# Nonlinear Model of the Hydraulic Automatic Gauge Control System: Conrollability Analysis and Observability

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Abstract: In the reversible cold rolling mill, it is critical to control the gauge or thickness of the cold-rolled steel strip for end-user requirements. To obtain a high precision in the output thickness of the strip, the automatic control system of the gauge is used. As one of the most important functions of the basic mill automation, there is the hydraulic roll gap controlsystem (HRGCS) which is the inner loop of the Hydraulic Automatic Gauge Control (HAGC).In order to improve the control performance of the strip mill, a theoretical nonlinear mathematical model of the complex HAGC has been established in this paper for a reversible cold rolling system.The new nonlinear model of the HAGC will be used to develop nonlinear control strategies to address the different control problems encountered.

## 1 INTRODUCTION

Nowadays, steel strips of various thicknesses are vitally important products in the iron and steel industry and are widely used in aeronautics, household appliances, car manufacturing, machines and many other areas.

The accuracy of the web thickness and flatness control is directly dependent on the performance of the web winding system (WWS) studied and controlled by nonlinear controllers presented in (akil, 2017), (akil, 2018a) and(akil, 2018b), the Automatic Gauge Control (AGC) and Automatic Flatness Control System (AFC) (John, 2010). In particular, the Hydraulic AGC System is one of the key techniques of modern mills to determine the quality of the web based on Hydraulic Roll gap controlsystem (HRGCS).

The HRGCS is an important part of the AGC system by pressing the position adjustment of work rolls in order to get a precision of the thickness and flatness of the web (Soszy´nski, 2012), (Kim, 2013) and(Hoshino, 1997). Therefore, the good dynamic characteristics of the HRGCS are very important for the AGC system (Zhang, 2012a).

The HRGCS is a typical of machine, electrical, hydraulic integrated system for complete control of the complex system. In nowadays, more and more scholars have devoted themselves to the study of hydraulic gap control (Zhou, 2007) and (Zhang, 2012b).

The cold rolling process has been taken as a research topic for many decades, and currently some theories are able to provide a valuable and detailed description of the roll gap (Grimble, 1978). First Siebel and von Karman (von Khmh, 1925),(Siebel, 1925) began studies on the subject; their analysis introduced the vertical segments concept of homogeneous compression of the sheet during rolling. Another fundamental supposition was the occurrence of a neutral plane in the length of the contact arc (Freshwater, 1996).

The models proposed by (Sims ,1945),(Bland and Ford ,1948) and (Nascimento, 2016) have solved analytically the problem by avoiding most numerical integrations, unlike the Orowan model (Orowan, 1943) which is much more complex and requires more of calculations. However, the simplifications of Bland and Ford led to a sacrifice of precision (Alexander, 1971).

In order to analyze and optimize the AGC, a precise model of the HRGCS is required first. Numerous studies on the control of the HRGCS in



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modeling and simulation have been carried out (Liu, 2012) and (Sun, 2014).Sun (Sun, 2015) established a mathematical model of the HRGCS that was used to design a controller with a predictive control theory.

d considering the nonlinear and saturation characteristics. However, it is difficult to determine the important parameters in the theoretical model with the mechanism analysis method. Moreover, real conditions play an important role in the decision on parameters (Ren, 2014), (Li, 2006) and (Dyja, 1996).

In order to adjust the parameters of the change of gap of the rolling mill, the authors carried out a number of experiments where they studied the rolling of the web with different correction parameters in each stand (Petryakov, 2011); (Galkin, 2011) and (Khramshin, 2010). The parameters were selected empirically and passed to the controller of the automatic process control system.

Based on the nonlinear equations of the elastic and plastic deformation of the web and work roll, and the rolling force, a nonlinear model of the HAGC for reversible cold rolling mill was developed in this paper.

The rest of the article is organized as follows: section 2 presents a description of the rolling process, Basic theories of the cold rolling mill introduced in Section 3, The model of the HAGC is established in Section 4 and the Simulation Results and Discussion in Section 5. Finally, the conclusion is given in section 6.

