Asynchronous Price Stabilization Model in Networks

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Abstract: We consider a multiagent network model consisting of nodes and edges as cities and their links to neighbors, respectively. Each network node has an agent and priced goods and the agent can buy or sell goods in the neighborhood. Though every node may not have an equal price, we can show the prices will reach an equilibrium by iterating buy and sell operations. We introduce a framework of protocols in which each buying agent makes a bid to the lowest priced goods in the neighborhood; and each selling agent selects the highest bid (if any). So far, we have just considered such a model in a synchronous environment. We, however, cannot represent the velocity of circulation of money in the synchronous system. In other words, we cannot distinguish the different speed of money movement if every operation is synchronized. Thus, we develop an asynchronous model which enables us to generalize the theory of price stabilization in networks. Finally, we execute simulation experiments and investigate the influence of network features on the velocity of money.

1 INTRODUCTION

Background. Conventionally, the topic of price determination has been discussed from microeconomics approach (N.G.Mankiw, 2018). In the presence of appropriate supply and demand curves, if the price is higher (resp. lower) than an equilibrium, there is excess supply (resp. excess demand) and thus the price moves to the equilibrium. At the equilibrium price, the quantity of goods sought by consumers is equal to the quantity of goods supplied by producers. Neither consumers nor producers have incentive to change the price / quantity at the equilibrium.

In contrast, we considered a multiagent network model (J.Kiniwa and K.Kikuta, 2011a; J.Kiniwa and K.Kikuta, 2011b; J.Kiniwa et al., 2017b), in which each agent repeatedly makes auctions and the price of goods is eventually determined. Our network model consists of nodes and edges as cities and their links to neighbors, respectively. Each node contains an agent which represents people living in the city. Agents who want to buy goods make bids to the lowest-priced neighboring node, if any. Then, agents who want to sell the goods accept the highest bid. We have shown the reason of price determination by using the idea of self-stabilization in distributed systems (S.Dolev, 2000). From any initial state, self-stabilizing algorithms eventually lead to a legitimate state without any aid of external actions. Such a self-stabilization resembles the price determination, where the price reaches an equilibrium without external operations.

Motivation. Our first work was motivated by an intuition that simulating trades between agents may stabilize the price instead of the supply-demand theory. We developed a trading model using auctions in which prices converge to a unique one (J.Kiniwa and K.Kikuta, 2011a; J.Kiniwa and K.Kikuta, 2011b). We, however, were not able to explain why such a unique price is determined. After that, we assumed a relation $p_i = m_i/q_i$ between the price p_i, goods q_i and money m_i at each node i, and each agent exchanges money and goods. Then, it enables us to estimate an equilibrium price $P_e = M/T$, where $M = \sum_i m_i$ and $T = \sum_i q_i$ (J.Kiniwa et al., 2017b). Further, we developed a method of expected optimal bidding and derived the difference between two presenting protocols (J.Kiniwa et al., 2017a). We, however, were not able to distinguish whether or not the convergence is fast because the velocity of money is always constant.

Problem. Irving Fisher's claim, $MV_m = P_eT$, has been accepted as the quantity theory of money, where V_m is the velocity of money. The correctness of our synchronous model was guaranteed by Fisher's quantity equation with $V_m = 1$. However, since the equation describes an arbitrary velocity V_m, there must exist some method which corresponds to such an exten-

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sion. So, our first issue was how to extend the model to an arbitrary velocity V_m. If it were possible, our network model could be guaranteed by the Fisher's quantity equation in general. Next, our second issue was how to compute the velocity of money. If it were possible, the velocity of money could be explicitly derived. Then, we could know whether the velocity of money is different in several network topologies and protocols.

Solution. For the first issue, we develop an asynchronous price stabilization model in order to express an arbitrary velocity V_m of money. We can use the concept of *asynchronous round* or simply *round*, an appropriate interval, and define the velocity as the basis of the slowest agent: In a round, the slowest agent trades only once, while the others do at least once. Then, the money payed by the slowest agent moves only distance 1, while the other money moves farther. So, the different speed between money gives the concept of velocity.

