Measuring Perceptual Similarity of Syntactically Generated Pictures

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Abstract: This paper shows how similar pictures can be generated using random and bag context picture grammars. An online survey was conducted to determine the similarity of the pictures generated by the picture grammars. Respondents were asked to rank pictures in order of similarity to the query picture. They were also asked to rank galleries of pictures from those containing pictures that are most similar to those containing pictures that are least similar. Furthermore, respondents were required to tell us how they determined the similarity of the pictures contained in the galleries. We then compared perceptual similarity with a chosen similarity measure — spatial color distribution descriptor (SpCD) — to determine if they are consistent. The spatial color distribution descriptor has provided excellent results in determining the similarity of computer-generated pictures, and so was seen as a good similarity measure for this research. The results show that there is a good correlation between the SpCD and perceptual similarity although in some instances humans do make different judgements.

1 INTRODUCTION

Determining picture similarity is a crucial element in many applications that require the comparison of pictures on different aspects, like color, texture, layout, and theme. Applications like search engines, picture database retrieval systems, picture generators and visual password schemes are some of the applications that may require determining the degree of similarity of pictures (Okundaye et al., 2013; Goldberger et al., 2003).

The problem with many picture retrieval systems is analyzing the relationship between how humans perceive similarity (perceptual similarity) and approaches used in content based image retrieval (CBIR). Humans judge the similarity of pictures by considering many features like color, semantics, luminance, texture and objects in the picture (Yamamoto et al., 1999; Zhou and Huang, 2003; Li et al., 2003; Neumann and Gegenfurtner, 2006). Most CBIR systems are based on one or more of these features. Mathematically based similarity measures are capable of finding similar pictures, but people may not find those pictures to be similar. Also, different people can have conflicting opinions on the similarity of a given set of pictures. Thus it is important, in some applications, to determine if the mathematical similarity measures on pictures in some set correspond with the human perceptual similarity measures applied to the same set of pictures.

Determining picture similarity is very important for our research as we focus on generating similar pictures using bag context picture grammars (BCPGs) (Ewert et al., 2017; Mpota, 2018) and random context picture grammars (RCPGs) (Ewert, 2009). An end goal of our work is in generating visual passwords, and appropriate distractors (pictures which are similar to the password picture) for a visual password system and it is thus necessary to evaluate if the generated pictures are similar.

In this work we generate similar pictures using BCPGs and RCPGs. We analyze how humans perceive the similarity of these generated pictures. Lastly we evaluate if perceptual similarity is consistent with the chosen mathematical similarity measure, the spatial color distribution descriptor (SpCD) (Chatzichristofis et al., 2010). The SpCD is a compact composite descriptor which combines color and spatial color distribution information (Chatzichristofis et al., 2010). This color model was observed as providing better retrieval results for syntactically generated pictures than color correlograms in (Okundaye et al., 2013).

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We conducted an online survey to determine how humans determine the similarity of syntactically generated pictures and to determine if the human view of similarity is consistent with the selected mathematical similarity measure. The results of the online survey are then compared with the results of applying the SpCD to the same pictures. We used cumulative discounted gain (DCG) to evaluate the consistency of ranking of perceptual similarity and the SpCD.

The rest of the paper is structured as follows: Section 2 presents the background information on perceptual similarity, picture grammars and the spatial color distribution descriptor. Section 3 presents the results of the online survey and the spatial color distribution descriptor in measuring the similarity of some pictures. Section 4 presents the evaluation of the results, and Section 5 provides the conclusion.

2 BACKGROUND

2.1 Perceptual Similarity

To understand visual perception, several researchers have tried to support their findings of mathematical similarity measures with human perception. For example, (Kiranyaz et al., 2010) tried to model the human perception of color. They observed that the human eye could not distinguish close colors well or identify a broad number of colors. Thus they showed that humans only use a few outstanding colors to judge similarity. In their research, they "have presented a systematic approach to extract such a perceptual (color) descriptor and then proposed an efficient similarity metric to achieve the highest discrimination power possible for color-based retrieval in general-purpose image databases". Moreover, (Yamamoto et al., 1999) conducted an experiment to evaluate the correlation between the similarity function and human perception. In addition, (Okundaye et al., 2014) conducted an online survey in which they required respondents to arrange pictures in the order of similarity to a given picture. This was important for their research, as the generated pictures were for a visual password system.

2.2 Picture Grammars

The pictures used in this study were generated using syntactic methods of picture generation, in particular bag context picture grammars and random context picture grammars. Both grammar classes are context-free grammars with regulated rewriting. In an RCPG each production rule has two sets of variables, the so-called permitting and forbidding context sets, which regulate the application of the rule during a derivation. A BCPG has a k-tuple of integers, called the bag, which regulates the application of rules during a derivation and changes during a derivation. Formal definitions of BCPGs and RCPGs are given below.

2.3 Definitions

In this section, we present notation and definitions. In particular, we define bag context picture grammars and random context picture grammars. Many of the definitions are from (Drewes et al., 2008; Ewert, 2009; Ewert et al., 2017), and have been modified where appropriate.

2.3.1 Preliminaries

Let $\mathbb{N} = \{0,1,2,\ldots\}$, $\mathbb{N}_+ = \{1,2,\ldots\}$ and $\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$. The sets $\mathbb{N}\cup\{\infty\}$ and $\mathbb{Z}\cup\{-\infty,\infty\}$ are denoted by $\mathbb{N}_\infty$ and $\mathbb{Z}_\infty$, respectively. Moreover, for $k \in \mathbb{N}_+$, let $[k] = \{1,2,\ldots,k\}$.

