Multi-dimensional Taylor Network Optimal Control of the Axisymmetric Cruise Missile Flight

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Abstract: In this paper, we present the design of the multi-dimensional Taylor network (MTN) optimal controller in the flight control of cruise missile. The MTN optimal control, which combines the classical architecture of feedback control system and the new controller structure, it is not only suitable for the analysis of the stability of the closed-loop system, but also for the control of nonlinear systems with mechanism known or unknown models. Firstly, this paper will briefly introduce the theoretical basis of the MTN optimal control. Secondly, the characteristics of the missile mathematical model and the theory of missile control will be explained, accompanied with the design of controller. Finally, the feasibility of the method is validated through numerical simulation of the PID controller, PIDNN controller and the MTN optimal controller. The results show that the MTN optimal controller has the best control effect of them.

1 INTRODUCTION

As a kind of high lethality tool the missile has played a very important role in the modern battle field. In a local war, high precision guided missiles may affect the war. Because of its powerful long-range destructive force and precise fixed point strike capability, the missile plays an irreplaceable role in the national defense security. The missile control system is an important part of the missile overall design, and its performance affects the capability of the missile directly.

The control system of missile is designed by PID in the literature (Liu, 2009), and the nonlinear model of missile is linearized, then the three channels controller is used in the missile control system. A new method for establishing the six degree of freedom simulation model of missile is proposed, and the general principles and methods of establishing the simulation model of the six degree of freedom of the large aircraft are presented in the literature (Yan, 1998). The paper (Wang, 2008) used the PID neural network algorithm to control the missile mathematical model linearized. This kind of linear model is difficult to accurately describe the actual nonlinear model. The PID neural network algorithm is used to control the attitude of the ballistic missile in the literature (Tang, 2012).

According to the above analysis, most design of missile control system in the past adopted the small disturbance linearization method, and then the model of the missile is linearized, which use the classical control method of transfer function for missile control. The defects caused by this method may lead to inaccurate simulation results. In addition, most research of missile control system in the past analyzed the channel in the missile control system. There are a few researches on the trajectory control of missile trajectory. In view of the current research status of the missile controller design, it is necessary to carry out the simulation research of the missile trajectory control based on the dynamic model of the missile.

In this paper, the multi-dimensional Taylor network (MTN) controller (Yan, 2010&2017) will be used in the design of missile control. The MTN controller is a nonlinear control technique. It uses the nonlinear model of the missile to design the controller to avoid linear behavior in various state points. In the meantime, it can be combined with intelligent control methods to directly identify the controller parameters through a set of desired output. This makes the design of the controller simpler and

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more efficient. Additionally, with the nonlinear terms of object considered, the control system can perform well in a wide range.

2 THE MTN CONTROL THEORY

A controlled system can be described as follows:

$$\begin{cases} x(k+1) = f(x(k), u(k), k) \\ y(k) = g(x(k), u(k), k) \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state variable, $u(k) \in R^m$ is the input variable, $y(k) \in R^m$ is the output variable. $f(x(k), u(k))$ and $g(x(k))$ are respectively smooth nonlinear function of relevant variables. We can also rewrite (1) as (2):

$$\begin{cases} x_1(k+1) = f_1(x_1(k), x_2(k), \cdots, x_n(k), u(k)) \\ x_2(k+1) = f_2(x_1(k), x_2(k), \cdots, x_n(k), u(k)) \\ \vdots \\ x_n(k+1) = f_n(x_1(k), x_2(k), \cdots, x_n(k), u(k)) \end{cases} \tag{2}$$

For all of the controllable system, under certain conditions, there must be an optimal control signal u*, which makes the system achieve the optimal control effect under the action of u*. For general control systems, feedback control is usually used to design the controller, which is based on the deviation between the expected value and the output feedback. At the same time, reference Wells Truss approximation theorem and the Taylor formula, can be used for quantitative and feedback the high-order error to infinite approximation of the optimal control signal. The whole control structure diagram can be expressed in Figure 1:

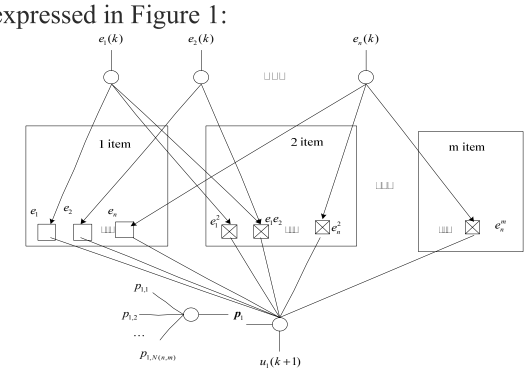

Figure 1: The structure of MTN.

The Taylor formula uses a series of infinite terms to approximate a function. These additional terms are obtained by the derivative of the function at a certain point.

The Taylor formula is defined as follows. For a positive integer *n*, if the function *f* (*x*) in the closed zone [a, b] can be continuously guided, either take [a, b] on a certain point *x*, then the function at the point where the approximate expression is as follows:

$$f(x) = \frac{f(a)}{0!} + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x) \tag{3}$$

In the formula, $f^{(n)}(a)$ is n derivative of *f*(*x*) at a point, polynomial $\frac{f^{(n)}(a)}{n!}(x-a)^n$ is *n*th Taylor expansion of *f*(*x*) at a point, R_n is the remainder term. The more Taylor's series, the more approximation function point value.

According to the Taylor formula and the Wells Dreads approximation theorem, we have the following expressions (Sun, 2014).

$$u_1(k+1) = \sum_{j=1}^{N(n,m)} p_j \prod_{i=1}^{n} e_i^{\lambda_{i,j}} + R_m(e_1, e_2, \cdots, e_n) \tag{4}$$

In the formula, p_j is the weight coefficient, $\lambda_{i,j}$ is the number of errors, R_m is the Taylor remainder which contains the number of the error terms are greater than m, $N(n,m)$ is the highest number of items is obtained through the combination with the highest number of dimensions. If the allowable error conditions and *m* value is larger, it can be simplified to formula (5).

$$u(k+1) = \sum_{j=1}^{N(n,m)} p_j \prod_{i=1}^{n} e_i^{\lambda_{i,j}} \tag{5}$$

In the formula, x_i can be expressed as follows:

$$\begin{aligned} & e_1(t) = e(t) \\ & e_2(t) = e_1(t) - e_1(t-1) = e(t) - e(t-1) \\ & e_3(t) = e_2(t) - e_2(t-1) = [e(t) - e(t-1)] - [e(t-1) - e(t-1)] \\ & \cdots \\ & e_n(t) = e_{n-1}(t) - e_{n-1}(t-1) \end{aligned} \tag{6}$$

As we can see, the structure of MTN controller is a forward network of three layers which consists of input layer, middle layer and output layer. All the

MTN elements of different power and order are multiplied by a given weight coefficient, adding all of them as to be the controller output. Combining with appropriate algorithm, it is easy to determine the weight of parameters. Detail analysis of MTN is described in (Zhou, 2013).

3 MISSILE MODEL ANALYSIS

In order to research the change regulation of the missile trajectory and the change of attitude, it is necessary to consider the missile motion as a rigid body motion in space. Missile kinematics equation is built on this assumption. It mainly describes the kinematics and motion rules of projectile body rotating around the center of mass. The position and attitude angle of the missile at a certain time point can be solved by using the missile kinematics equations. The missile motion equations are composed of missile dynamics, kinematics and quality variation equation. The relationship between these equations can describe the missile by process conveniently and accurately in force, torque and motion parameters of the missile. Then we can obtain the equations describing motion of the missile as shown in the following expressions (Qian, 2000):