For the second issue, we consider a variable $flow_i$ of money used at each node i. The sum of $flow_i$ through the network means the total quantity of money that was repeatedly used in a round. We consider the velocity of money as the sum of $flow_i$ divided by the amount of money supply. Since the money supply is constant, the velocity of money becomes large if the sum of $flow_i$ grows.

Related Work. The classical theory of price determination in microeconomics is introduced in (N.G.Mankiw, 2018). In contrast to the conventional work, we review the theory from a multiagent viewpoint. There exist a large body of literature on social economic networks (J.Benhabib et al., 2010) containing a network formation game (M.O.Jackson and A.Wolinsky, 1996) and a buyer-seller network (M.O.Jackson and A.Watts, 2010; R.Kranton and D.Minehart, 2001). The network formation game considers the choice of relationships between agents, and the buyer-seller network considers the competition and exchange in bipartite networks (E.Even-Dar et al., 2007; R.Kranton and D.Minehart, 2001). Unlike their interest in maximizing economic surplus, our work focuses on price stabilization. Auction theory has been comprehensively studied in (V.Krishna, 2002). Our protocol in Section 2.2 may be considered as a consensus algorithm. The consensus algorithm is described in (N.A.Lynch, 1996), and its self-stabilizing version is described in (S.Dolev et al., 2010). However, their work cannot be categorized as economics. Asynchronous systems have been extensively discussed in the area of distributed algorithms (N.A.Lynch, 1996). This is because most of the distributed algorithms must work in such an environment. Thus the multiagent system should be described as an asynchronous system.

Our previous work (J.Kiniwa and K.Kikuta, 2011a) considers a naive protocol in which each buyer makes a bid with an appropriate rate to a seller. Then, (J.Kiniwa and K.Kikuta, 2011b) and (J.Kiniwa et al., 2017b) analyze the best bidding price for a constant number of bidders, and (J.Kiniwa et al., 2017b) assumes the price is determined by the amount of money and goods.

Contributions. We propose an asynchronous price stabilization model in this paper. We consider the asynchronous system is not only as an extension of the synchronous system but also as a method of measuring the velocity of money. We define the velocity of money as the total spent money divided by the total supplied money in a round. To compare the velocity of money, we execute simulation experiments for two networks and three protocols. Then we obtain some reasonable results, that is, the velocity of money is fast if there is a lot of payment.

We organize the rest of this paper as follows. Section 2 states our model and protocols. Section 3 discusses how we can represent the velocity of money. Section 4 shows some results of simulation experiments for several networks and protocols. Finally, Section 5 concludes the paper.

2 MODEL

Here we describe our model consisting of a network in section 2.1, a protocol design in section 2.2, and the expected number of bidders in section 2.3.

2.1 Network

Our system can be represented by a connected network $G = (V,E)$, consisting of a set of nodes V and edges E, where the nodes represent cities and a pair of neighboring nodes is linked by an edge. Let N_i be a set of neighboring nodes of $i \in V$, and let $N_i^+ = N_i \cup \{i\}$. We assume that each node $i \in V$ has a good of one single type and their initial price may be distinct. Let p_i be the price of the goods at node i. Each node $i \in V$ has exactly one representative agent a_i who always stays at i and can buy goods in the neighborhood N_i. Each agent a_i has money m_i and the quantity q_i of goods. The price p_i is determined by the relation between the quantity of goods and the buying power, called a *supply-demand* balance. So we simply assume two properties at each node. First, the price is proportional to the amount of money for constant goods. Second, the price is inversely proportional to the amount of goods for constant money. That is,

$$p_i = \frac{m_i}{q_i}. \qquad (\star)$$

If the total amount of goods q_i are sold at each node i, then the total trade at i is equal to q_i. By summing up q_i for every node, we can verify the correctness of these assumptions by the Fisher's quantity equation (N.G.Mankiw, 2018).

The *buy operation* is executed as follows. Each agent a_i assigns a *value* v_i^j to the goods of any neighboring node $j \in N_i$, where the value means the maximum amount an agent is willing to pay. Agent a_i compares its own goods price p_i with the neighboring price p_j. If the cheapest price in N_i is p_j $(< p_i)$, agent a_i wants to buy it and makes a bid b_i^j to node j. We consider $v_i^j = p_i$ for any $j \in N_i$ because he can buy it at price p_i in his node (V.Krishna, 2002).

