Identifying Mathematics Education Students' Obstacles in Reading and Constructing Proofs in Real Analysis Courses

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Abstract: This study aims to find out how students' experiences and obstacle in learning the Real Analysis lectures that they have experienced. The aspects measured are: (1) the ability to read and understand a proof, and (2) The ability to construct a proof. Subjects in this study consisted of 43 students of Mathematics Education Study Program of FKIP Khairun University and data were collected through tests and interviews. Interviews were conducted with six students based on the work to identify the obstacles experienced in reading, understanding and constructing a proof. The data were analyzed by using the descriptive-qualitative approach. The results show that the ability to read, understand and construct a proof belong to the medium category.

1 INTRODUCTION

In general, obstacles are anything that hinders student learning (Moru, 2007), based on some writings on obstacles according to Tall (1991), which have identified three forms of obstacles, usually related to students' failure to accommodate new ideas. These obstacles are epistemological, cognitive, and didactic. Epistemological nature, more because of the internal reasons of mathematics itself (Brousseau, 1997; Sierpinska, 1987); cognitive traits, because of the abstraction process and conceptualization involved in it (Cornu, 1991; Dubinsky, 1991; Sfard, 1991; Tall & Vinner, 1981) while didactic traits, due to the nature of teaching and learning (Brousseau, 1997).

Proof activities are important in mathematics education, especially in Real Analysis, where most of the material is in the form of proof tasks related to lemma, theorems, and corollary. The activity of constructing a proof in real analysis is one of the obstacles that are often encountered by students, even though in constructing the proof some instructions have been given to facilitate the construction of proof.

According to Selden and Selden (2003) one of the important activities in mathematics is to read mathematical proof with the aim of determining whether the proof is valid or not, this activity is marked as proof validation and this is a complex process involving evaluation of arguments, proposing and answer questions, construct sub proof, remember definitions and theorems.

Furthermore Selden and Selden (Pfeiffer, 2010) assert that the ability to read proof is the ability to determine truth from mathematical proof and mental processes related to validation of proof. Validation not only determines the truth of the argument, but the validation of the proof includes: does the reader understand the argument provided?, the quality and clarity of the idea of the proof strategy, the clarity of the structure, the selection of appropriate, correct and adequate reasons, and what is the convincing argument? While the mental process when validating proof is asking or answering questions, constructing parts of proof or remembering theorems and other definitions.

Referring to "transactional theory of reading", Rosenblant (1988) suggests that during reading activities, the reader forms and is actively formed by the text. So the reader does not only recite the readings, but with the knowledge, interest and curiosity of the text being read, the reader will try to develop the meaning of the text. If this is associated with the ability to read proof from a student, then the student can express ideas/ideas contained in the proof both verbally and in writing using his own language and understand what is contained in the mathematical proof.

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Constructing proof requires the right idea at the right time, so that it requires some initial information (for example, assumptions, axioms, definitions) and applies inference rules (for example, remembering previous facts, and applying theorems) to desired conclusions to be concluded. In other words, constructing proof will occur mainly in a written format, so that the rules in writing, proof can be used synonymously with the construction of proof in a written format.

Pfeiffer (2010) states that validation processes can usually be managed in a linear sequence, such as constructing proofs. On the other hand the construction of proof and validation of proof requires each other, because during the process of constructing proof, how the proof will be validated, and as proof validation tends to require the construction of parts of proof or sub-proofs, this relationship can be seen in the following diagram:

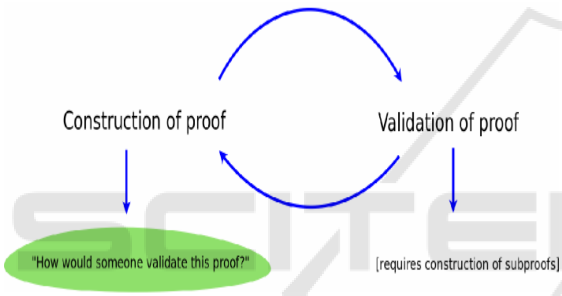

Figure 1: Construction Related to Proof Validation.