## 2 DESCRIPTION OF THE ROLLING PROCESS

Rolling is a manufacturing process by plastic deformation. In this method, the metal web undergoes a reduction in thickness by crushing between the two work rollers rotating in two opposite directions. While the larger diameter backup rolls (support rollers) serve to support the work rolls to prevent them from bending too much.

The rotary motion of the work rolls exerts a compressive force to continually decrease the initial thickness of the web by passing several times in the same rolling stand for the reversible single-roll mill.

A rolling mill cage is composed of:

- A pair of cylinders called "work roll" between which the material is elongated. The working cylinders are made of rectified cast iron;

In (Sun, 2014), the mathematical model of the HRGCS was established by adopting the mechanism modeling method and the model was dynamically simulated and analyze

- Another pair of cylinders called "backup roll" (a cylinder on each side of the pair of work rolls) to reduce the deformation of the work rolls.
- Two metal columns holding the cylinders together (one column on each side).

Figure 1 illustrates the rolling process where a web is engaged between the two rolls rotating in opposite direction. The thickness $h_{in}$ web must be able to engage between the rollers where it is deformed in compression to stand at a thickness $h_{out}$ corresponding to the adjustment of the clearance between the rolls.

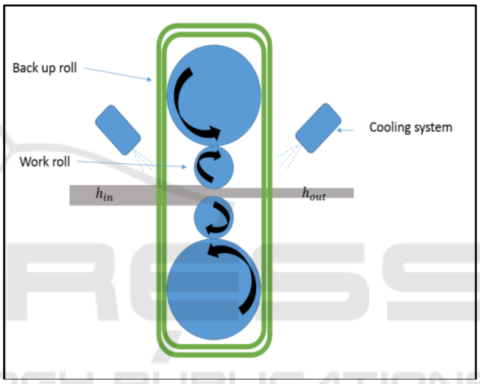


Figure 1: rolling Principle

The necessary compressive force is applied to the bearings of the backup rollers by hydraulic cylinders or by a screw arrangement driven by an electric motor. The thickness of the rolled web is mainly determined by the gap between two work rolls which is initially set by a pass line adjusting mechanism. The actual position control is performed by the exact and fast acting hydraulic control system called Hydraulic automatic gauge control (HAGC).

## 3 BASIC THEORIES OF THE COLD ROLLING MILL

### 3.1 Control System of the HAGC

The structure of the HAGC is shown in figure 2. The system is typically composed of thickness gauge, position sensors, position controller, servoamplifier, electrohydraulic servo valve, and hydraulic cylinder components.

