Unscented Model Predictive Control (UMPC) for Ship Heading Control with Stochastic Disturbance

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Abstract: This paper discussed a ship heading control problem with Unscented Model Predictive Control (UMPC). Rudder angle is controlled such that the heading angle follows the desired angle. This paper used model with three degrees of freedom, that is sway, yaw, and roll. UMPC is based on Model Predictive Control (MPC) for nonlinear system with stochastic disturbance. There are noises in the system model, therefore the system become dynamic stochastic with probabilistic constraint. These noises cause a changing in the state variable from a definite value to a distributed random variable. The objective function change from deterministic into the form of expectations of random variable. state variable constraints are changed from probabilistic to deterministic to address this issue. The objective function is changed into deterministic forms. System is linearized using stochastic linearization to approximate state transition. The Unscented Kalman Filter (UKF) is used as prediction process for MPC. The prediction process result is used by MPC algorithm by minimizing the objective function. The computation results showed that UMPC can handle problem with stochastic disturbance.

1 INTRODUCTION

Ship heading is one of the control problems in the marine field which have attracted great attention of researchers (Li and Sun, 2012, Fossen, 1994). A Ship requires a navigation system, guide, and control that is able to direct it to the desired angle (Perez, 2005, and Subchan and Zbikowski, 2009). One of the popular control method that have been applied in industry is Model Predictive Control (MPC) (Qin and Badgwell, 2003). MPC has some advantages compared to other controllers, such as the ability to predict the future process outputs, without ignoring the constraints (Bordons and Camacho, 2007, and Holkar 2010). MPC can also able to control wide range of process ranging from systems with relatively simple dynamics to systems with higher complexity, including system with long delay or unstable systems (Yoon, 2007 and Liuping, 2009).

Some works have been done to control the heading angle using MPC, using one degree of freedom (Subchan, 2014; Naveen, 2014), two degrees (Subchan, 2014), three degrees (Wang, 2010), and four degrees of freedom (Cahyaningtyas, 2014). In the previous studies, MPC is only able to control systems with measurable disturbance. In fact, the model's uncertainty of the model and stochastic disturbance are natural characteristic of the system model (Li, 2000, and Syafii, 2019).

Unscented Model Predictive Control (UMPC) is MPC for non-linear systems that can handle probabilistic constraints with approximating uncertainty in the model (Farrokhsiar, 2012). The Unscented Kalman Filter replaces the prediction process in MPC. System is linearized using stochastic linearization to approximate state transition (Wan and Merwe, 2000). In this work, UMPC is used to control the heading angle of non-linear stochastic system with three degrees of freedom.

2 LIFTING EQUIPMENT

This section explains the specifications of ship, mathematical model, and unscented model predictive control method.

2.1 Specification of Ship

The specification of ship is given in Table 1.

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Table 1: Ship specification.

Quantity (Symbol)	Value (Unit)
Length (L_{pp})	48 (meter)
Width (B)	8.6 (meter)
Draft (D)	2.2 (meter)
Mass (m)	359 × 10^3 (kg)
Volume of displacement (∇)	350 (meter^3)
Yaw Inertia (I_z)	33.7 × 10^6 (kg meter^2)
Roll Inertia (I_x)	3.4× 10^6 (kg meter^2)
Coordinate center of gravitation (x_G)	-3.38 (meter)
Coordinate center of gravitation (z_G)	-1.75 (meter)
Rudder area (A_R)	0.73 (meter)
Coefficient of lift force (C_L)	1.15
Distance CG-CP ($I_{\delta z}$)	1.2 (meter)
LCG ($l_{\delta z}$)	-23.5 (meter)
Metacenter (GZ)	0.776 (meter)
Density of water (ρ)	9.82 (meter/s^2)

2.2 Ship Dynamical Model

The kinematic model of ship as follows:

$$\dot{\phi} = p \quad (1)$$

$$\dot{\psi} = r\cos\phi \quad (2)$$

Where p, r, ψ, ϕ denote respectively roll rate, yaw rate, yaw angle and roll angle in inertial form.

