The Analysis of Students' Difficulties in Solving PISA Mathematics Problems

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Keywords: Student's Difficulties, Newman's Error Analysis, Pisa Problems, Mathematics

Abstract: This study aims to analyse students' difficulties in solving PISA mathematics problem. The mathematics tasks for this study was taken from PISA 2012 released mathematics question. For this research, 12 tasks were selected based on its content area, context, and mathematical process underlying the task. The participant of this research was 28 students from grade IX Mts Ma'arif Assaadah II Bungah-Gresik. To collect the data of student's error, only incorrect or partially correct answer were analysed using Newman's error categories namely comprehension, transformation, processing and encoding. The unknown' category was added for responses which provide too limited information to be analysed. In total, 201 incorrect and partially correct answer was gathered from 336 responses. The analysis result shows that 49.3% of error is located in comprehension, 42.7% in transformation, 7.2% in processing, and only 0.7% error is located in encoding. Based on mathematical process of the task, students made error the most in employ task. Based on mathematical content area of the task, students made error the most in change and relationship task. And the last, based on the context of the task, students made error the most in occupational and scientific task.

1 INTRODUCTION

The Program for International Student Assessment (PISA) is an international assessment coordinated by the Organization of Economic Cooperation and Development (OECD) to measure how well 15 years-old-student prepared to meet their futures challenges (OECD, 2009; 2013) (OECD, 2009)(OECD, 2013). PISA focuses on the competencies that indicate students' capability to continue learning through their lives by applying the skill that they acquire in school to non-school environment, evaluating their choices and making judgment (OECD, 2013). Hence, beside assessing students' knowledge, PISA also assesses students' capacity to reflect, and to apply their knowledge and experience in real life. 'Literacy' is used as an umbrella term of these knowledge and skill. PISA covers three domains: reading, science and mathematics literacy.

> Mathematics literacy in PISA is defined as:
> "An individual's capacity to formulate, employ and interpret mathematics in a variety of context. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizen" (OECD2013, p. 25).

The definition above highlights the three phases of mathematical process; formulate, employ and interpret. The term 'formulate' involves the capability to transform the presented problem or situation into mathematical model and recognise necessary variables and assumptions to solve the problem. The term 'employ' refers to the activities such as using mathematical reasoning, procedure, facts, concepts and tools to find the mathematical solution. As for the term 'interpret', it means interpreting the mathematical solution into the problem context or situation (OECD, 2013; OECD, 2016).

Based on these mathematical processes, PISA mathematics problems are classified into three categories: formulate, employ and interpret. In addition, PISA mathematics problems are also put into categories based on its context and content area.

Wildani, J.
The Analysis of Students' Difficulties in Solving PISA Mathematics Problems.
DOI: 10.5220/0008520202460252
In *Proceedings of the International Conference on Mathematics and Islam (ICMIs 2018)*, pages 246-252
ISBN: 978-989-758-407-7

Based on its context, the mathematics problems are grouped into four types of problem; personal, scientific, occupational and societal. As for its content area, the categories of the problems are space and shape, quantity, change and relationship, and uncertainty of data.

PISA results analysis shows that the skill measured by PISA predicts students' later success better than the competencies indicated by students' academic performance (Schleicher, 2007). Therefore, many countries including Indonesia use PISA surveys as reference when they reform their educational and educational practice (Grek, 2009; Liang, 2010). Indonesia's first participation in PISA was in 2000 (the survey was completed in 2002). The result was quite shocking as among 41 participating countries, Indonesia ranked in 38th place far below Thailand, a neighbourhood country (OECD, 2004). This result indicates that the education quality of Indonesia was low by international standard (The World Bank, 2015). The result of PISA 2000 evoked Indonesian government to invest massively for Indonesia's education system (The World Bank, 2015).

Despite the huge investment, there was no significant increase of Indonesian school children's performance in the next PISA, especially in mathematics literacy. In PISA 2012 where mathematics is the major domain, Indonesian school children mathematics performance ranked in 64th place among 65 participating countries lower then Malaysia, Thailand and Vietnam (OECD, 2013). 2012 was not the only year Indonesian students performed poorly in mathematics. In PISA 2009 only one third of Indonesian school children could solve mathematics task involving daily life context despite all the information needed to solve the task was given and the questions was clearly defined (OECD, 2010).