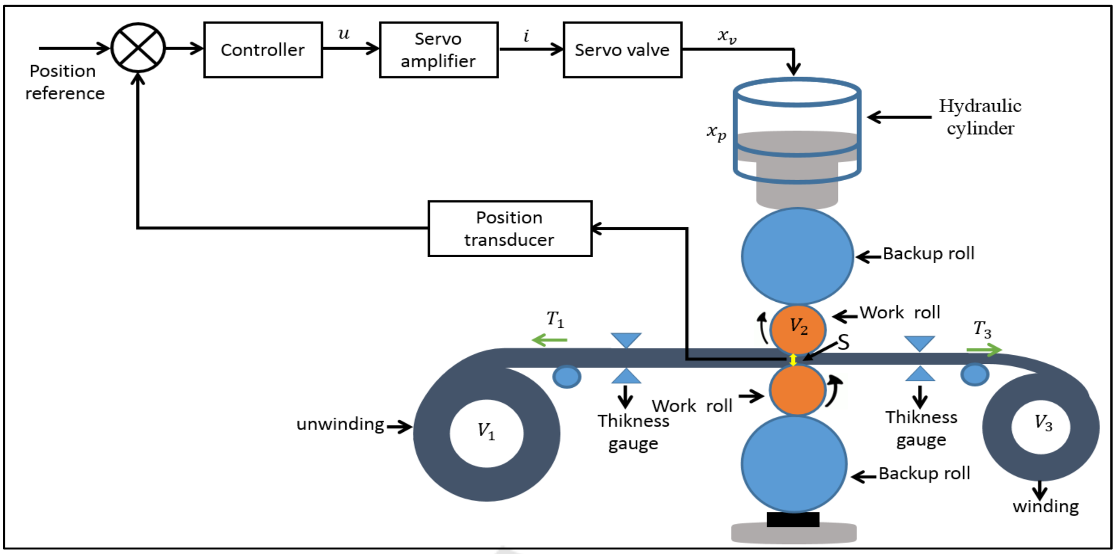


Figure 2: Basic configuration of HAGC

The position control system of hydraulic roll gap is constructed by the following process: the position difference is obtained by comparing the position reference and the actual position, and the difference is transmitted to the position controller; take the controller output values as servoamplifier input values; the amplified current signal is transferred to the degree of opening of the electrohydraulic servo valve; Finally, the hydraulic oil passes the servo valve by alternately driving the movement of the actuator (Zhanget al, 2009 , Wang, 2011 , Lv, 2007 and Andrew and Rui , 2000).

Table 1: Symbol description

| Symbol | Parameter |
|---|---|
| $x_v$ | the spool valve position |
| $K_v$ | Servo amplifier gain |
| $\tau_v$ | The associated time constants. |
| $i$ | servo valve input current |
| $Q_L$ | Servo valve flux |
| $C_d$ | the flow coefficient of the valve port |
| $\omega$ | Servo valve natural frequency |
| $P_s$ and $P_L$ | the inlet and outlet pressure of the servo valve respectively |
| $\rho$ | the oil density |
| $A_p$ | the active area of the cylinder piston |
| $C_t$ | the total leakage coefficient |
| $V$ | The oil pocket volume of the hydraulic cylinder |
| $\beta_e$ | the bulk modulus of elasticity |
| $M_t$ | the equivalent total mass of moving parts of the upper roller system |
| $B_p$ | the viscosity coefficient of cylinder |

## 3.2 Symbols

| | |
|---|---|
| $K_t$ | the elastic stiffness coefficient of load |
| $x_p$ | the cylinder piston displacement |
| $F_L$ | Other load force acting on the piston |
| $W_s$ | the plastic stiffness coefficient of the rolled piece |
| $h_{in}$ and $h_{out}$ | are the input and output thickness of the rolled piece respectively |
| T | The constant delay time. |
| $F_f$ | the unknown force including the coulomb friction force |
| $P$ | the rolling load |
| $M$ | the mill modulus |
| $S$ | the roll gap |
| $S_0$ | Unloaded roll gap |
| $A_d$ and $L_d$ | the magnitude and period of thickness deviation respectively |
| vin | the entry strip is passed through the roll stand with the velocity |
| $T_g$ | Time constant of hydraulic servo |
| $K_g$ | constants specified |

## 3.3 Basic Theoretical Equations

### 3.3.1 Servo Valve Torque Motor Equation

The servovalve used in this paper has a critical center (zero overlap). Its orifices are supposed symmetrical and matched. The internal leakage of the servo valve are neglected. The servo valve treats the current $i$ as the input and displacement of the valve core $x_v$ as the output.
The dynamic movement of the servovalve spool is described by the following dynamic equation:

$$\tau_v \dot{x}_v = -x_v + K_v i \qquad (1)$$

### 3.3.2 Servo Valve Flow Equation

The flow equation of the servo valve is a typical nonlinear loop. The outflow equation of the servovalve is:

$$Q_L = C_d \omega x_v \sqrt{\frac{P_s - sgn(x_v) P_L}{\rho}} \qquad (2)$$

In order to satisfy the Lipschitz condition to guarantee the existence and uniqueness of the solution to (2) for all initial conditions, the non-differentiable sign function is approximated by the continuously differentiable sigmoid function defined as:

$$sgn\big(x(t)\big) \approx sgm\big(x(t)\big) = \frac{1 - e^{-\delta x(t)}}{1 + e^{-\delta x(t)}} \; ; \; \delta > 0$$

By doing so, the system described by (1) becomes differentiable and allows the use of the feedback linearization approach

$$\frac{dsgm\big(x(t)\big)}{dt} = \frac{2\delta e^{-\delta x(t)}}{(1 + e^{-\delta x(t)})^2} \dot{x}(t); \; \delta > 0$$

$$\lim_{\delta \to \infty} \frac{2\delta e^{-\delta x(t)}}{(1 + e^{-\delta x(t)})^2} = 0$$

Furthermore, the use of the sigmoid function is required to ensure that the feedback linearization conditions on the Lie derivatives of the system dynamics are satisfied [4]. When$\delta \gg 1$, the sigmoid function behaves like the sign function and the model best approximates the real electrohydraulic system.

