An Evaluation System for Mathematical Models of Reservoir Sedimentation along the Yellow River

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Abstract. Based on theoretical research and measured data analysis, a multi-target and quantifiable assessment system for mathematical models of reservoir sedimentation was established. In this system, both typical physical model tested data and prototype materials were used to form a case database. The indexes were selected individually. Both analytic hierarchy process and structural equation models were adopted. The system can conduct quantitative assessments of numerical simulations for sediment levels within sediment laden river-reservoir systems.

1. Research objective

Mathematical models for reservoir sediment are commonly used to predict sediment transport and its accumulation within reservoirs. These models also provide important tools to study corresponding fundamental theories[1]. When assessing the suitability of mathematical models to real-world cases, expert consultation and review systems are traditionally used. However, subjectivity is virtually unavoidable. Up to the present, only a few comparison studies between similar mathematical models have been conducted. Many of these models are designed only for typical cases and have no established standards of evaluation which are based on benchmark model libraries. Therefore, a need exists for systematic research on how to assess the reliability, accuracy, and integrated performance of reservoir sediment models. The construction of such an assessment system has been applied to mathematical models used within the Yellow River reservoir network. This system promotes the quantitative assessment of respective models and advances the pursuit of reservoir sediment control. Thus, this research contributes to the overarching goal for sustainable utilization of the Yellow River reservoir system.

2. Evaluation system overview

At present, there are no widely-accepted research results that codify multi-objective and quantitative assessment systems for numerical simulations. To quantitatively assess the mathematical models, a case library should be established to provide standard cases with which to test model integration performance[2]. To fulfill this purpose, the key factors representing water-sediment transportation and its accompanying mechanisms are selected as the single indices. After dimensional scaling and

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weight assignment, the multi-objective and quantitative assessment system is established.

The assessment system for mathematical models used in sediment simulations of Yellow River reservoirs consisted of three parts: case library construction, indices selection, and quantitative assessment. The details are shown in Figure.1.

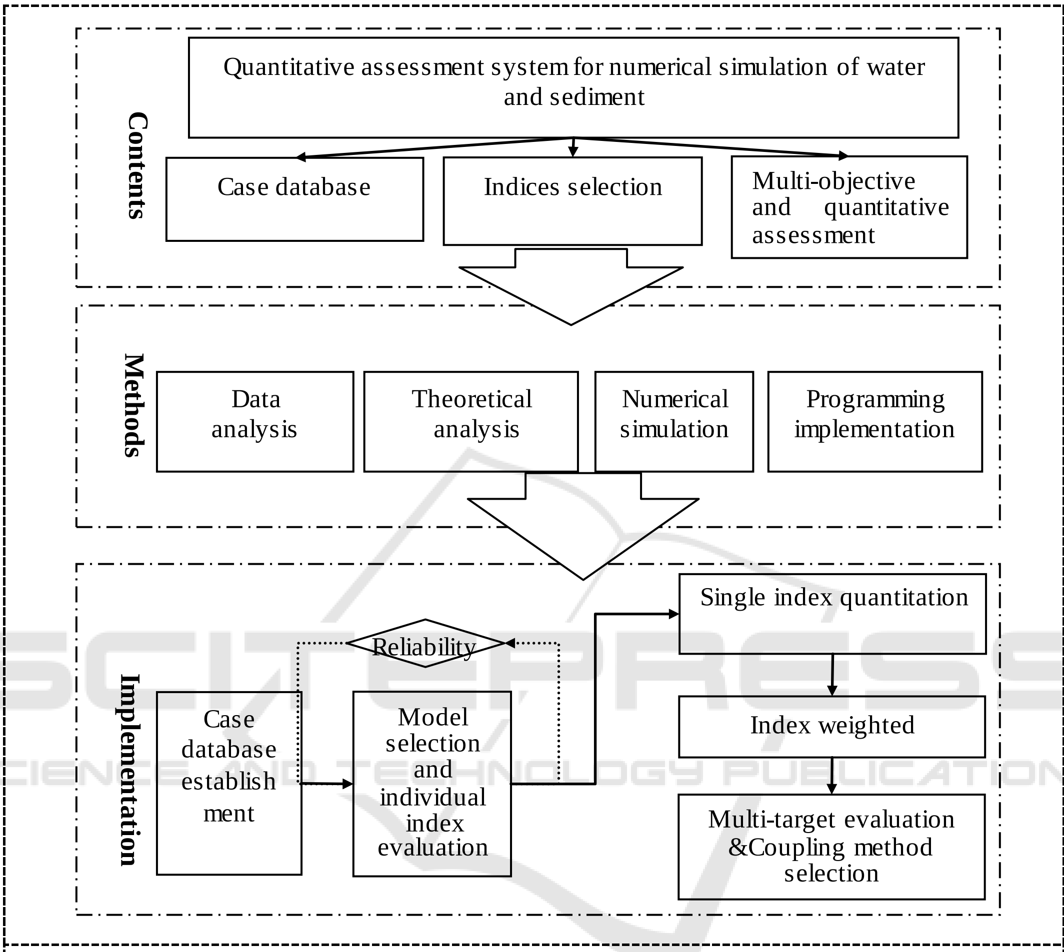

Figure.1 Evaluation system for mathematical model of reservoir sediment in the Yellow River.