The purpose of control is to control the rudder angle (δ) therefore the value of yaw angle as desired. Using kinematic model in Equation (1) and (2), and ship mathematical model with four degrees of freedom (Fossen, 1998), with assumption that surge velocity is constant, general dynamical model of ship with three degrees of freedom is shown below:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & 0 & 0 \\ b_1 & b_2 & 0 & 0 & 0 \\ c_1 & 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -mUr + Y_{hyd} \\ -mz_G Ur + K_{hyd} \\ -mx_G Ur + N_{hyd} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_{\delta_r} \\ K_{\delta_r} \\ N_{\delta_r} \\ 0 \\ 0 \end{bmatrix} \delta_r \right\} \quad (3)$$

Where:

$$a_1 = m - Y\dot{v}$$
$$a_2 = -(mzG + Y\dot{p})$$
$$a_3 = mxG - Y\dot{r}$$
$$b_1 = -(mzG + K\dot{v}),\ b_2 = Iz - K\dot{p}$$
$$c_1 = mxG - N\dot{v},\ c_2 = Iz - N\dot{r}$$

Equation (3) discretized using forward difference method with $\Delta t = 0.1$. From Equation (3), define state space of model as below:

$$x = [v, p, r, \phi, \psi] \quad (4)$$

Model in Equation (3) can be written as follow:

$$\dot{x} = f(x, u) \quad (5)$$
$$y = g(x, u) \quad (6)$$

Equation (5) stated mathematical model, with u is input system, that is rudder angle. While Equation (6) define output of model, in this case is yaw angle. Equation (5) and (6) is standard model system where there is no noise inherent in the system model. In the real, the presence of noise cannot be ignored. Therefore, system model can be defined as follow:

$$\dot{x} = f(x, u) + wk \quad (5)$$
$$y = g(x, u) + vk \quad (6)$$

Where $wk \sim N\ (0, \tau)$, that is normal distribution with mean 0 and variance τ and $vk \sim N(0, \Lambda)$, that is normal distribution with mean 0 and variance Λ.

The presence of noise causes system changes from deterministic to stochastic. MPC standard cannot be applied, therefore in the next section is formulated Unscented Model Predictive Control method to control system in Equation (5) and (6).

3 FORMULATION UNSCENTED MODEL PREDICTIVE CONTROL

Unscented Model Predictive Control method (UMPC) is stochastic MPC for non linear systems that approximates the transition from state variable using unscented transformation as statistic linearization method (Bradford, 2017). With assumption that control horizon equal to the prediction horizon ($N_c = N_p$), the objective function defined as follow:

$$\min_u J = \sum_{j=1}^{N_p} [(y(k+i \mid k) - y_d)^T\, Q\, (y(k + i \mid k) y_d) + u(k+i-1 \mid k)^T\, R u(k + i - 1 \mid k)] \quad (7)$$

With constraints:

$$xk+1 = f\ (xk, uk, k) \quad (8)$$

$$xmin \leq x(k+j \mid k) \leq xmax \quad (9)$$

$$umin \leq u\ (k+j-1 \mid k) \leq umax \quad (10)$$

Formulation Unscented Model Predictive Control (UMPC) is described below:

3.1 The Application of Unscented Kalman Filter (UKF) for the Prediction Process on MPC

The optimization problem solved in this research is in the form of dynamics stochastic, which the values of state variables represented by a distributed random variable. This condition prevents the MPC method from not being used. In this research, Unscented Kalman Filter (UKF) was used to replace the prediction process on MPC (Changchun, 2014). The UKF in the case of additive noise for (5) and (6) to approximate mean and covariance can be stated as follows (Subchan, 2019).

1. Definition of Sigma-points

$$\begin{gathered}\chi\ (k+j-1 \mid k) = [\hat{x}\ (k+j-1 \mid k) \\ \hat{x}\ (k+j-1 \mid k) + \sqrt{L+\lambda}\, P1/2(k+j-1 \mid k) \\ \hat{x}\ (k+j-1 \mid k) - \sqrt{L+\lambda}\, P1/2(k+j-1 \mid k)\end{gathered} \quad (11)$$

2. Covariance and mean approximation of predictions

$$\bar{X}^{[i]} = f\left(x^{[i]}, u_{k+j}\right) \quad (12)$$

$$\bar{X}_{k+j+1} = \sum_{i=0}^{2L} W_m^{[i]}\, \bar{X}^{[i]} \quad (13)$$

$$P_{k+j+i} = \sum_{i=0}^{2L} W_c^{[i]}\, \left(\bar{X}^{[i]} - \bar{X}_{k+j+1}\right)\left(\bar{X}^{[i]} - \bar{X}_{k+j+1}\right)^T + M_{k+j} \quad (14)$$