The outflow equation of the servovalve became:

$$Q_L = C_d \omega x_v \sqrt{\frac{P_s - sgm(x_v) P_L}{\rho}} \quad (3)$$

### 3.3.3 Hydraulic Flow Equation

The flow from servo valve into the cylinder, besides driving the piston movement, can be used to compensate various cylinder leaks and liquid compressed volume, etc. The continuous flow equation of the cylinder can be expressed as

$$Q_L = A_p \dot{x}_p + C_t P_L + \frac{V}{4\beta_e} \dot{P}_L (4)$$

### 3.3.4 Hydraulic Cylinder Load Force Balance Equation

The output rolling force of the cylinder keeps balance with the inertia force of the moving parts, viscous damping force, elastic load force and other load force. The dynamic equation can be written as

$$A_p P_L = M_t \ddot{x}_p + B_p \dot{x}_p + K_t x_p + F_L (5)$$

The $F_L$is other load force acting on the piston. And $F_L$ can be expressed by

$$F_L = W_s\big(h_{in} - h_{out}\big) + F_f$$

### 3.3.5 Elastic Deformation of the Mill and Spring Equation

Cold rolling is a processing method by passing the metal strip between two rolls (work rolls), rotating in opposite directions. Due to this rotational movement and the compression generated by the cylinders (deformation force P), there is a continuous reduction of the initial thickness by plastic deformation of the metal.

The simplified thickness model normally uses the equation gaugemeter (BISRA) or spring equation to derive the thickness of the strip. It is based on the relationship between the position of the roll gap, the gap of the work rolls and the thickness of the web.
A linear approximation of the grinding stretching characteristic is used to estimate the thickness at the exit of a cage as by the model "BISRA" or "gaugemeter" (Zhanget al, 2015):

$$h_{out} = \frac{P}{M} + S + \Sigma \qquad (6)$$

Where Σis the compensation including thickness compensation, e eccentricity compensation and thermal compensation.

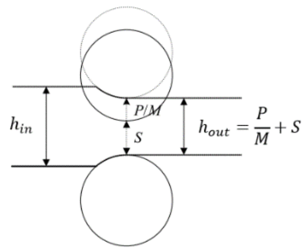


Figure 3: Equation gaugmeter

In the gaugemeter equation (see Figure 3) $\frac{P}{M}$ is the amount of the mill stretch when a roll separating force is applied. The output thickness is therefore equal to the position of the unloaded gap plus the stretch.When the entry strip is passed through the roll stand with the velocity$v_{in}$, the disturbance $\Delta h_{in}$ is expressed as:

$$\Delta h_{in} = A_d \sin\left(\frac{2\pi v_{in}}{L_d} t\right)$$

Where $A_d$and $L_d$ are the magnitude and period of thickness deviation respectively (Hwang et al, 1996).

There exists time delay between the output thickness $h_{out}$and the thickness $h$ detected by the thickness gauge, and it is a pure delay loop

$$h = h_{out}(t-T)$$

Where $T$ is the constant delay time.

### 3.3.6 Dynamic Equation of the Roll Gap

The rolling model at each stand consists of the dynamics of the roll gap and rolling direction, which are described below.

The roll gap S is assumed to be controlled by a local feedback loop of a hydraulic servo and is governed by the following equation (OZAKIet al, 2012):

$$T_g\ddot{S} + \dot{S} + K_g\left(h_g(S, h_{in}) - x_p\right) = 0 \qquad (7)$$

Where $x_p$denotes the command input, and $T_g$and $K_g$ are constants specified by the local feedback loop. The local feedback adjusts the roll gap S so that $h_g$ (S, h) coincides with the command input $x_p$.The function $h_g(S, h_{in})$ is defined in terms of the thickness h i and roll gap S i as

$$h_g = kh_{in} + (1-k)S$$

Which represents the thickness modified by the tuning parameter k (0 < k < 1).