The poor performance in mathematics literacy by Indonesian students in PISA should become concern of Indonesian government and educators as mathematical problems presented in PISA are related to everyday life where various contexts are involved. These problems require students to apply some fundamental mathematical capabilities such as communication, mathematising, representation, reasoning and argument, devising strategies to solve the problem, using symbolic, formal and technical language and operations, and using mathematical tools. These capabilities also become the focus of newest mathematics curriculum in Indonesia (namely curriculum 2013) as mentioned in the 3rd and 4th mathematics core competencies which reflect the qualities that must be mastered by students when they finish their education in primary and secondary level (Kementrian Pendidikan dan Kebudayaan Republik Indonesia, 2013) stating "understanding, applying and analysing factual, conceptual, and procedural knowledge…" and "experimenting, processing, reasoning, and presenting knowledge in concrete and abstract domain…". That being said, Indonesian students' constant poor performance in mathematics domain in PISA may suggest that school education failed to help students in achieving those competencies.

Considering the poor performance of Indonesian students in PISA, especially in mathematics domain, it is necessary to investigate students' difficulties in solving PISA mathematics problem. Hence, this study aims to analyse students' difficulties to solve PISA mathematics problem. Four research questions are derived from this aim:

1) What errors do students make when solving mathematics problem from PISA 2012?
2) Based on the mathematical content area, in what type of task do students make error the most?
3) Based on the context, in what type of task do students make error the most?
4) Based on the mathematical process, in what type of task do students make error the most?

2 METHODS

2.1 Participant

The participants of this study were28 students from grade IX in MTs Ma'arif Assa'adah II Bungah-Gresik-Indonesia with the age ranged from 14 to 15 years old (21 students were 14 years old, and 7 students were 15 years old). The selection of the age range of the subjects aimed to have the subject of this research close to the age range of PISA target. The gender of all participants was female as the school where the data was collected is a junior high school for girls.

2.2 Instrument

The mathematics problem used for this study was taken from released mathematics items by PISA 2012. The problems were selected such that each process, context, and content distributed equally in the questions given to the participant. In total, 12 questions were selected with each process consists of four questions, each content consists of three questions and each context consist of three questions. The selected questions then translated into Bahasa Indonesia, and then the researcher consulted the mathematical questions to experienced teacher to ensure that the language used in the questions will be understood by primary school students. Few adjustments regarding the language were made according to the suggestion given by the teacher. The participants were asked to solve these questions in 90 minutes. In this research, the term ‘mathematical task’ will be used instead of ‘mathematical question’.

2.3 Data Analysis

To analyse students’ error, only the incorrect and partially correct responses were analysed. The incorrect responses were coded based on Newman’s error categories. Newman (Newman, 1977) developed a model to classify the error student makes during problem solving. He proposed five types of error: error of reading, comprehension, transformation, process skill and encoding. The table 1 explain each type of error.

Table1: Description of Newman’s error analysis.

Type of error	Explanation
Error of reading	Error in simple recognition of words and symbols
Comprehension	Error in understanding the meaning of the problem
Transformation	Error in transforming a word problem into appropriate mathematical problem
Process skill	Error in performing mathematical procedure
Encoding	Error in presenting the mathematical solution into acceptable written form

In this study, the reading error from Newman’s error categories was left out because this type of error doesn’t indicate the difficulty/incapability to understand the meaning of the problem (Wijaya, et al., 2014). In addition, ‘unknown’ was added to error categories for responses which provide too limited information or no information at all to be analysed. The coding scheme used to analyse the error was adapted from ‘Coding Scheme for error types when solving context-based mathematics tasks’ (Wijaya, et al., 2014). The adaptation was conducted by adjusting the explanation of each type of error based on the nature of mathematical task used in this study and the error found during the analysis (table 2).

3 RESULTS AND DISCUSSION

In total, 201 (59.8%) incorrect and partially correct answers were gathered from 336 responses. Among these 201 answers, 74 answers couldn’t be analysed as the answer didn’t provide any information or the information was too limited which left 127 answers to be analysed.

Table 2: Coding scheme for error analysis.

Type of error	Explanation
Compre-hension	- Incorrect interpretation of the instruction - Misunderstood a keyword(s) which is usually mathematical term - Unable to select relevant information from the task - Unable to gather information which is not provided explicitly in the task - Use the irrelevant information from the task to solve the problem
Transformation	- Use mathematical procedure/formula/algorithm directly without analysis if it is necessary for the task - The answer only refers to the context or real-world setting without considering the mathematics perspective - Use incorrect mathematical procedure/concept/fact/algorithm which is not relevant to the task
Process skill	- Error in solving algebraic expression or function - Error in calculation - Incorrectly use the measurement - Use the correct formula/procedure but unable to finish the calculation
Encoding	The answer does not reflect to the context which is indicated by unrealistic answer

The analysis result of the incorrect and partially correct answer about the error that students made when solving PISA mathematics problem can be seen in the table 3.