3. Covariance and mean approximation of observations:

$$\bar{Y}^{[i]} = g\left(X_{k+j+1}\right) \quad (15)$$

$$\hat{y}_{k+j+1} = \sum_{i=0}^{2L} W_c^{[i]}\, \bar{Y}_{k+j+1}^{[i]} \quad (16)$$

$$S_{k+j+1} = \sum_{i=0}^{2L} W_c^{[i]}\left(\bar{Y}_{k+j+1}^{[i]} - \hat{y}_{k+j+1}\right)\left(Y_{k+j+1}^{[i]} - \bar{X}_{k+j+1}\right)^T + Cov_{k+j} \quad (17)$$

$$P_{k+j+1}^{x,y} = \sum_{i=0}^{2L} W_c^{[i]}\left(X_{k+j+1}^{[i]} - \bar{X}_{k+j+1}\right)\left(\bar{Y}_{k+j+1}^{[i]} - \hat{y}_{k+j+1}\right)^T \quad (18)$$

$$K_{k+j+1} = P_{k+j+1}^{x,y}\, S_{k+j+1}^{-1} \quad (19)$$

$$\bar{X}_{k+j+1} = \bar{X}_{k+j+1} \quad (20)$$

$$P_{k+j+1} = \hat{P}_{k+j+1} - K_{k+j+1} S_{k+j+1} K_{k+j+1}^T \quad (21)$$

3.2 Changing State Variable Constraint from Probabilistic to Deterministic

Objective function and constraints are defined in Equation (7)-(10) in deterministic form. The existence of noises causes a shift in the state variable from a definite value to a distributed random variable. The objective function change from deterministic into the form of random quantity expectations. The stochastic dynamic optimization problem can be written below.

$$\min_{u} J = \sum_{i=1}^{N_p} E[(y(k+1|k) - y_d)^T Q(y(k+i|k) - y_d) + u(k+i-1|k)^T Ru(k+i-1|k)] \quad (22)$$

With constrains:

$$x_{k+j+1} = f(x_k, u_k) + w_{k+j} \quad (23)$$

$$y_{k+j+1} = g(x_k, u_k) + v_{k+j} \quad (24)$$

$$P_r\left(x_{k+j}^i \ \leq\ \beta_j^i\right) \geq p_j^i, i = 1, \dots, n, j = 1, \dots, N_p \quad (25)$$

$$u_{k+j} \leq \mu_j, j = 0, \dots, N_p - 1 \quad (26)$$

Where x^i stated in the I th element from state variable x_{k+j}, β_j^i is state variable constraints, while β is probability of state constraints.

Input constraint in Equation (26) still in the form deterministic because its value not affected by state variable. State constraints in Equation (25) be changed in the form of deterministic. Suppose $\hat{x}k_{+j+1}$, mean value from random variable, where $_{k+j+1}$ ~, $N(0, P_{k+j|k})$, that is normal distribution with mean 0 and variance $P_{k+j|k}$. Given ξ_{k+j} = $P1/2$ $\hat{x}k{+}j|k$ has standard normal distribution N (0,1), Then, $P_r\left(x_{k+j} \leq \beta_j\right) \geq p_j <=> P\left(\varepsilon_{k+j} \leq \hat{\beta}_j\right) \geq p_j$ where $\hat{\beta}_j = \frac{j-\hat{x}_{k+j}}{\left(P^{ii}_{k+j|k}\right)^{1/2}}$. Suppose $\hat{\beta}^*_j$ solution from $\vartheta\left(\hat{\beta}^*_j\right) = p_j$, with $\vartheta(.)$ is the standard normal distribution function (Sahoo,2013). Then constrain in Equation (25) can be recast as:

$$\hat{x}_{k+j|k} \leq \beta_j - P^{\frac{1}{2}}_{k+j|k}\beta^*_j \qquad (27)$$

Constraint in Equation (27) have been in the form of deterministic.

3.3 Changing Objective Function from Random Quantity Expectation to Deterministic

Objective function that defined in Equation (22) is in the form of random quantity expectation. Based on probability theory, objective function in Equation (22) can be changed as follow.

$$J = \sum_{j=1}^{N_p} tr(E((y-y_d)(y-y_d)^T)Q) + u^T Ru \qquad (28)$$

Yan and Bitmead, 1990, showed that solving the stated in Equation (28) is equivalent to solving a deterministic dynamic programming:

$$\min_u J = \sum_{j=1}^{N_p} [(y(k+i|k) - y_d)^T Q(y(k+i|k) - y_d) + u(k+i-1|k)^T Ru(k+i-1|k)] \qquad (29)$$

Subject to Equation (11)-(21) and (26)-(27). In the next section, numerical evaluations of the Equation (29) are discussed.