The dynamics of the roll gap S i is governed by

$$\ddot{S} + \frac{1}{T_g}\dot{S} + \frac{1}{T_g}S = \frac{K_g}{T_g}\left(x_p - \frac{k}{M_0}P - k\Sigma\right) \text{ (8)}$$

### 3.3.7 Rolling Force Model

The mill model generates the roll force, exit strip thickness, entry strip velocity, and exit strip velocity using the roll gap, roll velocity, entry strip thickness, and the other factors. The roll force and the exit strip thickness are derived from Bland-Ford model (Bland et al., 1948 and Ford and Bland, 1951) and the gauge-meter equation respectively. The roll force can be expressed as:

$$P = (h_{in}, h_{out}, \dots)(9)$$

## 4 NON-LINEAR GLOBAL MODEL OF THE HAGC

### 4.1 Non-linear Model of the HAGC

The mathematical model of the HAGC can be described by the system of equations including nonlinear dynamic equations presented previously by posing $x_1 = S$ ,$x_2 = \dot{S}$, $x_3 = x_p$ , $x_4 = \dot{x}_p$ ,$x_5 = P_L$ and $x_6 = x_v$ as state variables:

$$sys\ 1)\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{T_g}x_2 - \frac{1}{T_g}x_1 + \frac{K_g}{T_g}\left(x_3 - \frac{k}{M_0}P - k\Sigma\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{1}{M_t}\left(A_p x_5 - B_p x_4 - K_t x_3 - F_L\right) \\ \dot{x}_5 = \frac{4\beta_e}{V}\left(C_d \omega x_6 \sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}} - A_p x_4 - C_t x_5\right) \\ \dot{x}_6 = \frac{1}{\tau_v}(-x_6 + K_v i) \\ with\ h_{out} = \frac{P}{M} + S + \Sigma \end{cases}$$

The state vector is $x(t) = [S, \dot{S}, x_p, \dot{x}_p, x_v, P_L]^T$ and control$u(t) = i(t)$.

The model of the HAGC presented in the above equation is nonlinear because of the sigmoid and square root functions. In this state representation, only the state variables are expressed as a function of time. However, hydraulic and mechanical parameters also vary during the operation of the HRGCS.

## 4.2 Controllability Analysis of HAGC

The controllable canonical form is to represent the dynamics of the system by a differential equation relating the output variable to the control variable HAGC. The state variable to be controlled and its successive time derivatives represent new HAGC state variables. The number of successive time derivatives is determined by the number of successive derivations performed on the output variable in order to obtain the control variable.

Nonlinear controllable canonical form are constructed from non-linear state representation of the HAGC. This direct relationship between the input variable and the state variable to enslave enables us to develop our control laws. Consider the non-linear state representation of the HAGC in the space where y (t) represents the output variable described below:

$$(sys\ 2)\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{1}{T_g}x_2 - \frac{1}{T_g}x_1 + \frac{K_g}{T_g}(x_3 - \frac{k}{M_0}P - k\Sigma) \\ x_4 \\ \frac{1}{M_t}\left(A_p x_5 - B_p x_4 - K_t x_3 - F_L\right) \\ \frac{4\beta_e}{V}\left(C_d \omega x_6 \sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}} - A_p x_4 - C_t x_5\right) \\ \frac{-x_6}{\tau_v} \end{bmatrix} + \begin{bmatrix} 0\\0\\0\\0\\0\\ \frac{K_v}{\tau_v}\end{bmatrix} i(t)$$

Or in compact form,

$$(sys3)\begin{cases}\dot{x} = f(x,t) + g(x,t)i(t) \\ \quad y(t) = h(x,t)\end{cases}$$

Where $x(t)$ is the n-dimensional state vector, $u(t)$ is the control input and $y(t)$ is the output.

Definition 1: the Lie derivative (Slotine et Li, 1991, p. 229): Let $h: \mathbb{R}^n \to \mathbb{R}$ be a smooth scalar function, and $f: \mathbb{R}^n \to \mathbb{R}^n$ be a smooth vector field on $\mathbb{R}^n$, then the Lie derivative of h with respect to $f$ is a scalar function defined by $L_f h = \frac{\partial f}{\partial x} f$

Definition 2: relative Degree (Khalil, 2002, p. 510): The nonlinear system describe by (sys 1) is said to have relative degree $l, 1 \le l \le n$, in a region$D_0 \subset D$ if $L_g L_f^{i-1} h(x,t) = 0$, $i = 1,2,\dots,l-2$ ; $L_g L_f^{\rho-1} h(x,t) = 0$ for all $x \in D_0$.