$$\begin{cases} m\frac{\mathrm{d}V}{\mathrm{d}t}=P\cos\alpha\cos\beta-X-mg\sin\theta \\ mV\frac{\mathrm{d}\theta}{\mathrm{d}t}=P(\sin\alpha\cos\gamma_V+\cos\alpha\sin\beta\sin\gamma_V)+Y\cos\gamma_V-Z\sin\gamma_V-mg\cos\theta \\ -mV\cos\theta\frac{\mathrm{d}\psi_V}{\mathrm{d}t}=P(\sin\alpha\sin\gamma_V-\cos\alpha\sin\beta\cos\gamma_V)+Y\sin\gamma_V+Z\cos\gamma_V \\ J_{xb}\frac{\mathrm{d}\omega_{xb}}{\mathrm{d}t}=M_{xb}-(J_{zb}-J_{yb})\omega_{yb}\omega_{zb} \\ J_{yb}\frac{\mathrm{d}\omega_{yb}}{\mathrm{d}t}=M_{yb}-(J_{xb}-J_{zb})\omega_{xb}\omega_{zb} \\ J_{zb}\frac{\mathrm{d}\omega_{zb}}{\mathrm{d}t}=M_{zb}-(J_{yb}-J_{xb})\omega_{yb}\omega_{xb} \\ \frac{dx}{dt}=V\cos\theta\cos\psi_V \\ \frac{dy}{dt}=V\sin\theta \\ \frac{dz}{dt}=-V\cos\theta\sin\psi_V \\ \frac{d\vartheta}{dt}=\omega_{yb}\sin\gamma+\omega_{zb}\cos\gamma \\ \frac{d\psi}{dt}=(\omega_{yb}\cos\gamma-\omega_{zb}\sin\gamma)/\cos\vartheta \\ \frac{d\gamma}{dt}=\omega_{xb}-\tan\vartheta(\omega_{yb}\cos\gamma-\omega_{zb}\sin\gamma) \\ \sin\beta=\cos\theta\left[\cos\gamma\sin(\psi-\psi_V)+\sin\vartheta\sin\gamma\cos(\psi-\psi_V)\right]-\sin\theta\cos\vartheta\sin\gamma \\ \sin\alpha\cos\beta=\cos\theta[\sin\vartheta\cos\gamma\cos(\psi-\psi_V)-\sin\gamma\sin(\psi-\psi_V)]-\sin\theta\cos\vartheta\cos\gamma \\ \sin\gamma_V\cos\theta=\cos\alpha\sin\beta\sin\vartheta-\sin\alpha\sin\beta\cos\gamma\cos\vartheta+\cos\beta\sin\gamma\cos\vartheta \\ \frac{\mathrm{d}m}{\mathrm{d}t}=-m_s \end{cases} \quad (7)$$

where among all of the sixteen state variables, m is the missile mass, V is the center of mass velocity; γ is the roll angle, ϑ is the pitch angle, ψ is the yaw angle; θ is the ballistic inclination, ψ_V is the trajectory angle, α is the angle of attack , β is the sideslip angle; J_{xb} , J_{yb} and J_{zb} are the moment of inertia of the three coordinate axes of the missile body; ω_{xb} , ω_{yb} and ω_{zb} are the component of the rotational angular velocity ω_b of the body coordinate system on three axes; M_{xb} , M_{yb} and M_{zb} are the components of the total moment of rotation in three coordinate systems of the missile body coordinate system; m_s is the amount of mass change per unit time, depending on the engine performance; (x, y, z) represents the coordinate value of the missile in the ground coordinate system.

4 DESIGN OF THE CONTROLLER

In this paper, we adopt three channels control method to design of the controller. The corresponding control system designed in the pitch channel, the yaw channel and the roll channel. The control method of the three channels as an example to introduce the missile three channels. The typical control principle diagram of three channels is given in Figure 2:

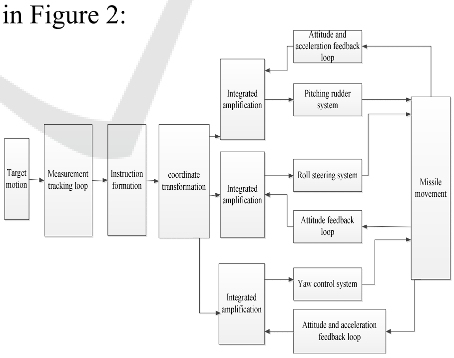

Figure 2: Missile Three Channels Control System.