Let $k \in \mathbb{N}_+$. If $I = [k]$, then elements of $\mathbb{Z}^I_\infty$ (which includes $\mathbb{Z}^I$) are written as k-tuples. On $\mathbb{Z}^I_\infty$, addition, subtraction, and scalar multiplication are defined componentwise in the usual way. Similarly, $\leq$ denotes componentwise ordering. An element q of $\mathbb{Z}_\infty$ which occurs in the place of a k-tuple, denotes the k-tuple of the appropriate size with all components equal to q.

2.3.2 Bag Context Picture Grammars

Bag context picture grammars generate pictures using productions of the form in Figure 1, where A is a variable, x_{11}, $x_{12},\ldots,x_{mm}$ are variables or terminals for $m \in \mathbb{N}_+$, and λ, μ and δ are k-tuples for some $k \in \mathbb{N}_+$. The interpretation is as follows: if a developing picture contains a square labelled A and if the bag is within the range defined by the lower limit λ and upper limit μ of the rule, then the square labelled A may be divided into equal squares with labels $x_{11},x_{12},\ldots,x_{mm}$ and the bag adjusted with δ.

We denote a square by a lowercase Greek letter, eg., (A,α) denotes a square α labelled A. If α is a square, then $\alpha_{11},\alpha_{12},\ldots,\alpha_{mm}$ denote the equal subsquares into which α can be divided, with, eg., α_{11} denoting the bottom left one.

Definition 1. A bag context picture grammar $G = (V_N,V_T,P,(S,\sigma),I,\beta_0)$ has a finite alphabet V of labels, consisting of disjoint subsets V_N of variables and V_T of terminals. P is a finite set of production rules. There is an initial labelled square (S,σ) with $S \in V_N$.

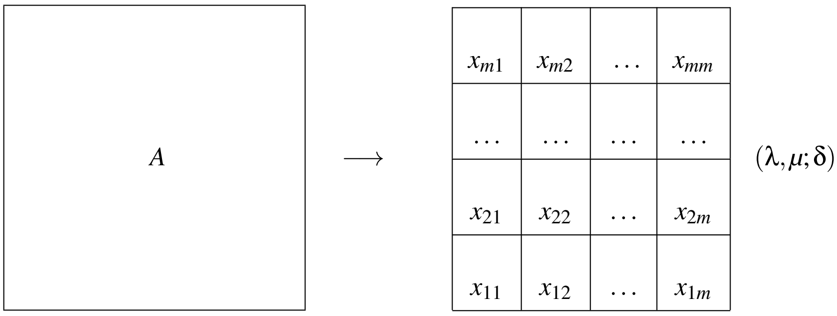


Figure 1: Production in BCPG.

Finally, I denotes a finite bag index set and $\beta_0 \in \mathbb{Z}^I$ the initial bag.

A rule in P is of the form $A \to [x_{11}, x_{12}, \ldots, x_{mm}]\ (\lambda, \mu; \delta)$, $m \in \mathbb{N}_+$, where $A \in V_N$, $\{x_{11}, x_{12}, \ldots, x_{mm}\} \subseteq V$, $\lambda, \mu \in \mathbb{Z}^I_\infty$, and $\delta \in \mathbb{Z}^I$. The k-tuples λ and μ are the lower and upper limits respectively, while δ is the bag adjustment.

Definition 2. A pictorial form is any finite set of non-overlapping labelled squares in the plane. The size of a pictorial form Π is the number of squares contained in it, denoted $|\Pi|$. If Π is a pictorial form, we denote by $l(\Pi)$ the set of labels used in Π.

Definition 3. Let $G = (V_N, V_T, P, (S, \sigma), I, \beta_0)$ be a BCPG, Π and Γ pictorial forms, and $\beta, \beta' \in \mathbb{Z}^I_\infty$. Then we write $(\Pi, \beta) \Longrightarrow (\Gamma, \beta') \in \prod \times \mathbb{Z}^I_\infty$. There is a derivation step from (Π, β) to (Γ, β') if there is a production $A \to [x_{11}, x_{12}, \ldots, x_{mm}]\ (\lambda, \mu; \delta)$ in P, Π contains a labelled square (A, α), $\lambda \le \beta \le \mu$, $\Gamma = (\Pi \setminus \{(A, \alpha)\}) \cup \{(x_{11}, \alpha_{11}), (x_{12}, \alpha_{12}), \ldots, (x_{mm}, \alpha_{mm})\}$, and $\beta' = \beta + \delta$. We denote the derivation step by $(\Pi, \beta) \Longrightarrow (\Gamma, \beta')$. As usual, $\Longrightarrow^*$ denotes the reflexive transitive closure of $\Longrightarrow$.

Definition 4. The (bag context) gallery generated by a BCPG $G = (V_N, V_T, P, (S, \sigma), I, \beta_0)$ is the set $\mathcal{G}(G) = \{\Phi \mid (\{(S, \sigma)\}, \beta_0) \Longrightarrow^* (\Phi, \beta), \text{with } l(\Phi) \subseteq V_T \text{ and } \beta \in \mathbb{Z}^I_\infty\}$. An element of $\mathcal{G}(G)$ is called a picture.

Definition 5. Let Φ be a picture in the square σ. For any $m \in \mathbb{N}_+$, let σ be divided into equal subsquares, say $\sigma_{11}, \sigma_{12}, \ldots, \sigma_{mm}$. A subpicture Ψ of Φ is any subset of Φ that fills a square σ_{ij}, with $i, j \in [m]$, i.e., the union of all the squares in Ψ is the square σ_{ij}; Ψ is called a proper subpicture of Φ if $\Psi \neq \Phi$.