Thus, by successively deriving the position y (t), we find:

$$\dot{y}(t) = L_f h(x,t) = x_2(t) \tag{10}$$

$$\ddot{y}(t) = L_f^2 h(x,t) = -\frac{1}{T_g}x_2 - \frac{1}{T_g}x_1 + \frac{K_g}{T_g}\left(x_3 - \frac{k}{M_0}P - k\Sigma\right) \tag{11}$$

$$y^{(3)}(t) = L_f^3 h(x,t) = \alpha_1 x_2 - \frac{1}{T_g^2}\left(-x_1 + K_g\left(x_3 - \frac{k}{M_0}P - k\Sigma\right)\right) + \frac{K_g}{T_g}x_4 \tag{12}$$

$$y^{(4)}(t) = L_f^4 h(x,t) = \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \frac{\alpha_1}{T_g}\left(-x_1 + K_g\left(-\frac{k}{M_0}P - k\Sigma\right)\right) + \frac{K_g}{T_g}\frac{1}{M_t}\left(A_p x_5 - F_L\right) \tag{13}$$

$$y^{(5)}(t) = L_f^5 h(x,t)$$

$$= \alpha_6 x_2 + \alpha_7 x_3 + \alpha_8 x_4 + \frac{\alpha_2}{T_g}\left(-x_1 + K_g\left(-\frac{k}{M_0}P - k\Sigma\right)\right) + \frac{\alpha_4}{M_t}(-F_L) + \alpha_9 x_5 + \alpha_5\left(C_d \omega x_6 \sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}}\right) \quad (14)$$

$$y^{(6)}(t) = L_f^6 h(x,t) + L_g L_f^5 h(x,t) i(t) \quad (15)$$

with $L_f^6 h(x,t) = f_x(x,t) + f_r(x,t)$

with $f_x(x,t) = \frac{-\alpha_6}{T_g}x_1 + \left(\frac{-\alpha_6}{T_g} - \frac{\alpha_2}{T_g}\right)x_2 + \frac{\alpha_6 K_g}{T_g}x_3 + (\alpha_8 - \alpha_9 A_p)x_4 - (C_t\alpha_9)x_5 + \alpha_9 \frac{4\beta_e}{V} C_d \omega x_6 \sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}}$,

$f_r(x,t) = \frac{\alpha_5}{2\rho}\frac{4\beta_e}{V} C_d \omega sgm(x_6) x_6^2 + \frac{\alpha_5 A_p}{2\sqrt{\rho}} \frac{sgm(x_6)}{\sqrt{P_s - sgm(x_6)x_5}} x_4 x_6 + \frac{\alpha_5}{2\sqrt{\rho}} \frac{sgm(x_6)}{\sqrt{P_s - sgm(x_6)x_5}} x_5 x_6 - \frac{\alpha_5}{\tau_v}\sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}} x_6$ and

$L_g L_f^5 h(x,t) = \frac{\alpha_5 K_v K_p}{\tau_v}\sqrt{\frac{P_s - sgm(x_6)x_5}{\rho}}$

$\alpha_1 = (\frac{1}{T_g^2} - \frac{1}{T_g})$ ;$\alpha_2 = (-\frac{\alpha_1}{T_g} + \frac{1}{T_g^2})$;

$\alpha_3 = \left(\frac{\alpha_1 K_g}{T_g} - \frac{K_g K_t}{T_g M_t}\right)$ ;$\alpha_4 = \frac{-K_g}{T_g^2} - \frac{K_g B_p}{T_g M_t}$ ;