The working principle of the three channels control mode: firstly, the measuring mechanism measures motion parameters of the missile and the target, then according to the relative motion between the missile and the target, the guidance and control system generates a series of control instructions, and control the attitude variable calculation, coordinate transformation, navigation calculation, error compensation calculation and control instruction the formation, the state variable feedback signal and the corresponding channel control instruction, and pass it to the executive body.

In this paper, the MTN control algorithm is used to design the missile longitudinal channel control system, and the pitch angle is controlled by using the MTN optimal control algorithm. Here the PID control as a MTN optimal control case, the proportion, integral and differential as three items of the MTN optimal control, then arranges and combines the three items, multiplied the corresponding control weight in the front of Taylor items, finally get the controller output. At the same time, combined with the experimental data, the parameters are optimized manually, and then complete the MTN control. Specific control framework as shown in Figure 3:

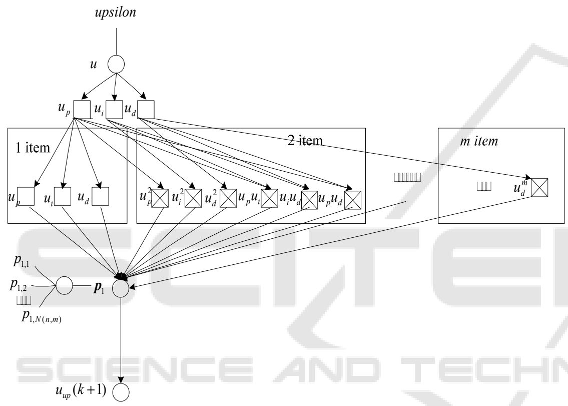

Figure 3: Control chart of pitch angle of MTN.

Due to limited time and laboratory hardware conditions, the controller only get the first term, the second term and the third term as the part number of the MTN control, the form can be written as (8):

$$\begin{aligned} u_{up}(k+1) = {} & w_1 u_p + w_2 u_i + w_3 u_d + w_4 u_p^2 + w_5 u_i^2 + w_6 u_d^2 \\ & + w_7 u_p u_i + w_8 u_p u_d + w_9 u_i u_d + w_{10} u_p u_i u_d \end{aligned} \quad (8)$$

In the formula, w_1, w_2 and w_3 are the control parameters of the first item, w_4, w_5, w_6, w_7, w_8, w_9 are the second terms of the control parameters, w_{10} is the third terms of the control parameter.

When the system is close to the state of equilibrium, the influence of high-order (8) type is far less than the linear term, such as u_p =0.01, u_p^2 =0.0001, and the high terms does not affect the stability of equilibrium. In other words, the influence of (8) on the stability of equilibrium is equivalent to PID. When the super unit ball system is out of balance, especially when it is far from the super unit ball, the impact of the high order term on the system is much larger than linear, so through parameter optimization can make the high terms object has better dynamic characteristics and anti-interference. In Figure 4, take the proportional link as an example, when the error is small, especially when it is close to the origin, the control effect of the higher order term is much smaller than that of the first term(Xia, 2016).

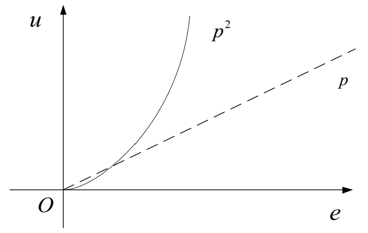

Figure 4: Comparison of high-order term and linear term.

In (8), MTN is composed of the linear and high terms of the error. The comparison of the linear and high terms is shown in Fig. 4.

According to (8) and Fig. 4, we have the following discussions:

1) MTN controller is a nonlinear controller, which includes proportional-integral-derivative (PID) control and linear controller as the special case, so we can take the PID parameters as the initial ones of MTN that makes the closed-loop system stable.

