show a change whenever the system change its behavior (at approximately 85-95 and 143-150). These changes may be detected and considered as anomalies.

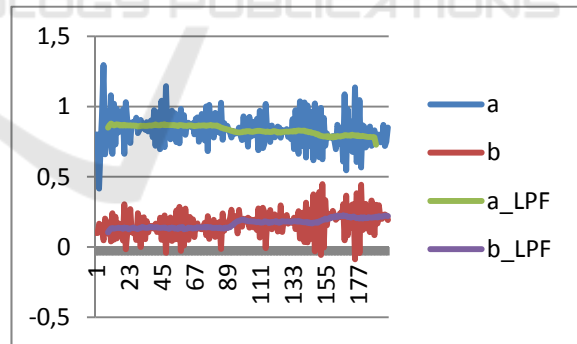


Figure 4: Values of a, b and low-pass filtered a_LPF and b_LPF.

In order to identify the position of the anomaly (of the change) we applied the following sequence of operations:

- Modified derivate of the a_LPF and b_LPF with the formula:
- $a_diff(n) = (a_LPF(n - 2) + a_LPF(n - 1)) - (a_LPF(n + 1) + a_LPF(n + 2)) \quad (8)$

- $b_diff(n) = (b_LPF(n - 2) + b_LPF(n - 1)) - (b_LPF(n + 1) + b_LPF(n + 2))$ (9)
- Threshold the a_diff and b_diff with values between the min and max of the two signals, obtaining a_thr and b_thr
- Detect the points of change with the condition:
- if($a_thr \neq 0$) AND ($b_thr \neq 0$) than detect(n)=1 else detect(n)=0

Figure 5 shows the filtered values (a_LPF , b_LPF) as well as the result of the detection process.

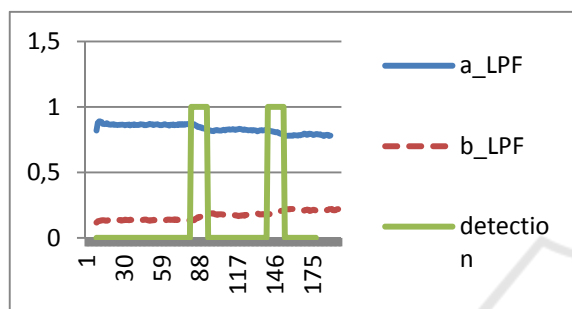


Figure 5: Detection of the anomaly.

This experiment shows that it is possible to detect slight changes in the behavior of a system that may be considered as anomalies. For this purpose, we measured the input and output signals, computed the momentary model of the system (coefficients a and b in the first order differential equations) and detected a significant change in the computed model. As the simulated experiment shows, some parameters of this method (e.g. filter length, threshold values) must be configured in accordance with some characteristics of the analyzed system (e.g. noise level, sampling rate, etc.).

In a very similar way, if the real system has two dominant time constants a second order differential equation may be used for the detection (as presented in equation 3). In this case using a system of 3 equations (similar with 5) we can compute the coefficients a , b and c . Then any significant change in these coefficients will indicate a possible anomaly.

3.3 Variation to the Proposed Detection Method

In a case when the input signal is not available (not known), only the output one is measured (e.g. temperature variations in a given environment, or a vibration on a mechanical device) the system model may be approximated as an autoregression model,

using the following equation:

$$y(n) = \sum_1^N c_k * y(n - k) + c_0 \quad (10)$$

Where: c_k are the coefficients of the model that must be computed

N - the order/length of the regression

In case of more complex systems different variations of autoregression models may be used such as ARMA, ARIMA or ARMAX. In these cases the execution time of the detection method may increase significantly.

In the followings, we present an example of an anomaly detection on a real case: the problem was to identify any damages on the bearing of an electric motor. For this purpose, we measured the vibrations on the chassis of the motor using an acceleration sensor. Figure 6 shows the acceleration signal measured on a motor with damaged bearing. The high frequency signals on the acceleration indicate an anomaly in the bearing.

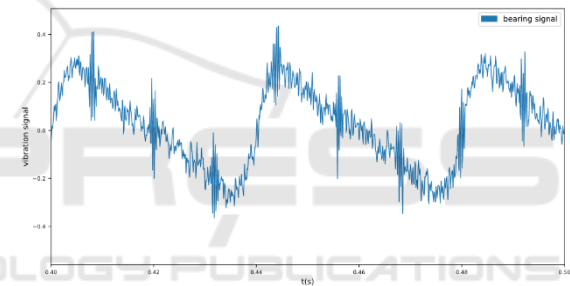


Figure 6: Acceleration signal measured on a faulty motor bearing.

In this case we used the following equation:

$$y(n) = c_2 * y(n - 1) + c_1 * y(n - 2) + c_0 \quad (11)$$

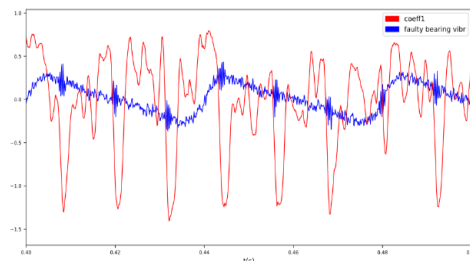


Figure 7: The variation of the c_1 coefficient.

We observed that the coefficient that best reflects the anomaly is in this case c_1 . Figure 7 shows the variation of coefficient c_1 over the original signal.

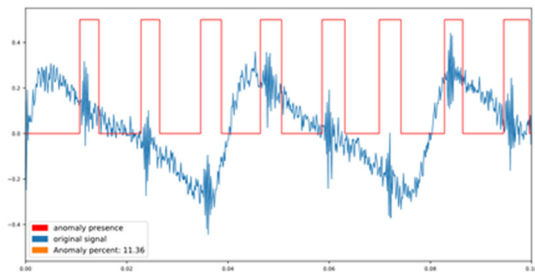


Figure 8: Labeled anomaly regions.

The next step was to apply a thresholding method in order to identify regions with faulty signals. The value of the threshold is determined in this case automatically from the histogram of the C1 values: the threshold value is the lowest point of a „valley” that separate the normal and abnormal c1 values. Figure 8 shows the final result with the labeled values.

4 CONCLUSIONS

This paper showed that anomaly detection methods may be derived from a system identification method. The first example considered a system that may be described with a first order differential equation. The coefficients a and b of the discrete equation show variations that can be exploited for anomaly detection. The second example considered a system where the input signal is not known. In this case an autoregression model was computed. Again, one of the coefficients of the discrete formula could be used for anomaly detection.

In both cases the actual sequence of processing steps needed for an accurate detection had to be adjusted with the specific characteristics of the analyzed system. So, from this point of view a single method cannot be generally applied to any real-life problems. But, with some adjustments the proposed method may solve a wider range of applications.

As it was demonstrated, the proposed anomaly detection method can detect slight changes in the behavior of a given system, that can be interpreted as anomalies and which may not be detected by more traditional methods or even by a human observer. The proposed method also eliminate false anomaly alerts which are caused by significant changes in the input signal that affect also the output; usually other anomaly methods ignore the input signal and its effect on the output signal.

The proposed method is rather simple and may be implemented on embedded devices with limited

computing or storage capabilities, such as microcontrollers or DSPs. It is also recommended for on-line anomaly detection.

As future work, we intend to apply pattern recognition and classification methods (e.g. neural networks and SVM) on the graph of the computed model coefficients in order to discriminate between normal and abnormal system behaviors.

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