$\alpha_5 = \frac{K_g A_p}{T_g M_t}\frac{4\beta_e}{V}$;$\alpha_6 = \frac{-\alpha_2}{T_g} - \frac{\alpha_1}{T_g}$; $\alpha_7 = \frac{-\alpha_2 K_g}{T_g^2} - \frac{\alpha_4 K_t}{M_t}$;$\alpha_8 = \alpha_3 - \frac{\alpha_4 B_p}{M_t} - \alpha_5 A_p$; $\alpha_9 = \frac{\alpha_4 A_p}{M_t} - \alpha_5 C_t$ ;$\alpha_{10} = \frac{-\alpha_6}{T_g} - \frac{\alpha_2}{T_g}$ ;

$\alpha_{11} = \frac{\alpha_6}{T_g}K_g$ ;$\alpha_{12} = \alpha_8 - \alpha_9 A_p$ ; $\alpha_{13} = -C_t\alpha_9$ ; $\alpha_{14} = \alpha_9 \frac{4\beta_e}{V\sqrt{\rho}} C_d \omega$and $\alpha_{15} = \frac{\alpha_5 4\beta_e C_d \omega}{2V\rho}$

The expression $L_g L_f^5 h(x,t) \neq 0$ if and only if $P_s > x_5(t)$ . According to the standard design of the electro-hydraulic servo systems explained by Merritt (1967), the pressure across the hydraulic actuator verifies $\frac{2}{3}P_s \geq x_5(t)$ . Thus, the control signal appears when the output is derived 6 times.Since the system is of the sixteenth order (six state variables), there is no internal dynamics. The nonlinear state representation in canonical and controllable form becomes:

$$\dot{y}_0(t) = y_1(t)$$
$$\dot{y}_1(t) = y_2(t)$$
$$\dot{y}_2(t) = y_3(t)$$
$$\dot{y}_3(t) = y_4(t)$$
$$\dot{y}_4(t) = y_5(t)$$
$$\dot{y}_5(t) = -a_0 y_0(t) - a_1 y_1(t) - a_2 y_2(t) - a_3 y_3(t) - a_4 y_4(t) - a_5 y_5(t) + F(x,t) + L_g L_f^5 h(x,t) i(t) \quad (16)$$

With $a_0 = \frac{\alpha_6}{T_g} - \frac{\alpha_{11}}{K_g} - \frac{\alpha_{13} T_g M_t}{K_g A_p}\left(-\frac{\alpha_3}{K_g} + \frac{\alpha_1}{T_g}\right) - (\alpha_{14} - \alpha_{16})\left(-\frac{\alpha_7}{K_g} + \frac{\alpha_2}{T_g}\right)$,

$$a_1 = -\alpha_{10} - \frac{\alpha_{11}}{K_g} + \alpha_{12}\left(\frac{\alpha_1 T_g}{K_g} + \frac{1}{K_g T_g}\right) - \frac{\alpha_{13} T_g M_t}{K_g A_p}\left(-\alpha_2 - \frac{\alpha_3}{K_g} + \alpha_4\left(\frac{\alpha_1 T_g}{K_g} + \frac{1}{K_g T_g}\right)\right) - (\alpha_{14} - \alpha_{16})\frac{\sqrt{\rho}}{\alpha_5 C_d \omega}\left(-\alpha_6 - \frac{\alpha_7}{K_g} + \alpha_8\left(\frac{\alpha_1 T_g}{K_g} + \frac{1}{K_g T_g}\right)\right),$$

$$a_2 = -\frac{T_g \alpha_{11}}{K_g} + \frac{\alpha_{12}}{T_g} - \frac{\alpha_{13} T_g M_t}{K_g A_p}\left(\frac{-\alpha_3 T_g}{K_g} + \frac{\alpha_4}{T_g}\right) - (\alpha_{14} - \alpha_{16})\frac{\sqrt{\rho}}{\alpha_5 C_d \omega}\left(\frac{-\alpha_7 T_g}{K_g} + \frac{\alpha_8}{K_g}\right),$$

$$a_3 = -\frac{\alpha_{12} T_g}{K_g} + \frac{\alpha_{13} T_g^2 M_t}{K_g^2 A_p} + (\alpha_{14} - \alpha_{16})\frac{\sqrt{\rho}\alpha_8 T_g}{\alpha_5 C_d \omega K_g},\ a_4 = -\frac{\alpha_{13} T_g M_t}{K_g A_p},\ a_5 = -(\alpha_{14} - \alpha_{16})\frac{\sqrt{\rho}}{\alpha_5 C_d \omega} \text{ and}$$

$$F(x,t) = \left(\frac{-k}{M_0}P - k\Sigma\right)\left[-\alpha_{11} + \frac{\alpha_{13} T_g M_t}{K_g A_p}\left(\alpha_3 - \frac{\alpha_3 K_g}{T_g}\right) + (\alpha_{14} - \alpha_{16})\left(\alpha_7 - \frac{\alpha_2 K_g}{T_g^2}\right)\right] - \frac{\alpha_{13}\alpha_1}{T_g A_p}F_L + f_r(x,t)$$