In the following, we give a brief example of a BCPG. Detailed examples of BCPGs and bag context galleries can be found in (Ewert et al., 2017; Mpota, 2018).

Example 1. Consider the BCPG $G_{\text{carpet}} = (V_N, V_T, P, (S, \sigma), [8], (1,0,0,0,0,0,0,0))$, where $V_N = \{S, T, U, F, C\}$, $V_T = \{w, b, g\}$ and P is the set of rules in Figure 2. Terminals w, b and g represent white, purple and green circles, respectively.

G_{carpet} generates a variation on the sequence of pictures approximating the Sierpiński carpet (Bhika et al., 2007). The corresponding gallery contains, amongst others, the pictures in Figures 4–7.

Rule 2 divides every square labelled S into nine equal subsquares, eight of which are labelled T and the central one w. All occurrences of T can turn into U (Rule 3) and then S again (Rule 6). Therefore the initial square is divided into increasingly smaller subsquares. All subsquares are of the same size, apart from those that are labelled by the terminal w. The cycle of rules, Rules 2–3–6–2 … cannot be repeated arbitrarily often. On the contrary, Rule 3 can be applied maximally 72 times, as bag position 8 of the upper limit is set to 71. Therefore the subsquares cannot become arbitrarily small.

Once this cycle has stopped, every label T is eventually turned into C, which becomes one of b, g or C in a specific order. Consider Rules 8, 9 and 10. Rule 8 has to be applied exactly five times before Rule 9 can be applied. Similarly, Rule 9 has to be applied exactly three times before Rule 10 can be applied. The latter rule resets the counters for terminals b and g (bag positions 6 and 7) to zero. Once Rule 10 has been applied exactly once, Rule 8 is enabled again. This cycle is enforced by positions 6 and 7 of the lower and upper limits in these rules. This ensures that, for every white circle on the lowest level of refinement, there are five purple and three green circles. □

2.3.3 Random Context Picture Grammars

Random context picture grammars generate pictures using productions of the form in Figure 3, where A is a variable, $x_{11}, x_{12}, \ldots, x_{mm}$ are variables or terminals for $m \in \mathbb{N}_+$, and $\mathcal{P}$ and $\mathcal{F}$ are sets of variables. The interpretation is as follows: if a developing picture contains a square labelled A and if all variables of $\mathcal{P}$

$$
\begin{aligned}
S &\rightarrow [T,T,T,T,w,T,T,T,T]\,((1,0,0,0,0,0,0,0)\,,(\infty,\infty,0,\infty,\infty,\infty,\infty,\infty)\,; && (1)\\
&\qquad (-1,8,0,0,0,0,0,0)) && (2)\\
T &\rightarrow U\,((0,1,0,0,0,0,0,0)\,,(0,\infty,\infty,0,\infty,\infty,\infty,71)\,;(0,-1,1,0,0,0,0,1))\ | && (3)\\
&\quad F\,((0,1,0,0,0,0,0,0)\,,(0,\infty,0,0,\infty,\infty,\infty,0)\,;(0,-1,0,1,0,0,0,0))\ | && (4)\\
&\quad C\,((0,1,0,1,0,0,0,0)\,,\infty;(0,-1,0,0,1,0,0,0)) && (5)\\
U &\rightarrow S\,((0,0,1,0,0,0,0,0)\,,(\infty,0,\infty,\infty,\infty,\infty,\infty,\infty)\,;(1,0,-1,0,0,0,0,0)) && (6)\\
F &\rightarrow C\,((0,0,0,1,0,0,0,0)\,,(\infty,0,\infty,\infty,\infty,\infty,\infty,\infty)\,;(0,0,0,-1,1,0,0,0)) && (7)\\
C &\rightarrow b\,((0,0,0,0,1,0,0,0)\,,(\infty,\infty,\infty,\infty,\infty,4,\infty,\infty)\,;(0,0,0,0,-1,1,0,0))\ | && (8)\\
&\quad g\,((0,0,0,0,1,5,0,0)\,,(\infty,\infty,\infty,\infty,\infty,\infty,2,\infty)\,;(0,0,0,0,-1,0,1,0))\ | && (9)\\
&\quad C\,((0,0,0,0,1,5,3,0)\,,\infty;(0,0,0,0,0,-5,-3,0)) && (10)
\end{aligned}
$$

Figure 2: Rules for grammar G_{carpet}.

and none of $\mathcal{F}$ appear as labels of squares in the picture, then the square labelled A may be divided into equal squares with labels $x_{11}, x_{12}, \ldots, x_{mm}$.

Definition 6. A random context picture grammar $G = (V_N, V_T, P, (S, \sigma))$ has a finite alphabet V of labels, consisting of disjoint subsets V_N of variables and V_T of terminals. P is a finite set of productions of the form $A \rightarrow [x_{11}, x_{12}, \ldots, x_{mm}](\mathcal{P}; \mathcal{F})$ with $m \in \mathbb{N}_+$, where $A \in V_N$, $x_{11}, x_{12}, \ldots, x_{mm} \in V$ and $\mathcal{P}, \mathcal{F} \subseteq V_N$. Finally, there is an initial labelled square (S, σ) with $S \in V_N$.