To clarify the link between construction and proof validation, Pfeiffer developed the diagram above as a development of the impact of learning through proof validation, as in the next diagram:

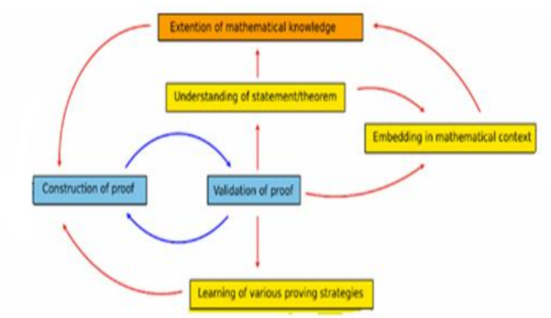

Figure 2: Validation of Proof in the Process of Learning about Mathematical Proof.

Based on Figure 2, it appears that the link between validation of proof and construction of proof, validation of proof requires an understanding of statements/theorems that are appropriate to the mathematical context and additional mathematical knowledge and learning from various strategies for constructing proof, and vice versa. It is further expected that the ability to validate proof can improve the ability to construct proofs, develop a deeper understanding of the meaning and the meaning of the theorem that is proven and develop knowledge, methods or strategies in mathematical proof.

According to Selden and Selden (2014) three actions that are useful in constructing proof are: (1) *Exploring*. The act of constructing a part of proof, one may understand what must be proven and what is available to use without having an idea of how to proceed. Such a situation, people might try to prove something new from an unknown value; (2) *Reworking arguments in cases of suspected error or wrong direction*. Constructing proof, perhaps one should be suspected that someone made a mistake or made an argument that was not in the direction and did not help. The thing to do is to re-respond to part of the argument; (3) *Validating a proof of completion*. After completing the proof, we must read and examine carefully the truth in each row from top to bottom and each of the following lines of what has been said above.

The mathematical knowledge related to the activity of understanding and validating proof and constructing the proof, carried out by teachers and students, including the one presented by Rogers and Steele (2012) is "how to verify the truth" which is checking or confirming the truth of a known idea, "explaining why" is opening up the thoughts and reasons behind why this statement is true including giving reasons to support the conjecture (Hanna, 1995), "communicating mathematical knowledge" that is helping others understand mathematical ideas and disseminating mathematical knowledge to others, "creating new mathematical knowledge, "He meant developing new ideas in mathematics," confirming conjectures/conjectures ", building mathematical ideas, and" systematizing domains "namely applying logical structures to the domain of mathematics, organizing and cataloging the results in relation to an axiom and prior knowledge (Knuth, 2002). In line with this, Hanna (Magajna, 2013) states that the function of proof and proof is verification that something is true, explanation of why something is true, systematization of concepts, variations in results and theorems, discovery of new results, communication of mathematical knowledge, constructing theories empirically, exploration of the meaning of definitions, and incorporation of known facts into new frameworks.

2 METHOD

Subjects in this study consisted of 43 students of mathematics education FKIP Khairun University, and data collected through tests and interviews. Interviews were conducted on 6 (six) students based on the work to identify the obstacle experienced in reading, understanding and constructing a proof. The data were analyzed by the descriptive-qualitative approach.

3 RESULTS AND DISCUSSION

Based on the results of the study of the aspects observed in the mathematical proof ability of students, namely: (1) The ability to read and understand the proof, and (2) The ability to construct proofs, presented several examples of student answers that have not been perfect in carrying out proof. Errors that often appear in answering questions and types of errors from the results of student answers.

To solve the question no. 1, You are welcome to read carefully the examples of proof presented starting from the table of Informal Arguments to formal proof, then you are asked to prove the matter in the space provided. If the space provided is not sufficient in the verification process, you can use the additional paper provided. In this study, students were asked to read the given example, "prove that $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$". Next is given the problem as follows: Problem 1. Prove that $\lim_{x\to 0} \sin x \ \cos\left(\frac{1}{x}\right) = 0$.