Table 3: Distribution of error

Type of error	n	Percentage
Comprehension	68	49,3%
Transformation	59	42,7%
Process skill	10	7,2%
Encoding	1	0,7%
Total	138	

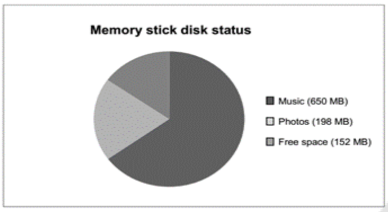
The total number of errors exceeds the number of analysed answer because multiple codes was applied where it was possible there were more than one type of error in one answer.

The table shows that according to Newman's error analysis result, the most common error occurred was comprehension error followed by transformation error. This implies that the most difficult stage in solving PISA mathematics problem for students is in understanding the meaning of the problem and transforming the word problem into appropriate mathematical model. The figure 1 is the example of students work and the error they made.

MEMORY STICK

A memory stick is a small, portable computer storage device.

Ivan has a memory stick that stores music and photos. The memory stick has a capacity of 1 GB (1000 MB). The graph below shows the current disk status of his memory stick.



Question 1: MEMORY STICK PM00AQ01 –

Ivan wants to transfer a photo album of 350 MB onto his memory stick, but there is not enough free space on the memory stick. While he does not want to delete any existing photos, he is happy to delete up to two music albums.

Ivan's memory stick has the following size music albums stored on it.

Album	Size
Album 1	100 MB
Album 2	75 MB
Album 3	80 MB
Album 4	55 MB
Album 5	60 MB
Album 6	80 MB
Album 7	75 MB
Album 8	125 MB

By deleting at most two music albums is it possible for Ivan to have enough space on his memory stick to add the photo album? Circle "Yes" or "No" and show calculations to support your answer.

Jawab: YA/TIDAK

karena 2 album foto semisal 100 + 75 MB hanya cukup 175 MB sedangkan Ivan akan memindahkan sebesar 350 MB jadi perkiraan Ivan itu tdk akan memiliki ruang yg cukup.

English translation:
Answer: NO
Because by deleting two albums, for example 100+ 75 MB will only result 175 MB free space whereas Ivan needs to transfer 350 MB album. Hence, deleting two albums will not give Ivan enough space

Figure 1: Student's work where she made comprehension error.

In this work, student failed to select relevant information from the task showing that there is 152 MB space in Ivan's memory stick which leads to the wrong answer. This type of error is included in comprehension error.

Question 1: PENGUINS PM921Q01

Normally, a penguin couple produces two eggs every year. Usually the chick from the larger of the two eggs is the only one that survives.

With rockhopper penguins, the first egg weighs approximately 78 g and the second egg weighs approximately 110 g.

By approximately how many percent is the second egg heavier than the first egg?

A. 29%
B. 32%
C. 41%
D. 71%

Perbandingan = Berat telur kedua – Berat telur pertama
= 110 g – 78 g
= 32 %

English Translation
Ratio: the weight of second egg−the weight of first egg

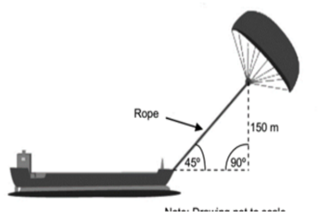
Figure 2: Student's work where she made transformation error.

Figure 2 above shows that student was unable to perform the correct mathematical procedure where she should have divided the difference by the weight of the first egg and then multiplied it by 100% to get the correct percentage. Instead, she directly put "%" in her answer which results the wrong answer. This error is included in transformation error category.

Question 3: SAILING SHIPS PM923

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?

A. 173 m
B. 212 m
C. 285 m
D. 300 m



Jawab:
Tali $= \sqrt{150^2 + 150^2}$
Tali $= \sqrt{22500 + 22500}$
$= \sqrt{45000}$

Figure 3: Student's work where she made process skill error.

In this task, student was able to perform the correct mathematical procedure, but she was unable to finish the calculation which results in unfinished answer. The inability to finish the calculation is included in process skill error.

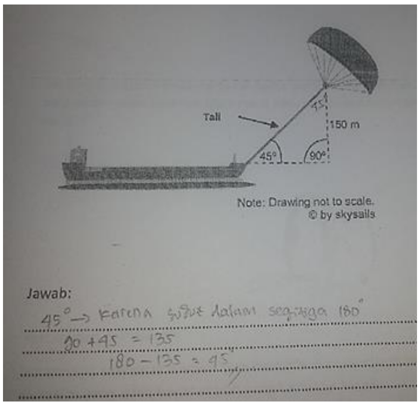


Figure 4: Student's work where she made encoding error.