Definition 7. For an RCPG G and pictorial forms Π and Γ, we write $\Pi \Longrightarrow \Gamma$ if there is a production $A \rightarrow [x_{11}, x_{12}, \ldots, x_{mm}](\mathcal{P}; \mathcal{F})$ in G, Π contains a labelled square (A, α), $l(\Pi \setminus \{(A, \alpha)\}) \supseteq \mathcal{P}$ and $l(\Pi \setminus \{(A, \alpha)\}) \cap \mathcal{F} = \emptyset$, and $\Gamma = (\Pi \setminus \{(A, \alpha)\}) \cup \{(x_{11}, \alpha_{11}), (x_{12}, \alpha_{12}), \ldots, (x_{mm}, \alpha_{mm})\}$. As above, $\Longrightarrow^*$ denotes the reflexive transitive closure of $\Longrightarrow$.

Definition 8. The (random context) gallery $\mathcal{G}(G)$ generated by a grammar $G = (V_N, V_T, P, (S, \sigma))$ is $\{\Phi \mid \{(S, \sigma)\} \Longrightarrow^* \Phi \text{ and } l(\Phi) \subseteq V_T\}$. An element of $\mathcal{G}(G)$ is called a picture.

Examples of RCPGs and random context galleries can be found in (Ewert, 2009). It has been shown that every RCPG can be written as a BCPG (Ewert et al., 2017; Mpota, 2018).

2.4 Spatial Color Distribution Descriptor

One of the key elements in this research is to determine the similarity of the generated pictures. There are many content based image retrieval systems that measure the similarity of pictures based on different features, like color, texture, content and layout. The main feature of the pictures generated in this research is color, and hence color descriptors were considered more appropriate. There exist many color descriptors for measuring similarity. We have considered several CBIR systems and decided to use the spatial color distribution descriptor (Chatzichristofis et al., 2010), since (Okundaye et al., 2013) observed that the SpCD provided better retrieval results for syntactically generated pictures than correlograms (Huang et al., 1997), color histograms (Swain and Ballard, 1991) or other color features. Although tree edit distance, which was introduced in (Pawlik and Augsten, 2011), was found to generate good results for pictures generated by tree grammars (Okundaye et al., 2013), we chose not to use it, as the pictures in this research were not generated using tree grammars. There also exist many CIBR systems that include spatial information. The SpCD is a compact composite descriptor which combines color and spatial color distribution (Chatzichristofis et al., 2010). This descriptor is suitable for colored pictures that contain a small number of colors and texture regions, eg., hand-drawn sketches and colored graphics such as the ones generated by picture grammars. We calculated similarity according to this descriptor with the Img(Rummager) application (Chatzichristofis et al., 2009).

3 RESULTS

For this research, it is important to measure if perceptual similarity correlates to the SpCD, because we need to be sure that the results from the mathematical measure reflect what people think. For this, we conducted an online survey to evaluate the level of consistency between perceptual similarity and the SpCD.

We obtained 408 responses through the online survey. Most of the respondents were staff members or students from the University of the Witwatersrand, Johannesburg. Other respondents were contacts of the

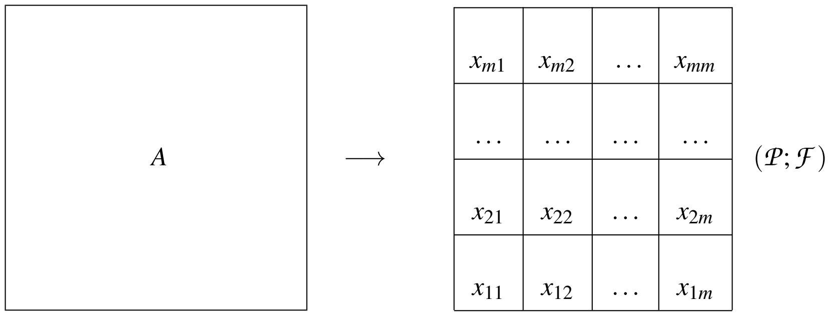


Figure 3: Production in RCPG.

authors.

The survey contained the following points:

Ranking of Pictures in Gallery: We showed the respondents the galleries in Figures 4–10. For each gallery a respondent had to rank the pictures in the gallery in terms of how similar they felt the picture was to a given picture, in particular the picture with label (c) in each gallery. In the ranking, the value 1 was given to the most similar picture and 5 to the least similar picture. The picture (c) was used both as the query picture and as a picture in the gallery to check for outliers.

Similarity of Pictures in Gallery: For Figures 4–10, we asked respondents to select the statement that best described the similarity of the pictures in that gallery. The statements were:

- not at all similar,
- somehow similar,
- similar,
- very similar, and
- identical.

Ranking of Galleries: For Figures 4–7 and Figures 8–10, respectively, we asked respondents to rank the galleries from the gallery with the pictures that are most similar to each other to the gallery with the pictures that are least similar to each other.

Factors that Determine Similarity: We asked respondents which factor they considered the most important when determining the similarity of the pictures in a gallery. We provided them with the following options:

- colors present in the picture,
- distribution of the colors in the picture,
- objects in the picture,
- distribution of the objects in the picture, and
- other (specify).

3.1 Ranking of Pictures in Gallery

As stated above, picture (c) was used as the query picture in each gallery, i.e., all pictures were compared to picture (c) to determine their similarity to it.

The results of the SpCD and the online survey are presented in Tables 1–7. Each table is structured as follows:

Rank: The first column presents the picture ranking from 1 (most similar) to 5 (least similar).

SpCD: The second column presents the SpCD. It is divided into two columns, the first giving the picture label and the second its SpCD value.

Perceptual: The third column presents the perceptual similarity. It is divided into two columns, the first giving the picture label and the second its average perceptual ranking.