Problem number 1 above relates to the indicator of the ability to verify the steps in formal proof from the examples provided to be applied in solving problems similar to the example. Students are expected to begin verification by verifying and completing the informal argument table that has been provided, which then follows the stages as a basis for probing the question in question. Errors made by students, including those presented below:

Soal 1. Buktikan bahwa $\lim_{x\to 0} \sin x \ \cos\left(\frac{1}{x}\right) = 0$

Lengkapi tabel Argumen Informal berikut ini:

Konstruksi Diagram Bukti	Potongan Bukti	Kategori	Pengkodean	Alasan
Yang akan dibuktikan dengan cara: Kita mencari δ yang bergantung pada ε. $\left\lvert \sin x \ \cos\left(\frac{1}{x}\right) - 0 \right\rvert = \cdots$ ⋮ $\left\lvert \sin x \ \cos\left(\frac{1}{x}\right) - 0 \right\rvert < \delta = \varepsilon$	Berdasarkan definisi limit, kita akan mencari δ untuk $\lvert x-0 \rvert$ yang bergantung pada ε,	Definsi & Asumsi Pilihan	DEF&AC	Berdasarkan definisi limit dan memilih atau menetapkan $\delta > 0$
	$\left\lvert \sin x \ \cos\left(\frac{1}{x}\right) - 0 \right\rvert$ $= \left\lvert \sin x \cos\left(\frac{1}{x}\right) \right\rvert$ $= \lvert \sin x \rvert \left\lvert \cos\left(\frac{1}{x}\right) \right\rvert$	Definisi	DEF	Berdasarkan definisi limit
	$\lvert \sin x \rvert \left\lvert \cos\left(\frac{1}{x}\right) \right\rvert \leq \lvert \sin x \rvert$ karena menggunakan sifat $\left\lvert \cos\left(\frac{1}{x}\right) \right\rvert \leq 1$	Referensi Interior & Definisi	IR & DEF	Memasukan potongan bukti sebelumnya untuk memenuhi definisi limit
	Sedemikian sehingga $\lvert \sin x \cos\left(\frac{1}{x}\right) - 0 \rvert$ $= \lvert \sin x \cos\left(\frac{1}{x}\right) \rvert$ $= \lvert \sin x \rvert \lvert \cos\left(\frac{1}{x}\right) \rvert$ $\leq \lvert \sin x \rvert$ $= \left\lvert \sin x \cdot \frac{x}{\sin x} \right\rvert$ $= \lvert x \rvert < \delta$ artinya, $\delta = \varepsilon$	Asumsi Pilihan dan depinisi	AC dan DEF	Berdasarkan asumsi pilihan yakni dengan memilih atau menetapkan $\delta > 0$, untuk $\lvert x-0 \rvert$, dan berdasarkan definisi limit, maka dapat dipilih $\delta = \varepsilon$.
	$\lvert \sin x \cos\left(\frac{1}{x}\right) - 0 \rvert < \varepsilon$	Definisi dan kesimpulan	Def dan C	Berdasarkan definisi limit dan menyimpulkan bukti

Figure 3: Results of Student Work in Completing Informal Arguments.

Bukti Formal:

Ambil sebarang $\varepsilon > 0$, pilih $\delta = \varepsilon$ sedemikian sehingga untuk setiap x dengan $0 < \lvert x - 0 \rvert < \delta \Rightarrow \lvert f(x) - L \rvert = \lvert \sin x \cos\left(\frac{1}{x}\right) - 0 \rvert$

$= \lvert \sin x \cos\left(\frac{1}{x}\right) \rvert = \lvert \sin x \rvert \lvert \cos\left(\frac{1}{x}\right) \rvert \leq \lvert \sin x \rvert$

$= \left\lvert \sin x \quad \frac{x}{\sin x} \right\rvert = \lvert x \rvert < \delta = \varepsilon$