2) For asymptotically stable systems, the high-order terms of the MTN controller are high-order infinitesimal in the vicinity of the error equilibrium point, and do not affect the stability of the equilibrium point. On the other hand, the farther away from the equilibrium point the error state, the greater role the high-order terms are playing. The effect of the higher order term is much greater than that of the linear term when the error state is far away from the equilibrium point-centric hyper-unit sphere. As the sudden input or large disturbances can cause large errors and large fluctuations, the main effect of higher order terms is to affect the dynamic performance and anti-disturbance performance of the closed loop system, especially performance of resisting strong disturbance.

3) MTN optimal controller is equivalent to the linear controller because it is dominated by linear parts when the error state is in the vicinity of the error balance point. When the error state is far from

the equilibrium point-centric hyper-unit sphere, the nonlinear parts will play a leading role. When the error is in between, the linear parts and the nonlinear ones will take effect in the same time. Therefore, the MTN can be regarded as a kind of sliding mode control (SMC) without chattering.

In the design of longitudinal channel control system, the attitude angle of the missile's climbing section and the last dive section is controlled by PID, and the double closed loop compound control is adopted in the flying section. In this paper, the pitch angle control is used as the inner loop, and the MTN is used to optimize the control. The outer loop uses a high degree of feedback, and the PID control algorithm is adopted. After optimizing the control parameters of the pitch angle controller, the outer loop is added to the longitudinal control system. After adjusting the inner loop parameters, the controller parameters of the outer loop are adjusted.

The height control structure is shown in figure 5:

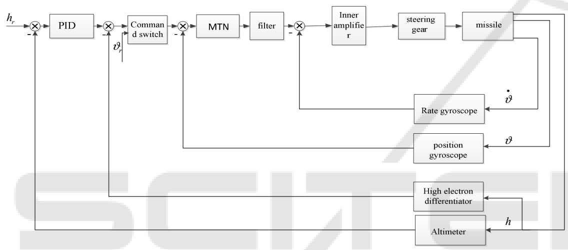

Figure 5: MTN height control block diagram.

5 RESULTS AND ANALYSIS

Four feedback signals are used in the design of the height control system, such as the height, the height change rate, the pitch angle and the pitch angular velocity. If the flight height is 50m, height control chart of missile can be obtained as Figure 6 showed.

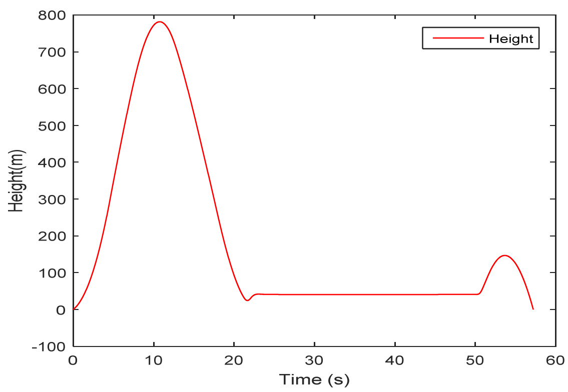

Figure 6: MTN height control.

As can be seen from figure 6, the missile flights smoothly from the glide segment to the transition period, and quickly converge to about 50m. The result shows that the double closed-loop control method designed by PID control as the outer loop and MTN optimal control as the inner loop can control the missile better.

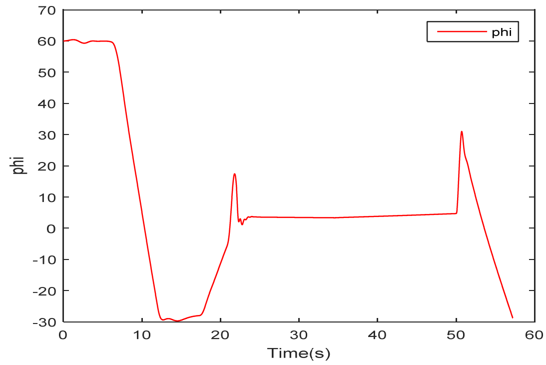

Figure 7: Pitch angle control of MTN control.