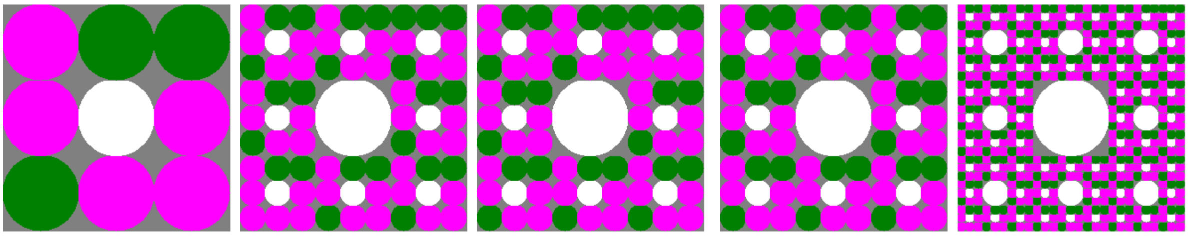
The average perceptual ranking (or score) AV over all the respondents was calculated as:

$$\mathrm{AV} = \frac{\sum_i^n w_i x_i}{\sum_i^n x_i} \ , \tag{11}$$

where

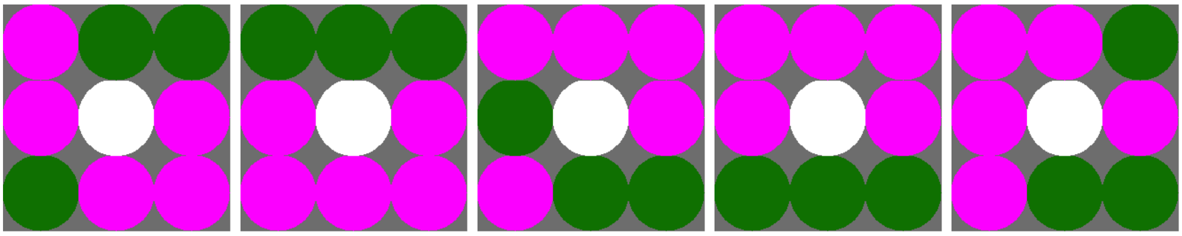
- n is the number of ranks,
- w_i is the weight of the rank, where the picture that was ranked as the most similar is given the weight of 5 and the least similar picture is given the weight of 1, and
- x_i is the number of responses for each possible answer.

For the ranking according to the SpCD, the picture with the smallest value is the most similar to the query picture, while the picture with the largest value is the least similar to the given picture. On the other hand, for the ranking according to the perceptual similarity, the picture with the largest value is the most similar to the query picture and the picture with the smallest value the least similar.



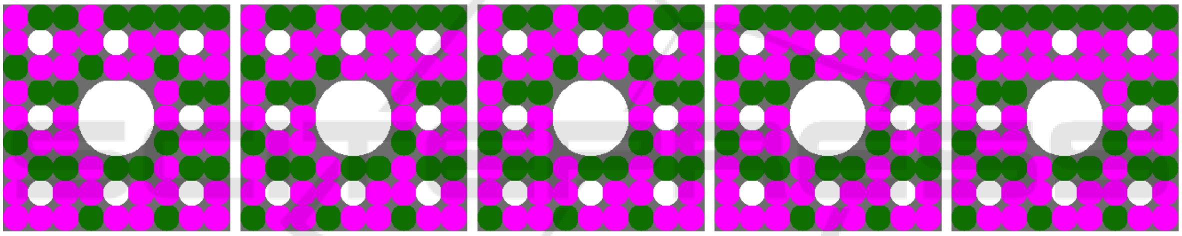
(a) (b) (c) (d) (e)

Figure 4: Gallery A: Sierpiński carpet, different refinements.



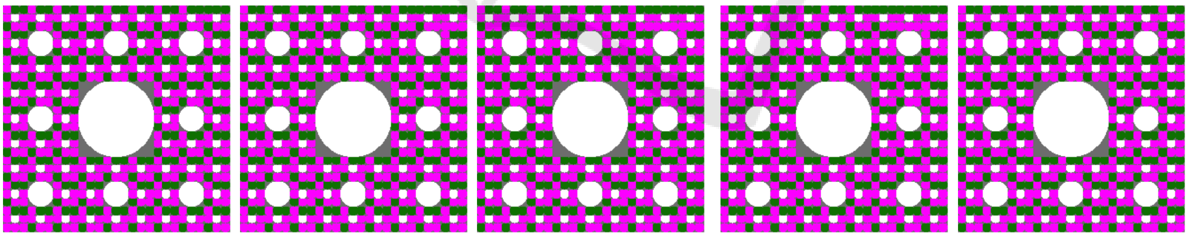
(a) (b) (c) (d) (e)

Figure 5: Gallery B: Sierpiński carpet, first refinement.



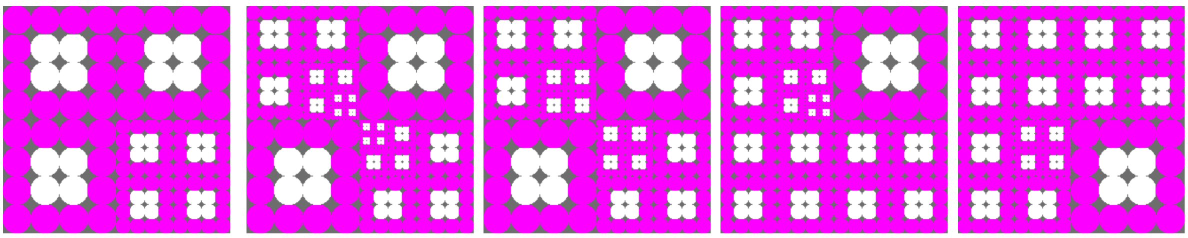
(a) (b) (c) (d) (e)

Figure 6: Gallery C: Sierpiński carpet, second refinement.



