A Cyclical Model for Waste Products Inventory-Routing at Power Stations

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Abstract: The inventory routing problem (IRP) is one of the challenging optimization problems in supply chain management (SCM). Inventory Routing Problems currently get a lot of attention, in which a typical property is that inventory control and vehicle routing tasks are taken into account simultaneously. The objective of the IRP is to jointly determine optimal quantities of the product to be delivered to the customers, delivery times and optimal vehicle routes for the shipment of these quantities. At power enterprises, SCM strategies are more and more interesting for the resource optimization. In this paper, a cyclical inventory routing problem is modeled and accounted for one distribution system of waste products at the power stations. This proposed problem is formulated as a linear mixed-integer program, in which the demand rates of customers are assumed to be constant. As to the resource optimization, a practical instance is presented and thoroughly discussed, to illustrate the behavior of the proposed model.

1 INTRODUCTION

Waste Products at power stations, e.g. coal fly ash, plaster *etc.*, are comprehensively utilized in building industry. However a challenging problem when ones intend to deliver and store such waste products, e.g. coal fly ash, due to the fact that it is dispersive mixed-solid and is scattered very easily,. As a result, special vehicles (i.e. tank trucks) have to be employed for its delivery, and high costs arise.

In this paper, a cyclical multi-period inventory routing problem (IRP) is discussed, in which the delivery and storage of coal fly ash can be represented. Inventory routing problems involve the integration of inventory management and vehicle routing optimization. The typical main objective in such problems is to determine an optimal distribution policy, consisting of a set of vehicle routes and delivery quantities that minimize the total inventory holding and transportation costs. The IRP arises in distribution systems implementing a 'Vendor Managed Inventory' (VMI) policy. Compared with the traditional non-integrated inventory replenishment and vehicle scheduling, in which customers manage their inventories themselves and call in their orders, overall inventory and routing performances throughout the supply chain are by far superior when VMI is implemented. Nowdays, the IRP has been one of the most challenging and interesting optimization problems in supply chain and logistics management.

2 A BRIEF LITERATURE REVIEW

Since Bell *et al.* (1983) first investigated the integrated inventory management and vehicle scheduling, various versions of the inventory routing problems have been extensively studied. A large variety of solution approaches have also been proposed for the solution of these problems. Inventory routing problems can be modeled and approached in different ways depending on the characteristics of its parameters. Different models can be obtained for example, when customers consume the product at a stable or at a variable rate; when the planning horizon is finite or infinite, and so on. Dror and Ball (1987) decompose a multi-period IRP into series of single period problem. They study the problem with constant demands and then propose and compare two solution approaches for the resulting single period problem. Campbell *et al.* (2002) and Campbell and Savelsbergh (2004) also worked on the multi-period IRPs where the decisions are executed over a finite horizon. For recent research devoted to the multi-period IRPs, we refer to e.g. Yu *et al.* (2008), Taarit *et al*. (2010), *etc.*

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Other fundamental contributions in the class of infinite inventory routing problems are for example Anily and Federgruen (1990) and Hall (1992). Aghezzaf *et al*. (2006) address a special case of infinite inventory routing problems, namely, the cyclic inventory routing problem (CIRP), in which a single distribution center, supplying a single product, serves a set of customers, each being visited by an assigned vehicle in a cyclical manner and in such a way that at no moment a stock-out should occur at any of the customers. Later this cyclic problem and its variants are further discussed in e.g. Raa and Aghezzaf (2009) and Zhong and Aghezzaf (2011).

The strategy presented in inventory routing problems allows to reduce costs if a supplier or a third-party logistics server regards the delivery and storage of coal fly ash as a model of the IRP, due to the fact that the supplier or the logistics provider may coordinate both inventory control and transportation policy. More specially, a multi-period inventory routing problem (MP-IRP) is concerned with a distribution system using a fleet of homogeneous vehicles to distribute the product from a single depot to a set of customers having stable demands. The considered distribution policies are executed over a given finite horizon, for example on a set *T* of consecutive periods (or days). The objective is to determine the quantities to be delivered to the customers, the delivery time, and to design the vehicle delivery routes, so that the total distribution and inventory costs are minimized. The resulting distribution plan must prevent stockouts from occurring at all customers during the planning horizon. A mixed-integer model is built up for this MP-IRP, in which the distribution pattern of 'multi-tour' is employed, i.e. a vehicle can make a set of different tours when it is used (see e.g. Aghezzaf *et al.* 2006, Zhong and Aghezzaf (2011). In addition, the presented model considers the vehicle fleet size as part of the optimization problem and has to be determined. Also, the initial inventory levels at the customers have to be determined in this model, instead of predefined amounts as done in other similar works (see for example Taarit *et al*. 2010 and references therein).

The remainder of this paper is organized as follows. In Section 3, a linear mixed-integer formulation for the MP-IRP is presented. In Section 4, a practical case is studied to illustrate the behavior of the presented model. Finally, some concluding remarks are provided in Section 5.