Jadi; $\lvert \sin x \cos\left(\frac{1}{x}\right) - 0 \rvert < \varepsilon$

dengan demikian terbukti $\lim_{x\to 0} \sin x \cos\left(\frac{1}{x}\right) = 0$

Figure 4. Example of a Formal Proof Job Preparing Students to Question 1

The results of student work in Figures 3 and 4 are one example of an incomplete and inappropriate answer. Before describing the work results of students, researchers conducted interviews with students with the aim of uncovering errors or difficulties encountered in carrying out the verification of the questions above. The results of the interview are presented as follows:

Researcher: *In completing the argument table (3rd row of the 2nd column), you only use properties* $\left|\cos\left(\frac{1}{x}\right)\right| \leq 1$ *, but not yet using properties* $|\sin x| \leq |x|$ *, why not use properties* $|\sin x| \leq |x|$*?*

$M_{Eks\text{-}16}$: *I see in the example only properties* $\left|\cos\left(\frac{1}{x}\right)\right| \leq 1$*, written by Sir*

Researcher: *Oh yes, but as soon as you complete the next column, there appear properties* $|\sin x| \leq |x|$*, what underlies you write like that?*

$M_{Eks\text{-}16}$: *When I write like that, I think* $\sin x$ *with* $\sin x$ *can share one another, Sir*

Researcher: *If you look at the time you compile a formal proof, election* $\delta = \varepsilon$ *it's right, but once compiling the next step appears* $\cdots \leq |\sin x| = \left|\sin x \,.\, \frac{x}{\sin x}\right| = |x| < \delta \cdots$*, this step is illogical, why do this?*

$M_{Eks\text{-}16}$: *After I tried using it … I mean what I fill in the table of informal arguments, I think this is correct Sir, it turns out this is wrong, sir, I have a bit of difficulty utilizing the traits* $|\sin x| \leq |x|$ *and manipulate signs of inequality sir*

Researcher: *Ok, thank you, I think you have shown a persistent effort to do proofs, hopefully for the next question you are better.*

From the results of the interview, it shows that the third row of the second column, students have tried to include the previous pieces of evidence with categories and coding, but the pieces of evidence in the form of arguments that have not guaranteed the next trait, students have not used the properties of $|\sin x| \leq |x|$. Then for the next column, students have used the previous step, but the pieces of evidence for the choice assumption category (AC) and (DEF) are " $\cdots \leq |\sin x| = \left|\sin x \,.\frac{x}{\sin x}\right| = |x|$" becomes illogical, even though in the selection δ = ε as in the column it shows that the student is right to choose. Next in compiling formal evidence, the steps in the first row up to the third row of students have done correctly, by using triangular inequality and properties $\left|\cos\frac{1}{x}\right| \leq 1$, but in the fourth row the students repeated, making mistakes entering "$\cdots \leq |\sin x| = \left|\sin x \,.\frac{x}{\sin x}\right| = |x|$",so this formal proof becomes invalid.

In general, from the results of the analysis of student work for question number 1, several obstacles can be found which cause student difficulties in proving, as follows: (1) When completing the informal argument table, students cannot yet utilize the general nature of $|\cos x| \leq 1$ and $\left|\frac{\sin x}{x}\right| \leq 1$, (2) Make mistakes in selection δ and manipulate the nature of inequality ≤ to be < (obstacle to manipulating algebraic forms), (3) Students have difficulty utilizing the concepts related to the questions to be proven, and (4) Students have difficulty connecting informal arguments and rewriting them into formal proof.

4 CONCLUSIONS

In general, from the results of the analysis of student work, several obstacles can be found which give rise to student difficulties in proof, namely when completing the informal argument table, students were not able to take advantage of the general nature of |cos x | ≤ 1, making mistakes in the selection δ do proof construction into formal proof, manipulate the nature of inequality ≤ to be (obstacle to manipulating algebraic forms), students have difficulty utilizing concepts related to the questions to be proven, students have difficulty connecting informal arguments and rewriting them into formal proof, and in the final settlement in constructing proofs of students experiencing obstacles in using the previous steps to formulate formal proof.

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