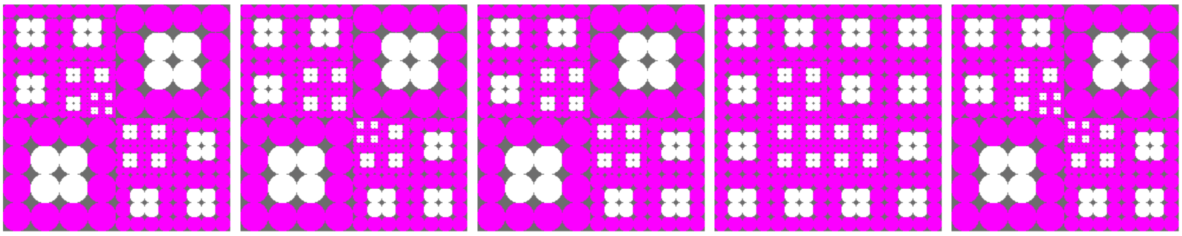
(a) (b) (c) (d) (e)

Figure 7: Gallery D: Sierpiński carpet, third refinement.



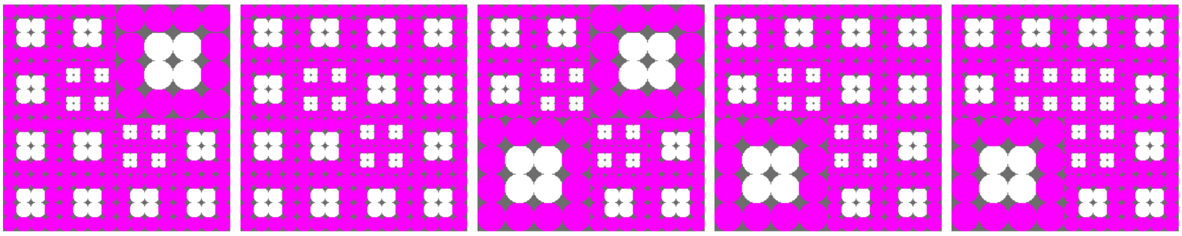
(a) (b) (c) (d) (e)

Figure 8: Gallery E: Flowers.



(a) (b) (c) (d) (e)

Figure 9: Gallery F: Flowers.



(a) (b) (c) (d) (e)

Figure 10: Gallery G: Flowers.

Table 1: Similarity of Figure 4(c) to pictures in Figure 4.

	SpCD		Perceptual	
Rank	**Picture**	**Value**	**Picture**	**Score**
1	c	0	c	4.53
2	d	1.932	b	3.80
3	b	2.782	d	3.31
4	e	8.601	e	1.89
5	a	33.425	a	1.62

The SpCD values and the average perceptual ranking cannot be compared directly, because they use different unit measures. We therefore compare the ranking of the pictures by the two measures.

In the following, each of Tables 1–7 is discussed briefly.

Consider Table 1, which gives the results for Gallery A in Figure 4. For this gallery, the SpCD is to a degree consistent with human perceptual similarity as three of the five pictures were ranked the same for both similarity measures.

Consider Table 2, which gives the results for Gallery B in Figure 5. For this gallery, the SpCD is to a degree consistent with human perceptual similarity. Three of the five pictures were ranked at the same positions. There is a small score difference of 0.17 in the perceptual similarity of the remaining two pictures, implying that respondents found these pictures to be very similar.

Consider Table 3, which gives the results for Gallery C in Figure 6. For this gallery, the SpCD is to a degree consistent with human perceptual similarity. Both measures ranked the first picture on the same position. However, there is a difference at Ranks 2 and

Table 2: Similarity of Figure 5(c) to pictures in Figure 5.

	SpCD		Perceptual	
Rank	**Picture**	**Value**	**Picture**	**Score**
1	c	0	c	4.79
2	d	23.437	e	3.00
3	e	30.126	d	2.83
4	a	67.653	a	2.79
5	b	73.297	b	1.59

Table 3: Similarity of Figure 6(c) to pictures in Figure 6.

	SpCD		Perceptual	
Rank	**Picture**	**Value**	**Picture**	**Score**
1	c	0	c	4.67
2	a	2.394	b	3.51
3	b	3.426	a	3.30
4	e	4.830	d	1.83
5	d	5.291	e	1.79

3. Both measures place pictures (a) and (b) at these ranks, but in different orders. However, the difference of the weighted score of 0.21 suggests that respondents found these pictures to be very similar. There is a similar situation at Ranks 4 and 5. Both measures place pictures (d) and (e) at these ranks, but in different orders. Also in this case, the difference of the weighted score of 0.04 suggests that respondents found these pictures to be very similar.

Consider Table 4, which gives the results for Gallery D in Figure 7. It shows no correlation between the two measures. In fact, the ranking of the pictures in the online survey suggests that the respondents

Table 4: Similarity of Figure 7(c) to pictures in Figure 7.

	SpCD		Perceptual	
Rank	Picture	Value	Picture	Score
1	c	0	d	4.13
2	e	0	b	3.76
3	a	0.433	c	3.29
4	b	0.433	a	2.62
5	d	0.433	e	1.39

Table 5: Similarity of Figure 8(c) to pictures in Figure 8.