The picture 4 shows the solution for the same mathematical task indicated in Fig. 3 where students were asked to calculate the length of the rope. She answered that the length of rope is 45°. The error in this answer is the use of the unit system for length. She used degree instead of meter. The response of student indicated in Fig. 4 shows that students gave unrealistic answer within the context of this task. Hence, this error is included in encoding error.

The type of error in Neman's error category reflects the stages in answering mathematics word problem (Newman, 1977). If the type of error in Newman classification is viewed as stages, then comprehension and transformation are included in earlier stages of problem solving. This suggests that students already faced the difficulties in the earlier stage of solving PISA mathematics problem when they had to understand the meaning of the problem and transform the word problem into appropriate mathematical model.

Distribution of incorrect and partially correct answer in mathematics task based on its content area, context and mathematical process can be seen in the table 4. The table 4 indicates that among four types of mathematics task based on its content area, students had most difficulties in the problems which content area is change and relationship. This type of problem requires student to understand about various types of changes and know their occurrence so that students can use appropriate mathematical models to describe and predict the change (OECD, 2013). Being less literate in this area implies that students lack understanding about these knowledge. Furthermore, it also implies students lack skill in mathematics modelling and representation which are important aspect of change and relationship.

Table 4: Distribution of error based on content area, context, and mathematical process.

Type of task	n (in %)		
	Correct	Partially correct	Incorrect
1. Based on its content area			
Change and relationship	10.7	3.6	85.7
Space and shape	42.9	4.8	52.4
Quantity	45.2	0	54.8
Uncertainty and data	61.9	20.2	17.9
2. Based on its context			
Personal	39.3	3.6	57.1
Societal	77.4	0	22.6
Occupational	16.6	25	58.3
Scientific	27.4	0	72.6
3. Based on its mathematical process			
Formulate	34.8	17.8	47.3
Employ	24.1	3.6	72.3
Interpret	61.6	0	38.4

The second types of task with huge percentage of error are quantity task. To solve this type of task, students must understand about units, magnitudes, count, measurements and numerical trend and pattern. Students also must use numerical reasoning which is the essence of mathematical literacy relative to quantity (OECD, 2013). Having difficulties in this area suggest the lack of skill in number sense, mental calculation, multiple representation of numbers, estimation and assessment of reasonableness of result and elegance of computation as the aspects of quantitative reasoning as well as lack of understanding about units, magnitudes, count, measurements and numerical trend and pattern.

For the type of mathematics task based on its context, students found it most difficult when dealing with occupational and scientific problems. The problem context itself is the students' world aspect in which the problems are situated (OECD, 2013). Usually the context of the problem determines how the problem solver chose their strategies and representation (OECD, 2016). In PISA 2012 (which items were used for this study), occupational context problems encompass the problems centred around the world of work.

As for scientific problems, it covers the problems relate to the use or application of mathematics to the natural world and issues related to science and technology. The huge numbers of students having difficulties in these contexts suggest the unfamiliarity with these kinds of problems which lead to the possibilities that students were rarely exposed to problems related to work and science.

For type of mathematics task based on its mathematical process, students encountered most difficulties when solving employ problems. The term 'employ' refers to the ability to apply mathematical reasoning, procedures, fact, and concept to solve problems formulated mathematically. It covers some activities such as devising and implementing strategies to find mathematical solution, manipulating numbers, data, and algebraic expressions, and making generalization (OECD, 2016). The difficulties in the employing process indicate the lack of mentioned abilities above.

4 CONCLUSIONS

This study aimed to get better insight about students' difficulties while solving PISA problems, focusing on the type of error, mathematical process, content area, and the context.

The result of this study shows that the most common difficulties encountered by the students in solving PISA mathematics problem was comprehension which is the difficulty in understanding the meaning of the problem. Students also had major difficulty in transforming word problem into mathematical model. On the other hand, students only made minor error in processing the mathematical procedure and presenting it in formal form.

The other finding of this study shows based on the content area of the problem, students made error the most when solving change and relationship problem. Based on the context of the problem, students found it most difficult to solve occupational and scientific problem. Meanwhile, based on mathematical process underlying the mathematical problem, students had the most difficulties in problem which mathematical process is 'employ'.