4 SIMULATIONS

In this section, the simulation results are displayed followed by a discussion of the systems performance analysis. The purpose of control in this research is to control the rudder angle (δ) therefore the value of yaw angle (y) as expected (yd) with minimum energy (u). In simulation is used a discrete system with discretion time Δt = 0.1. Given constraint in rudder angle and yaw velocity respectively $|\delta| \leq 35°$ and $|r| \leq 0.0932$ rad/s. Total time for simulation are 300 seconds. The weighting matrix Q = 200, R = 10, with noise $wk \sim N(0,10{-}4)$ and $vk \sim N(0,10{-}2)$. Initial value of state variable described as follow.

$$x_0 = [0\ 0\ 0.0853\ rad/s\ 0\ 30°] \qquad (30)$$

In the first simulation was simulated UMPC with different value of prediction horizon, that is Np = 20,25, and 30. Figure 1 shows that the heading angle can reach the desired heading angle, that is 0°. From Figure 2 and Figure 3, the yaw rate and rudder angle satisfy the given constraints.

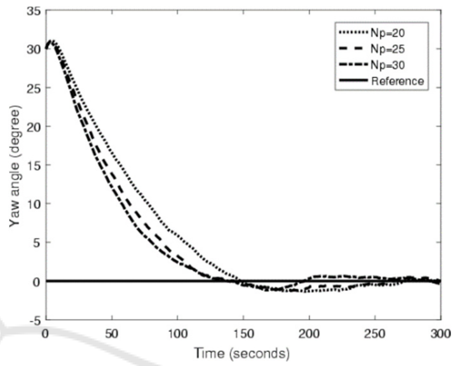

Figure 1: Yaw angle with UMPC method.

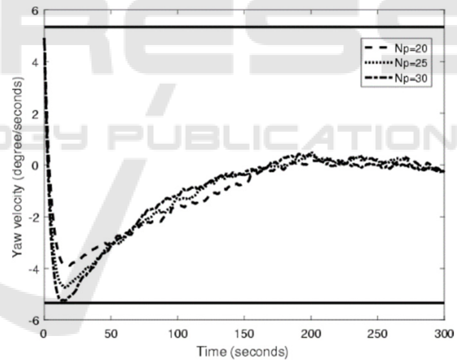

Figure 2: Yaw velocity with UMPC method.

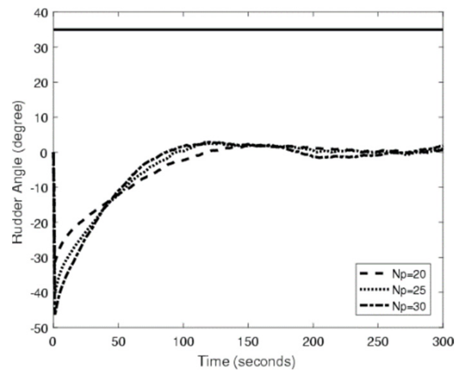

Figure 3: Rudder angle with UMPC method.

Then the comparison of the RMSE for each prediction horizon is considered. The following equation is used to compute the RMSE.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{Total\ time}(y_i - y_{di})^2}{Total\ time}} \qquad (31)$$

Where y_i and y_{di} are the heading angle at time i and the desired heading angle at time i.

Table 2: Comparison of the RMSE for each prediction horizon.

Prediction Horizon (N_p)	RMSE
20	11.7418023
25	10.3627875
30	9.3963874

According to Table 2, the smallest RMSE is reached when Np = 30. In the next simulation is used Np = 30.

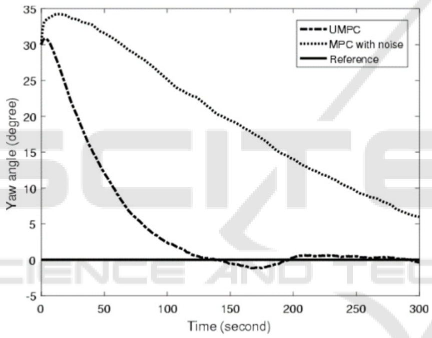

Figure 4: Comparison yaw angle between UMPC and MPC with noise.