	SpCD		Perceptual	
Rank	Picture	Value	Picture	Score
1	c	0	c	4.68
2	b	0	b	3.97
3	d	0.894	a	2.51
4	a	0.907	d	2.24
5	e	1.865	e	1.63

were not able to tell the difference between the pictures. For example, the respondents ranked Figure 7(c), which is the query picture, third instead of first. This might be because the pictures in this gallery have very small subpictures, which might have made it difficult for respondents to distinguish one picture from another. It is worth noting that the SpCD values for this gallery are very small, and that the difference between values in Table 4 is small compared to that in Tables 1, 2 and 3. Moreover, we observe that the SpCD could not measure the difference between pictures which are different. For example, it ranked pictures (c) and (e) as identical, and similarly pictures (a), (b) and (d). We assume the underlying reason is that the SpCD cannot measure the difference between pictures that have such small subpictures.

Consider Table 5, which gives the results for Gallery E in Figure 8. For this gallery, the SpCD is to a degree consistent with human perceptual similarity. Three pictures were ranked the same by both measures. The measures differed at Ranks 3 and 4. However, the difference of the weighted score of 0.27 suggests that respondents found these pictures to be very similar.

Consider Table 6, which gives the results for Gallery F in Figure 9. In this case, the SpCD is consistent with human perceptual similarity as both measures ranked the pictures in the same order.

Consider Table 7, which gives the results for Gallery G in Figure 10. For this gallery, the SpCD is to a degree consistent with human perceptual similarity. Two pictures, namely pictures (c) and (b), were ranked the same by both measures. Moreover, both measures ranked picture (d) higher than picture (e). However, human perceptual similarity ranked picture (a) higher than pictures (d) and (e) whereas the SpCD

Table 6: Similarity of Figure 9(c) to pictures in Figure 9.

	SpCD		Perceptual	
Rank	Picture	Value	Picture	Score
1	c	0	c	4.63
2	a	0	a	3.48
3	b	0	b	3.31
4	e	0	e	2.44
5	d	1.276	d	1.31

Table 7: Similarity of Figure 10(c) to pictures in Figure 10.

	SpCD		Perceptual	
Rank	Picture	Value	Picture	Score
1	c	0	c	4.76
2	d	0	a	3.46
3	e	0.610	d	2.95
4	a	0.625	e	1.97
5	b	0.625	b	1.81

ranked picture (a) lower than pictures (d) and (e). Moreover, the SpCD assigned pictures (a) and (b) the same values, whereas perceptual similarity did not consider these pictures to be identical.

3.2 Similarity of Pictures in Gallery

In the second section of the survey, respondents were shown Figures 4–10 and asked to select the statement that best described the similarity of the pictures within that gallery. The options were: not at all similar, somehow similar, similar, very similar, and identical.

Consider Table 8, which shows how humans evaluated the similarity of the pictures within each gallery. The value 1 indicates that the pictures are not at all similar, while 5 indicates that the pictures are identical. All the values in Table 8 are higher than 1, which implies that the respondents found the pictures to be similar to some degree. The highest values are for Galleries C and D in Figures 6 and 7, which implies that the respondents considered these galleries to have the pictures that are most similar to each other.

Table 8: Perception of similarity of pictures in each gallery.

Rank	Gallery	Perceptual value
1	A	1.82
2	B	2.40
3	C	3.12
4	D	3.64
5	E	2.17
6	F	2.66
7	G	2.24

Table 9: Ranking of galleries in Figures 4–7 according to similarity of pictures in gallery.

Rank	Gallery	Perceptual value
1	D	3.64
2	C	3.12
3	B	2.40
4	A	1.82

3.3 Ranking of Galleries

In the third section of the survey, respondents were asked to rank the galleries in Figures 4–7 and Figures 8–10, respectively, from the gallery containing pictures that are most similar to each other to the gallery containing pictures that are least similar to each other.

Consider Table 9, which gives the results for Figures 4–7. Humans ranked Gallery D in Figure 7 highest, i.e., as the gallery with pictures that are most similar to each other. This view correlates with the SpCD measures for this gallery (Table 4), which are very low (0 or 0.433), which implies that the pictures are very similar to the query picture and each other. Humans ranked Gallery C in Figure 6 second. This view correlates with the SpCD measures for this gallery (Table 3), which are the second lowest for the four galleries under consideration. Humans ranked Gallery A in Figure 4 last, i.e., as the gallery with pictures that are least similar to each other. This does not correlate with the SpCD measures for the four galleries. The SpCD values are the highest for Gallery B in Figure 5. Moreover, they differ a great deal from one picture to another, implying that Figure 5 is the gallery containing the least similar pictures. A possible explanation for this discrepancy might be that humans considered it important that the objects in Figure 5 have the same size, whereas the SpCD measures the distribution of colors.

We observe that the SpCD values for Gallery A (Table 1) differ greatly from one picture to another. This implies dissimilarity between the pictures, but these differences are not bigger than those for Gallery B (Table 2), rather the opposite.

Consider Table 10, which gives the results for Figures 8–10. Humans ranked Gallery F in Figure 9 highest, i.e., as the gallery with pictures that are most similar to each other. This view correlates with the SpCD measures for this gallery (Table 6). Four pictures have the value 0, which means that the SpCD measure found them to be identical to the query picture. The pictures are not identical, but this result shows that both measures found these pictures to be very similar. Humans ranked Gallery G in Figure 10 second. This view correlates with the SpCD measures for this gallery (Table 7), which are the second highest for the three galleries under consideration. Humans ranked Gallery E in Figure 8 third. This view correlates with the SpCD measures for this gallery (Table 5), which are the highest for the three galleries under consideration.