As shown in Figure 7, the pitch angle is maintained at 60 degrees at first, then decreased rapidly, which corresponds to the missile in flight down. In 22s, after a smaller overshoot, the pitch angle is stable at 0 degree, and the missile gets into the cruise flight. At the last time in the 50s, the pitch angle first increases rapidly at less time, and then decreased. This corresponds to the missile's climb to a height, and then quickly dive down hit the target flight process.

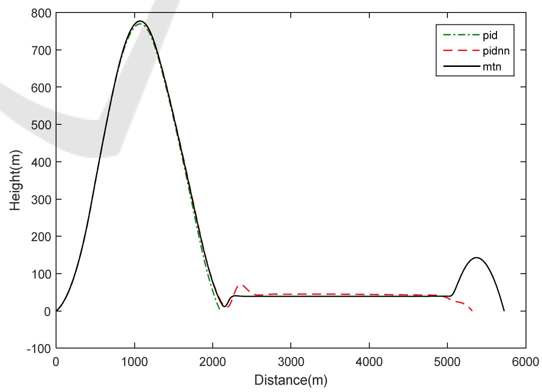

Figure 8: Height adjustment comparison curve.

Fig.8 illustrates that when the constant wind is 9m/s, the optimal control effect of multi-dimensional Taylor network is better than that of PID control and PID neural network.

In addition, a good many simulations of the cruise missile counteracting wind disturbance by three kinds of control methods are conducted, and their results are summarized as follows: 1) anti-disturbance capacities of PID control for constant wind, gust and random wind respectively are 8m/s, 9m/s and 19m/s; those of PID neural network control (PIDNN) for the three winds respectively are 9m/s, 10m/s and 10m/s; those of MTN optimal control for the three winds respectively are 9m/s, 11m/s and 22m/s. Under the same conditions, the MTN optimal control has the best wind anti-disturbance effect. 2) Under the condition of the shear wind ratio k1= 10^{-7} , the maximum shear wind stresses b1 borne by MTN, PIDNN and PID respectively are 16×10^{-3} , 16×10^{-3} and 13×10^{-3} ; under the condition of shear wind stress b1= 16×10^{-3} , the maximum shear wind ratio k1 borne by MTN and PIDNN respectively are 16×10^{-7} and 4×10^{-7} ; Under the condition of b1= 13×10^{-3} , the maximum k1 of MTN, PIDNN and PID can be 108×10^{-7} , 97×10^{-7} and 23×10^{-7} , respectively. 3) Target striking accuracy: under the same conditions, the average target hitting impact degrees of PID, PIDNN and MTN respectively are 2.020313, 2.364979 and 0.032239. If results for random wind without waves (in which the target hitting impact degree of PID is particularly large and reaches 18.5181) are neglected, the average target hitting impact degrees of PID, PIDNN and MTN respectively are 0.520514, 2.390054 and 0.033152. Their corresponding target deviations respectively are 52.0514m, 239.0054m and 3.3152m, i.e., the average target hitting accuracy of MTN is 14.70 and 71.09 times more than those of PID and PIDNN respectively. Neglecting the results of flight path divergence by PID and PIDNN (i.e. out of control), the maximum target hitting impact degrees of PID, PIDNN and MTN respectively are 18.5181, 6.85152 and 0.186743. Their corresponding target deviations respectively are 1851.81m, 685.152m and 18.6743m, i.e., the worst target hitting accuracy of MTN is 98.16 and 35.69 times more than those of PID and PIDNN respectively (Yan, 2017).

6 CONCLUSIONS

In this paper, we illustrate the optimal control theory of MTN, establish the missile flight dynamics model and analyze its characteristics. Among them, the scheme guidance is adapted in the missile flight path in the take-off, horizontal and dive directions, in which gradient method combined with hand adjustment is used to optimize MTN controller parameters. The above results show that the MTN optimal control has better dynamic performance and external stability, stronger anti-disturbance performance and 1~2 magnitudes higher target striking accuracy than PID and PIDNN.

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