Establishing a case database denotes the compiling and selection of cases which either have analytical solutions or are representative of laboratory and prototype data. After this procedure comes index selection. When selecting indices and conducting dimensional analysis, assessment points should be established according to model characteristics. Indices should be able to properly assess the accuracy of the model in simulating water-sediment transportation and its accompanying processes. The final step is selecting the multi-target and quantitative assessment method. Based on the quantitative criterion of the individual index, an appropriate assessment method is selected to carry on the weighted coupling processing. Following processing, the assessment system is complete. In this study, both analytic hierarchy processing and structural equation modelling were used to optimize the coupling process of the model index. In this way the optimal multi-objective assessment method for reservoir sediment mathematical models was established.

3. Case database construction

Carefully filtered data was collected from established benchmark solutions, experimental results, and field observations. Cluster and discriminant analysis methods were used to prepare the metadata for model evaluation. Afterward, cases were categorized by data source, spatial scale, flow characteristics, sediment, and calculation dimensions. Finally, a standard case database for reservoir sediment numerical models was established. The database consists of more than 20 different field tests of reservoirs in the Yellow River basin. In each case, both the boundary conditions and measured data were copiously recorded. Thus, these cases can be easily applied to the calibration and validating of 1D and 2D numerical models. The cases are listed in Table 1.

Table1. Cases for model evaluation.

Category	Case name	Description
Real regulation case (1,2D)	Water and Sediment Regulation Test of the Lower Yellow River (WSRT of LYR) in 2004	Can be used to calibrate or evaluate model performance on flood routing, sediment transport, concentration and the state of deposition and erosion in natural river.
	WSRT of LYR 2005	
	WSRT of LYR 2006	
	WSRT of LYR 2007	
	WSRT of LYR 2008	
	WSRT of LYR 2009	
	WSRT of LYR 2010	
	Flood in August, 1996 (Huayuankou~Jiahetan)	
	Flood in August, 1996 (Gaocun~Sunkou)	
Laboratory experiment	Gravity current experiment	Test models' ability of simulating gravity current
	The backward erosion experiment for fine sediment deposition in a flume	The changing process of bed and surface in flume caused by backward erosion
	Water and Sediment Regulation of Xiaolangdi Project in 2009	Flood routing,Sediment scouring and deposition process, gravity current, tributary backward flow and sedimentation
	Water and Sediment Regulation of Xiaolangdi Project in 2010	
	Water and Sediment Regulation of Xiaolangdi Project in 2011	
	Water and Sediment Regulation of Xiaolangdi Project in 2012	
	Field observations at Sanmenxia 1964~1965	Sediment backward erosion and deposition morphology in reservoirs
	Field observations at Sanmenxia 1972~1973	Sediment backward erosion and deposition morphology in reservoirs
	Xiaolangdi Reservoir Operation in 2002~2010	Long-series calculation
	Liujiaxia Reservoir Operation in 1996~2010	Sediment transportation and the deposition morphology changing in mainstreams and tributaries

4. Index selection and quantitative evaluation model

Under the proposed evaluation framework for Yellow River reservoir sediment models, the selected performance indices were: impoundment curve, reservoir sedimentation morphology, sediment deposition volume, deposition thickness, reservoir sediment concentration, outflow sediment concentration, gravity-current outflow sediment volume, and maximum gravity-current outflow sediment concentration. Indexes either with very large compatibility factors or very small impact factors were removed from consideration to obtain a more reasonable set of indexes. In this framework, we first needed to obtain the dimensionless form of the individual index. Then, the dimensionless index value for each set of observed data was determined. These observed index values were compared with numerical model index values to calculate the relative difference. Finally, each individual index was given a weighted value to scale its importance in model evaluation.

4.1. Individual index for model evaluation a subsection

The Delphi method is used to analyze the reliability of each index. First, more than 30 experts individually ranked the primary indexes based on importance. Then, based on these scores, indexes with very large compatibility factors or very small impact factors were removed. This process optimized the selection of individual indexes. The selected primary indexes were: reservoir sediment deposition, sediment flow patterns, backward erosion and tributary pouring. The evaluation system is illustrated in Table 2.