Table 10: Ranking of galleries in Figures 8–10 according to similarity of pictures in gallery.

Rank	Gallery	Perceptual value
1	F	2.66
2	G	2.24
3	E	2.17

3.4 Factors that Determine Similarity

In the last section of the survey, respondents were asked which factor was most important to them when determining the similarity of the pictures in a gallery. Table 11 shows the factors that respondents considered important, and the percentage of respondents for each factor.

4 EVALUATION

Only one gallery, namely Gallery D in Figure 7, showed no correlation at all between the SpCD and perceptual similarity in ranking the pictures. This gallery was treated as an outlier as humans failed to rank the picture which was used as the query picture correctly. One gallery, namely Gallery F in Figure 9, had the same ranking for both the SpCD and perceptual similarity. For four galleries, namely the galleries A–C and E (Figures 4–6 and 8), the correlation was high, in that there were more pictures that were ranked the same by both measures than pictures that were not. In the remaining gallery, Gallery G in Figure 10, there were more pictures that were ranked differently by both measures than pictures that were ranked the same.

It is important to evaluate the effectiveness of the SpCD in representing perceptual similarity. Such an evaluation will aid us in determining whether or not the SpCD is consistent with perceptual similarity and direct the future research. In this evaluation, we use cumulative discounted gain (DCG) (Järvelin and Kekäläinen, 2000), which evaluates the ranking of documents. The key feature in DCG is that highly relevant documents should be ranked higher than the less relevant ones. Since, in this survey, the main focus was on the ranking of pictures, cumulative discounted gain was deemed to be the best method to evaluate the consistency between perceptual similarity and the SpCD. We furthermore present the evaluation by the

Table 11: Most important factor when determining similarity of pictures.

Rank	Factor	%
1	Distribution of the objects in the picture	46.46
2	Distribution of the colors in the picture	28.54
3	Objects in the picture	14.39
4	Colors present in the picture	6.31
5	Other: symmetry; both distribution of colors and objects in the picture; subshapes; patterns	4.29

Table 12: DCG calculation for Table 1.

	SpCD (DCG)			Perceptual (iDCG)		
i	**Picture**	**rating** (i)	**DCG**	**Picture**	**rating** (i)	**iDCG**
1	c	5	$\frac{5}{\log_2(1+1)}$	c	5	$\frac{5}{\log_2(1+1)}$
2	d	3	$\frac{3}{\log_2(1+2)}$	b	4	$\frac{4}{\log_2(1+2)}$
3	b	4	$\frac{4}{\log_2(1+3)}$	d	3	$\frac{3}{\log_2(1+3)}$
4	e	2	$\frac{2}{\log_2(1+4)}$	e	2	$\frac{2}{\log_2(1+4)}$
5	a	1	$\frac{1}{\log_2(1+5)}$	a	1	$\frac{1}{\log_2(1+5)}$
	$\mathrm{DCG} = \sum_{i=1}^{5} \frac{\mathrm{rating}(i)}{\log_2(i+1)} = 10.138$			$\mathrm{iDCG} = \sum_{i=1}^{5} \frac{\mathrm{rating}(i)}{\log_2(i+1)} = 10.269$		

Table 13: NDCG results.

Table	DCG	iDCG	NDCG
1	10.138	10.269	0.987
2	10.138	10.269	0.987
3	10.095	10.269	0.983
5	10.200	10.269	0.993
6	10.269	10.269	1.000
7	10.006	10.269	0.974

normalized cumulative discounted gain (NDCG) (Le and Smola, 2007), which normalizes the values to lie between 0 and 1, to aid the comparison.

The cumulative discounted gain for a given query is

$$\mathrm{DCG} = \sum_{i=1}^{n} \frac{\mathrm{rating}\,(i)}{\log_2(1+i)} \quad , \tag{12}$$

where

- n is the number of ranks,
- i is the rank of a picture from 1 (most similar to the query picture) to 5 (least similar), and
- rating (i) is the value assigned to a picture according to its perceptual similarity, from 5 (most similar) to 1 (least similar).

The ideal cumulative discounted gain (iDCG) for a given query is the DCG according to the perceptual ranking.

The normalization (NDCG) is calculated by dividing the DCG by the iDCG, i.e.,

$$\mathrm{NDCG} = \frac{\mathrm{DCG}}{\mathrm{iDCG}} \quad . \tag{13}$$

For example, Table 12 gives the DCG calculation for Table 1.

Table 13 presents the DCG and NDCG values for Tables 1–7, except for the outlier Table 4. The average NDCG is 0.987. The closer the NDCG value is to 1, the higher the correlation between the ranking of the pictures by the SpCD and by human perception.

5 CONCLUSION

In this paper we show how similar pictures can be generated by bag context picture grammars and random context picture grammars. We then present the results of an online survey that we conducted to determine how humans determine the similarity of syntactically generated pictures. We applied the spatial color distribution descriptor to the same images and we present results which compare the human view of similarity to the selected mathematical similarity measure.

The humans seemed to have very different opinions regarding the similarity of pictures. A reason may be that different people compare pictures using different measures, some placing more emphasis on color while others place more emphasis on objects. However, the majority of respondents agreed on the similarity of individual pictures compared to the query picture. Most respondents found the given galleries of pictures to contain similar pictures which is very important as this research is about the generation of similar pictures. When comparing the results of the survey with the results of the spatial color distribution descriptor similarity measure, perceptual similarity seemed to correlate to the spatial color distribution descriptor measure. This implies that the spatial color distribution descriptor can be used to judge the similarity of pictures generated by bag context picture grammars and random context picture grammars.

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