3 PROBLEM FORMULATION

To be more precise, the discussed multi-period IRP consists of a single distribution center *r* using a fleet of homogeneous trucks to distribute a single product to a set of geographically dispersed customers *S* over a given planning horizon. It is assumed that customer-demand rates and travel times are stable over time. Thus, the objective of this MP-IRP is to determine the quantities to be delivered to the customers, the delivery time, and to design the vehicle delivery routes, so that the total distribution and inventory costs is minimized while preventing stockouts from occurring at all customers during the whole planning horizon.

To build up this mixed-integer model for the MP-IRP, some main assumptions are made below:

1) The time necessary for loading and unloading a truck is neglected in the model;

2) Inventory capacities at the depot are assumed to be large enough so that the corresponding capacity constraints can be omitted in the model;

3) Transportation costs are assumed to be proportional to travel times;

4) Split delivery is not allowed, such that a customer is always replenished by one vehicle, in the same tour in each period of the planning horizon.

A more formal description and a proposed linear mixed-integer formulation of the MP-IRP are given in the following paragraphs:

Let $H=\{1,2,...,T\}$ be the planning horizon set of consecutive periods indexed by *t*. Let π be the size in time unit of one period,, for example 8 working hours. Let *S* be the set of customers indexed by *i* and *j* and $S^+ = S \cup \{0\}$ where *0* represents the depot. A homogeneous fleet of trucks *V* is used to serve these customers. The other necessary parameters of the model are given below:

ψ^v: the fixed operating cost of truck $v \in V$ (in RMB per truck);

κ^v: the capacity of truck $v \in V$ (in ton);

η_{jt}: the holding cost of per unit per period of product at customer $j \in S$ (in RMB per ton per period);

η_{j0}: the initial holding cost of per unit of product at customer $j \in S$ (in RMB per ton);

D_{jt}: the demand at customer $j \in S$ in period $t \in H$ (in ton), i.e. $D_{jt} = d_{jt} \cdot \pi$ for $t \in H$, where

d_{jt} represents the demand rate at customer *j* (in ton per hour);

σ : travel cost of truck (in RMB per km per hour);

v : truck speed (in km per hour);

tr_{ij} : duration of a trip from customer $i \in S^+$ to customer $j \in S^+$ (in hour);

C_j : the inventory capacity at customer *j* (in ton).

The variables of the model are defined as follows:

I_{jt} : the inventory level of customer $j \in S$ at the end of period $t \in H$ (in ton);

I_{j0} : the initial inventory level of customer $j \in S$ (in ton);

Q^v_{ijt} : the quantity of product remaining in truck $v \in V$ when it travels directly to the customer $j \in S^+$ from customer $i \in S^+$ in period $t \in H$. This quantity equals zero when the trip *(i, j)* is not on any tour made by truck $v \in V$ in period *t* (in ton);

q_{jt} : the quantity that is delivered to customer $j \in S$ in period $t \in H$, and 0 otherwise (in ton);

x^v_{ijt} : a binary variable sets to 1 if customer $j \in S^+$ is visited immediately after customer $i \in S^+$ by truck $v \in V$ in period $t \in H$, and 0 otherwise;

y^v : a binary variable sets to 1 if truck $v \in V$ is being used, and 0 otherwise;

Thus, the linear mixed-integer formulation for the multi-period IRP is presented as follows:

$$CV = \sum_{v \in V} \psi_v \cdot y_v + \sum_{v \in V} \sum_{t \in T} \sum_{i \in S^+} \sum_{j \in S^+} \sigma v tr_{ij} \cdot x^v_{ijt} + \sum_{t \in T} \sum_{j \in S} \eta_{jt} I_{jt} + \sum_{j \in S} \eta_{j0} I_{j0} \quad (1)$$

Subject to:

$$\sum_{v \in V} \sum_{i \in S^+} x^v_{ijt} \le 1 \quad \forall j \in S, t \in T \quad (2)$$

$$\sum_{i \in S^+} x^v_{ijt} - \sum_{k \in S^+} x^v_{jkt} = 0 \quad \forall j \in S^+, t \in T, v \in V \quad (3)$$

$$I_{j1} = I_{j0} + q_{j1} - D_{j1} \quad \forall j \in S \quad (4)$$

$$I_{jt} = I_{j,t-1} + q_{jt} - D_{jt} \quad \forall j \in S, t \in T, t \ge 2 \quad (5)$$

$$\sum_{v \in V} \sum_{i \in S^+} Q^v_{ijt} - \sum_{v \in V} \sum_{k \in S^+} Q^v_{jkt} = q_{jt} \quad \forall j \in S, t \in T \quad (6)$$

$$Q^v_{ijt} \le \kappa^v \cdot x^v_{ijt} \quad \forall i, j \in S^+, t \in T, v \in V \quad (7)$$

$$\sum_{i \in S^+} \sum_{j \in S^+} tr_{ij} \cdot x^v_{ijt} \le \pi \quad \forall t \in T, v \in V \quad (8)$$

$$I_{j0} \le I_{jt} \quad \forall t \in T, t = T, j \in S \quad (9)$$

$$\sum_{i \in S^+} x^v_{ijt} \le y^v \quad \forall j \in S, t \in T, v \in V \quad (10)$$

$$x^v_{ijt}, y^v \in \{0,1\}, 0 \le I_{j0}, I_{jt} \le C_j, Q^v_{ijt} \ge 0, q_{jt} \ge 0 \quad \forall i, j \in S^+, t \in T, v \in V \quad (11)$$