Table 2. The evaluation index system for sediment mathematical model for reservoirs.

Targets	Primary Index	Secondary Index
The Evaluation System For Mathematical Models of Reservoir Sedimentation.	Reservoir Sediment Deposition	Impounding Curve
		Sediment Deposition Volume
		Deposition Thickness
		Outflow Sediment Concentration
	Sediment Flow Patterns	Gravity-Current Outflow Sediment Volume
		Maximum Outflow Sediment Concentration By Gravity-Current
		Process Of Sand Group By Gravity-Current
	Backward Erosion	Erosion
	Tributary Pouring	Sediment Deposition By Tributary Pouring

4.2. Value of single index and its weight

After primary indexes were selected, the dimensionless index values were calculated based on field experiments. These were compared with model outputs and ranked based on relative errors. Finally, the overall weight of the index was calculated to quantify its importance in model evaluation. The relative-error rankings and weights for each secondary index are shown in Table 3.

Table 3. Index Rank and Weight Allocation.

Category			**Score** **weight (1~10)**				
			5	**4**	**3**	**1**	1D 2D model
Reservoir sedimentation	impounding curve		≤5%	5%~8%	8%~10%	10%~15%	8.79 8.61
	reservoir sedimentation volume	Long-term series (10^8 m^3)	≤10%	10%~15%	15%~20%	20%~30%	8.72 8.53
		grouping sediment	≤20%	20%~25%	25%~30%	30%~50%	8.55 8.39
	Reservoir deposition thickness	Flood events (daily-averaged)	≤10%	10%~15%	15%~20%	20%~30%	7.37 7.22
		multi-year averaged	≤15%	15%~20%	20%~25%	25%~40%	7.18 6.94
	Outflow sediment concentration		≤20%	20%~25%	25%~30%	30%~50%	7.63 7.46
Flow pattern for sediment transportation	Gravity-current and outfall	Outflow sediment concentration (kg/m^3)	≤20 %	20%~25%	25%~30%	30%~50%	6.81 6.65
		Maximum outflow sediment concentration (kg/m^3)	≤20 %	20%~25%	25%~30%	30%~50%	6.29 6.10
		Grouping sand	≤20 %	20%~25%	25%~30%	30%~50%	5.77 5.53
backward erosion	erosion volume (10^8m^3)		≤20 %	20%~25%	25%~30%	30%~50%	6.62 6.83
tributary pouring	tributary pouring Sediment deposition (10^8m^3)		≤20 %	20%~25%	25%~30%	30%~50%	6.33 6.57

Note:the value of % represents the relative errors between simulated results and real value (measured data); the weight reflects the importance of index to the models' evaluation.

4.3. Multi-target evaluation coupling model

Once the indexes were ranked and weighted, a number of individual indexes were compiled into a multi-target model. In this study, an analytic hierarchy process (AHP) and structural equation model (SEM) were applied to compile the indexes[3].

4.3.1. Analytic hierarchy process. The overall objective of the AHP is to utilize the calculated indexes to perform a comprehensive evaluation. The process is defined as follows: first, the research objective is divided into several analysis criteria, i.e., reservoir sedimentation, sediment flow transport, backward erosion and tributary backflow. Next, the importance of each individual criterion was determined through use of the secondary indexes. Expert rankings were used to determine the index weights, as mentioned in section 4.2. The corresponding evaluation system R_I is acquired by comparing simulation results using simulated results and measured data. Through corresponding weight matrix, the second-level indexes are weighted statistically processed and A_I is calculated:

$$A_I=B_i*R_I \tag{1}$$

Where R_I is the expert scores of second-level indexes, B_i is weight matrix of layer P_i~ P_{ij}. The value of E can be acquired by coupling B and A. Based on the overall evaluation and the ranking of different indexes, the evaluation result under different objectives can be acquired.

$$E=B*A \tag{2}$$

This process combines statistical and error analysis theory with expert index rankings to create a weighted index matrix. Once the weighted index matrix is established, the weighted treatment of qualitative index fuzzy quantification method is used to establish the membership function to describe the differences and connections of each index, which can better resolve the relevance and ambiguity of comprehensive evaluation.

4.3.2. Structural equation modelling. Structural equation modelling is a recently developed statistical modeling method. It is particularly suitable for factors that are more subjective and difficult to quantify, such as model comprehensive performance, visualization effects, and evaluating curve-fitting processes. First, the structural equation for the comprehensive model evaluation is constructed. Applying the structural equation modelling method includes five main steps: model construction, fitting, evaluation, correction and application. This model combines measurable flow and sand observation variables with potential variables that are difficult to measure. Thus, a multi-objective evaluation method for decision-making is constructed. Variables which are difficult to directly measured are terms "latent variables". In this assessment, parameters such as reservoir siltation, sediment transport, backward erosion, and tributary backflow are used as the primary assessment criteria. Currently, there are no direct methods to measure these criteria. Thus, observable variables must be utilized to quantify them. In SEM, "observed variables" are variables that can be quantified using measured data, such as mass of sediment released from the reservoir, deposition thickness, etc[4].