The second simulation will compare performance system between UMPC and MPC with noise. In the simulation using MPC with noise, stochastic model system with deterministic constraints is simulated. Figure 4 shows in the end of the simulation, heading angle when controlled using MPC with noise has not reached the reference yet. But using UMPC, it reaches reference angle in time 148 seconds. From Figure 5 and Figure 6, the yaw rate and rudder angle satisfy the given constraints.

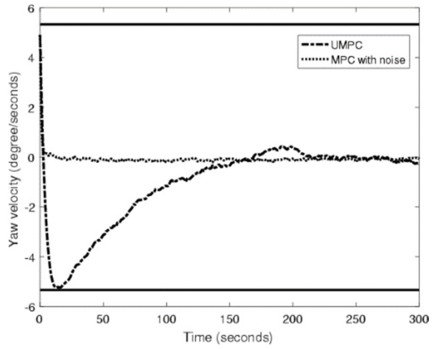

Figure 5: Comparison yaw velocity between UMPC and MPC with noise.

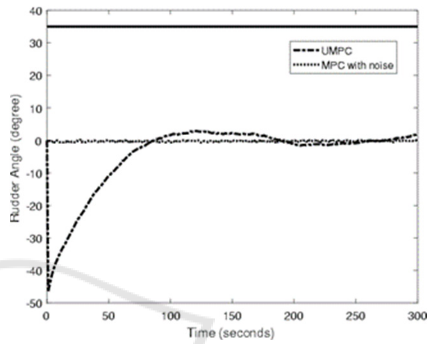

Figure 6: Comparison rudder angle between UMPC and MPC with noise.

The third simulation is carried out by varying the noise values. The scenarios is given in Table 3, where w_k is noise in the system model and v_k is noise in the measurement model.

Table 3: Noise variations for simulation using UMPC.

Scenario	w_k	v_k
1	$N(0,10^{-4})$	$N(0,10^{-2})$
2	$N(0,10^{-2})$	$N(0,10^{-2})$

In Figure 7, heading angle in Scenario 1 can reached the reference point faster than Scenario 2. In The yaw angle in Scenario 1 reaches the reference at 148 seconds. While in Scenario 2, the yaw angle reaches the reference at 390 seconds. A large noise value of system model in Scenario 2 causes the additional time which needed to reach the reference. From Figure 8 and Figure 9, the yaw rate and rudder angle satisfy the given constraints.

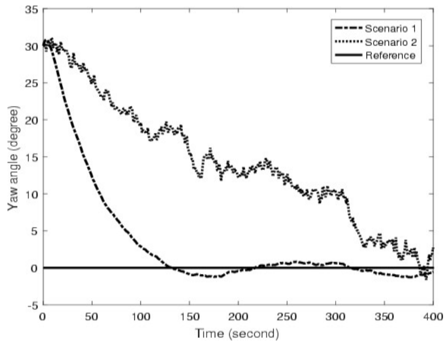

Figure 7: Comparison yaw angle.

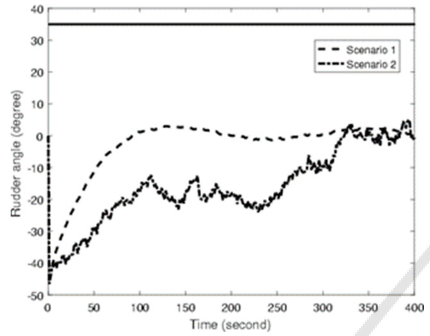

Figure 8: Comparison yaw velocity.

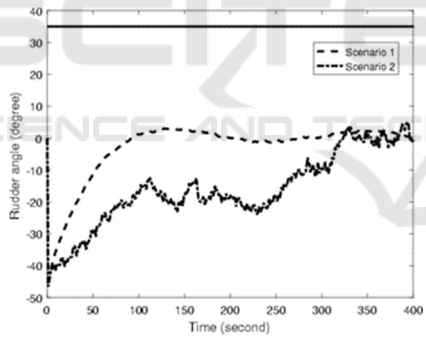

Figure 9: Comparison rudder angle.

5 CONCLUSIONS

In this paper Unscented Model Predictive Control (UMPC) used to solve the ship heading control problem. This approach uses the Unscented Kalman Filter (UKF) to replace prediction process in MPC. UMPC can handle problem with stochastic disturbance. The simulation results show that the whole constraints are satisfied with variation in noise value and prediction horizon. In this work, the weighting matrices are Q = 200 and R = 10. From simulation, best performance reached with Np = 30.

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