With the new selected state variables such as$y_0(t) = y(t)$,$y_1(t) = \dot{y}(t)$,$y_2(t) = \ddot{y}(t)$, $y_3(t) = \dddot{y}(t)$ , $y_4(t) = y^{(4)}(t)$ and $y_5(t) = y^{(5)}(t)$.

The mathematical representation of equation (16) has the previous state variables because the system is non-linear. For the design of the exact linearization controller, the controllable canonical form of equation (16) is used.

### 4.3 HAGC Observability

The problem of designing an observer for a nonlinear system has been widely studied. Many different approaches have been considered to design observers. However, the observability property of the system must be verified before to design an observer.

It is well-known that the observability of a nonlinear system can be lost under some conditions. Now, we recall the conditions for checking that a nonlinear system is observable (Hermann, 1977).

For the system (sys3), let us define the vector of output derivatives, H (x), as follows:

$$H(x) = \begin{pmatrix} h_1 \\ h_2 \\ . \\ . \\ . \\ h_3 \end{pmatrix} = \begin{pmatrix} h(x) \\ L_f h(x) \\ . \\ . \\ . \\ L_f^{n-1} h(x) \end{pmatrix} \quad (17)$$

and the Observability matrix, O (x), as:

$$O(x) = \frac{\partial H(x)}{\partial x} = \begin{pmatrix} h(x) \\ L_f h(x) \\ . \\ . \\ . \\ L_f^{n-1} h(x) \end{pmatrix} \quad (18)$$

where $L_f^i h(x)$represents the i-th Lie derivative of h (x) in the f vector field direction. In addition, let us suppose the following condition:

Condition 1: The system (sys3) is globally observable, in the sense that the observability rank condition

$$rank\big(O(x)\big) = n \qquad (19)$$

is fulfilled for all x $\in \mathrm{R}^n$.

Remark 1. In the following, the observability property is studied for the case of the HRGCS. Then, from Condition 1 and by considering $h = x_1$ the output of system (17).Then,

$$rank\big(O(x)\big) = 6$$

Thus system (17) is weakly observable.

## 5 SIMULATION RESULTS AND DISCUSSION

The hydraulic system of the roll gap that we modeled is simulated using the MATLAB SIMULINK software and the simulation is performed over 40 seconds. Figure 4 shows the time performance obtained for HAGC to study that a step

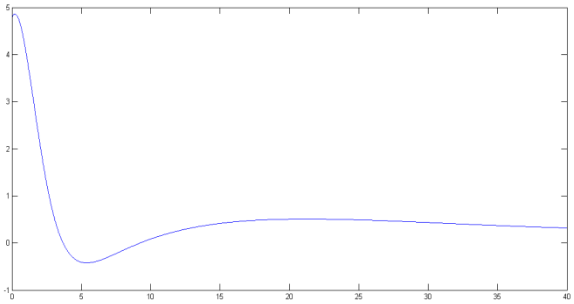

Figure 4: step response of HAGC

is applied. The HRGCS index response shows a significant overshoot and stabilizes after 20 s.

The hydraulic HAGC is very uncertain, so it should really be used by control algorithms with strong robustness.

## 6 CONCLUSIONS

In this paper, a dynamic nonlinear mathematical model of the HRGCS of the reversible cold rolling mill is developed primarily from the first principles governing mechanical and electrical components, associated with the theory of rolling for make the resulting model fit for decision-making and control analysis.

The dynamic mathematical model developed can be used to analyze and design futurement nonlinear order to fully control the various reversible cold rolling mill systems thus contributing to improving the final product quality broadband mills.

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