The result of this study can be used as a basis for future studies or designing an intervention to solve educational problem, especially in mathematics education field

For instance, considering the fact that the most difficulties encountered by students in solving PISA mathematics problems is in comprehension and transformation phase, the future study can be focused on the efforts of how to improve students' comprehension and transformation skill. It should be also noted that reading comprehension has a positive correlation to student's mathematics and science performance in PISA (Akbaşlı, et al., 2016).

In English department, student's reading comprehension can be improved using text structure task (Rohman, 2017). Text structure refers to how ideas in text are interconnected to deliver message to reader (Meyer & Rice, 1984). Mathematics teachers may adapt this strategy into mathematics classroom to enhance students understanding of mathematics problem. For future study, mathematics education researchers may also investigate the effect of these strategies in students' mathematics literacy.

Transformation skill is closely linked to mathematics modelling. Hence, to improve students' ability in transforming word problem into mathematics problem, teachers may need to engage and familiarize students with mathematics modelling activities. In fact, modelling activities within classroom has been proven to be effective to improve students' mathematical representation (Hanifah, 2015) which is important aspect of transformation skill.

The data about the error students made based on problem's content area, context and mathematical process especially provide a basis to design a learning material (such as students worksheet, handbook or learning module) to develop or improve students' mathematics literacy.

Finally, some limitation of this study should be considered when readers want to make use of this study as this study evidently has some limitations. For example, the finding from the students in this study does not automatically apply to students in other school or region. Given the same PISA questions, the result may differ in the distribution of error types. The other limitation is in mathematical task selection. In this study, only 12 tasks were chosen from 100 tasks in total. If more mathematical task were used to collect data, there is a possibility that this study would give different result.

REFERENCES

Akbaşlı, S., Şahin, M. & Yaykiran, Z., 2016. The Effect of Reading Comprehension on the Performance in Science and Mathematics. *Journal of Education and Practice*, 7(16), pp. 108-121.

Grek, S., 2009. Governing by numbers: the PISA 'effect' in Europe. *Journal of Education Policy,* 21(1), pp. 23-37.

Hanifah, 2015. Penerapan Pembelajaran Model Eliciting Activities (MEA) dengan Pendekatan Saintifik untuk Meningkatkan Kemampuan Representasi Matematis

Siswa. *Kreano Jurnal Matematika Kreatif-Inovatif*, 6(2), pp. 191-198.

Kementrian Pendidikan dan Kebudayaan Republik Indonesia, 2013. *Kompetensi Dasar. Sekolah Menengah Pertama (SMP)/Madrasah Tsanawiyah (MTs)*, s.l.: Kementrian Pendidikan dan Kebudayaan Republik Indonesia.

Liang, X., 2010. Assessment use, self-efficacy and mathematics achievement: Comparative analysis of PISA 2003. Data of Finland, Canada and the USA. *Evaluation & Reasearch in Education*, 23(3), pp. 213-229.

Meyer, B. & Rice, G., 1984. *The Structure of Text.* In P.D Pearson ed. New York: Longman.

Newman, M. A., 1977. An anlysis of sixth-grade pupils' errors on written mathematical task. *Victorian Institute for Educational Research Bulletin*, Vol. 31, pp. 31-43.

OECD, 2004. *Learning for tomorrows' world: First results from PISA 2003*, Paris: OECD Publishing.

OECD, 2009. *PISA 2009 Assessment framework: Key competencies in reading, mathematics, and science*, Paris: OECD Publishing.

OECD, 2010. *PISA 2009 results: What students know and can do. Students' performance in reading, mathematics, and science (Vol. I)*, Paris: OECD Publishing .

OECD, 2013. *PISA 2012 Assessment and analytical framwork: Mathematics, reading, science, problem solving and financial literacy*, Paris: OECD Publishing.

OECD, 2016. *PISA 2015 Assessment and analytical framework: Science, reading, mathematics and financial literacy*, Paris: OECD Publishing.

Rohman, A., 2017. Improving Students' Comprehension Through Text Structure Task. *Journal of Linguistic and English Teaching* , 2(1), pp. 1-12.

Schleicher, A., 2007. Can competencies assessed by PISA be considered school knowledge 15-years-old should possess?. *Educational Change*, 8(4), pp. 349-357.

The World Bank, 2015. *Indonesia: Teacher certification and beyond. An empirical evaluation of the teacher certification program and education quality improvements in Indonesia*, Jakarta: The World Bank.

Wijaya, A., Heuvel-Panhuizen, M. v. d. & Robitzch, M. D. A., 2014. Difficulties in solving context-based PISA matehmatics task: An analysis of students' error. *The Mathematics Enthusiast*, 11(3), pp. 555-584.