Three cost components are taken into account in the model: the total fixed operating cost of using the truck(s); the total transportation cost; the total inventory holding cost, including the initial inventory holding cost and the inventory holding cost at every end of period $t \in H$. For the restrictions, constraints (2) guarantee that each customer is visited by each truck at most once in period *t*. Constraints (3) are the usual flow conservation constraints assuring that if a truck arrives at a customer, it must leave after it served this customer to a next customer or to the depot. Constraints (4), (5) are the inventory balance constraints of each customer. Constraints (6) are the delivered load balance constraints. These constraints eliminate possible formation of sub-tours. Constraints (7) ensure that the quantity carried by a vehicle should not exceed the truck's maximum capacity. Constraints (8) indicate that the total travel time of a vehicle should not exceed the considered horizon length in each period. Constraints (9) indicate that the initial inventory level at customer j at the end of period T shall cover its initial inventory. Constraints (10) indicate that at every period, a vehicle cannot be used to serve any customer unless this vehicle is already selected. Constraints (11) are the integrality and sign constraints to be imposed on the variables, in which the capacity constraints at all customers are taken into account.

4 CASE STUDY

To gain a better understanding of the MP-IRP's model, this section provides a small practice example to illustrate different behaviors of the MP-IRP's model, in which variable initial inventories are taken into account. This example corresponds to a third-party logistics company delivers the coal fly ash from one power station to a set of 7 customers, including 5 cement plants and 2 brickyards. To be more precise, let us consider this small instance for the MP-IRP, consisting of 7 customers indexed by $\{1, 2, ..., 7\}$, and a single depot (i.e. the power plant) indexed by 0. These 7 clients are distributed uniformly over a square of 100 by 100 km. A fleet of homogeneous trucks with capacity 35 ton is available for the distribution of the product. The fixed using cost of the truck ψ is 800RMB per day. The truck's average speed ν is 50 km per hour, and the travel cost σ is 8RMB per km. In this case the planning horizon set contains 3 consecutive periods (i.e. 3 days), and the size of one period π is assumed to be 8 hours. Demands of these clients and the inventory holding costs for each client are shown in Table 1 below.

Table 1 Parameters for a case of 7-clients

	$D_{jt}(t)$			η_j (R/t)	d_j(t/h)
Client	t=1	t=2	t=3		
1	8.07	9.27	9.57	0.32	2.99
2	3.93	4.53	3.33	1.28	1.31
3	7.05	7.65	8.70	1.18	2.60
4	6.54	5.28	5.91	1.17	1.97
5	5.61	5.85	6.09	0.41	1.95
6	6.27	5.97	6.57	0.70	2.09
7	9.42	8.28	9.06	0.86	2.94

By solving the instance, the optimal solution shows that a fleet of 2 trucks is required to replenish the customers. The delivery routes in each period are shown below, with the total cost around 9195RMB (for 3 days).

A. t=1: Vehicle 1 :={(3), (7)}; Vehicle 2:={(2, 4, 5)}.

B. t=2: Vehicle 1 :={(4, 6)};Vehicle 2:={(1)}.

C. t=3: no delivery occurs.

Due to the fact that variable initial inventory arises in the model, it is allowed the solver to find a best three-way tradeoff for each customer, among the initial inventory, the inventory at the end of every period and the quantity to be delivered to each, such that the total distribution and inventory costs can be reduced as much as possible. According to the financial statistics of the company, without the logistics optimization presented above, the average running cost is around 5000RMB per day in this case, where the operation manner corresponds to that the company receives the orders given by the customers, the trucks are sent and the deliveries occur then. As a result, one can observe that the MP-IRP model for this coal fly ash distribution system might achieve a cost saving around 38.7% when this optimization model is used.

5 CONCLUSION

This paper provides a specific multi-period inventory routing problem (MP-IRP) that might be addressed for the distribution of coal fly ash. In general, it consists in a single depot distributing a single product to a set of customers having stationary demands, using a fleet of homogeneous vehicles over a given finite horizon. The objective is to determine the quantities to be delivered to the customers, the delivery time, and to design the vehicle delivery routes, so that the total distribution and inventory costs are minimized. The presented MP-IRP is formulated as a linear mixed-integer program with some side constraints. A practical case is studied to illustrate the behaviors of the MP-IRP and the merits of such a model. The extensions to this problem, if for example, the variants by taking stochastic inventory into account or some efficient meta-heuristics for the model, are worthy being investigated further.

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