The SEM evaluation is constructed with the following system of equations:

$$\begin{cases} \omega = B\eta + \Gamma\xi + \zeta \\ y = \Lambda_y + \varepsilon \\ x = \Lambda_x \xi + \delta \end{cases} \tag{3}$$

Where denotes the impact of the latent and observational variable matrices on the final evaluation. The matrix elements were weighted and random error was minimized. The definition of equation parameters and variable names are shown in Table 4.

Table 4. Structural Equation Modelling Variables.

η	Potential endogenous variables, refer to model comprehensive performance score
ξ	Potentially exogenous variables that characterize some unmeasured first-level indicators in the model's overall performance evaluation, such as reservoir siltation, sediment transport status, backward erosion, etc.
y	Observations of variable η, ultimately represent quantitative representations of these unmeasurable secondary indicators, such as program visualization, result curve fitting, and so on.
x	The observation value of the variable ξ, indicate the secondary indicators that can be directly measured, such as the sedimentation thickness of the reservoir, the sediment load of the reservoir, the maximum sediment concentration in the gravity flow, etc.
ζ	Random error of latent variable equation
ε	Measurement error of y
δ	Measurement error of x
B	Weight coefficient of η
Γ	Weight coefficient of ξ
Λ_y	Regression coefficients of η
Λ_x	Regression coefficients of ξ

Once model parameters are estimated, it is necessary to evaluate whether the model can be fit to the data. The statistical evaluation indexes commonly used in the SEM equations include the goodness-of-fit test λ^2, the goodness-of-fit index, and the adjusted goodness-of-fit index.

The test of goodness of fit λ^2 can be calculated using the following equation:

$$\lambda^2 = (n\text{-}1)F \tag{4}$$

Where F is the fitting function and n is the size of the sample. If λ^2 has less than 2 degrees of freedom, the goodness-of-fit is satisfactory. If there is not a satisfactory fit between the model and the data, the model needs to be revised until the model passes the test.

The goodness-of-fit index and the adjusted goodness-of-fit index are formulated as follows:

$$\text{GFI} = 1 - \frac{F[S,\Sigma(\hat{\theta})]}{F[S,\Sigma(0)]} \tag{5}$$

$$\text{AGFI} = 1 - \frac{(p+q)(p+q+1)/2}{df}(1 - GFI) \tag{6}$$

Where F represents the fitting function, df represents the degree of freedom, S is the variance-covariance matrix of the observed variable, Σ represents the variance-covariance matrix of the model estimation, p represents the total number of endogenous variables, and q represents the total number of exogenous variables.

From the GFI formula, it is observed that the value of GFI is <1. In practical applications, it is generally considered that the model exhibits good fit when the value of GFI is greater than 0.90. The AGFI index adjusts the GFI by the number of degrees of freedom and the number of parameters within the model. The value of AGFI ranges between 0 and 1. The more degrees of freedom within the model, the greater the value of AGFI. Generally, when the AGFI is greater than 0.90, the model is considered to exhibit good fit with the data.

The sample data obtained in this survey were verified to meet the conditions required for normal distribution and maximum likelihood estimation as a matter of experience. The maximum likelihood estimation method produced in the statistical software AMOS 17.0 was used to analyze and test the input data. The calculation method of the structural equation model can compare and analyze the degree of matching between the model and the collected sample data as a whole[5-6]. The SEM can then determine the mutual influence of variables within the model by analyzing the fitted index values. The results of the AMOS 17.0 fitted index output is shown in Table 5:

Table 5. Partial Fitting Indexes of Structural Equation Model.

Fitting index	GFI	AGFI
Judgement standard	> 0.9	> 0.9
Index of model	0.958	0.987

Table 5 shows that the model GFI and AGFI values are between 1 and 0.9, indicating that the model exhibits good fit with the data. The results show that this model does not need to be corrected.

5. Conclusions

In this paper, a basin-oriented evaluation system for mathematical models of reservoir sedimentation is initially developed, and a case database for evaluating reservoir sediment models in the Yellow River is established. In addition, both Delphi and reliability analysis methods were adopted to propose an evaluation index of mathematical models of reservoir sedimentation and obtain non-dimensionalization of individual indexes. An analytic hierarchy process and structural equation model were used to establish weighted quantifications of evaluation criteria. Thus, the initial steps for a comprehensive and quantifiable evaluation of Yellow River reservoir sediment models are established.

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