The *sell operation* is executed as follows. After accepting bids, agent a_j *contracts* with $a_i \in N_j$, who made the highest bid b_i^j at some appropriate time, called a *contract time*. Agent a_j passes a_i goods, and conversely agent a_i passes a_j money. Such trades are repeated until the price p_j becomes equal to p_i caused by the exchange of goods and money between them. We do not take the carrying cost of goods into consideration but focus on the change of prices. Each node $i \in V$ has a state Σ_i represented by a tuple — the goods and the money (q_i, m_i).

We assume an *asynchronous model*, that is, every agent aperiodically executes operations, exchanges messages, and knows the states of neighboring agents. We call the state of all nodes a configuration. A configuration is legitimate if every node has equally priced goods. In the asynchronous system, there is no bound on the rate of step-execution. However, it is convenient to use the number of *asynchronous rounds* or *rounds* in order to evaluate the system. The first round in an execution E is the shortest prefix E' of E such that each agent executes at least one step in E'. Let E'' be the suffix of E that follows E', that is, $E = E'E''$. The second round of E is the first round of E'', and so on. Intuitively, we can regard a round as the time interval between the two operations of the slowest agent.

2.2 Protocol Design

In this section, we first consider a protocol model, called a first-price protocol (**FirstPrice**). In the protocol, each agent a_i asynchronously makes a bid b_i^j to an agent $a_j \in N_i$ with the lowest price in the neighborhood. However, all the bids to a_j may not be submitted yet when a_j chooses the highest one. The following assumption means that once a contract is made, it is known to neighbors and a new submission of bid is suppressed until the agents complete the trade.

Assumption 1. *Once a buyer and a seller have made a contract, they complete the trade until their prices are balanced without interference.* □

<u>FirstPrice</u>

1. Each agent a_i makes a bid b_i^j to node $j \in N_i$ which has the lowest-priced goods in N_i and less than p_i.
2. At a contract time, the agent a_j contracts with the neighboring a_{h_1} who has made the highest bid $\max_{h_1 \in N_j} b_{h_1}^j$ at the time. The goods moves from q_j to q_{h_1} and the money moves from m_{h_1} to m_j at h_1's bidding price $b_{h_1}^j$ as long as $p_{h_1} > p_j$. The new prices p_{h_1} and p_j after the exchange are determined by the amount of money/goods.
3. If several agents make bids to node j with the same highest price, agent a_j makes deals with one of them at random.
4. (*priority rule:*) If concurrent buy (b_j^k to $k \in N_j$) and sell (b_h^j from $h \in N_j$) operations occur at agent a_j, he gives priority to the sell over the buy.

If 2 above is replaced by the following $2'$, we call it a second-price protocol (**SecondPrice**). Let agent a_{h_2} have made the secondly highest bid to node j, called a secondly bidder.

$2'$. At a contract time, the agent a_j contracts with the neighboring a_{h_1} who has made the highest bid $\max_{h_1 \in N_j} b_{h_1}^j$ at the time. The goods moves from q_j to q_{h_1} and the money moves from m_{h_1} to m_j at the secondly bidder a_{h_2}'s bidding price $b_{h_2}^j$ as long as $p_{h_1} > p_j$.

In summary, if buyer a_i pays his bidding price to seller a_j, we call the protocol a first-price protocol. In contrast, if buyer a_i pays the secondly highest (i.e., other buyer's) bidding price to seller a_j, we call the protocol a second-price protocol.

Example 1. *Figure 1 shows an example of our network system consisting of 4 nodes $V = \{1,2,3,4\}$. At first, the prices of goods are $(p_1, p_2, p_3, p_4) = (50, 110, 70, 100)$ as shown in Figure 1(a). Each agent a_i wants to buy the lowest-priced goods at node $j \in N_i$ if its price is lower than p_i, that is, $p_i > \min_{j \in N_i} p_j$. Thus, both a_2 and a_3 make bids to node 1. The action of a_4, however, is too slow to attend the a_1's contract time (The anticipated operation is depicted as a dotted arrow). Since agent a_2 beats a_3, agent a_2 makes a contract with agent a_1. Let x units be the number of a_2's buying goods.*

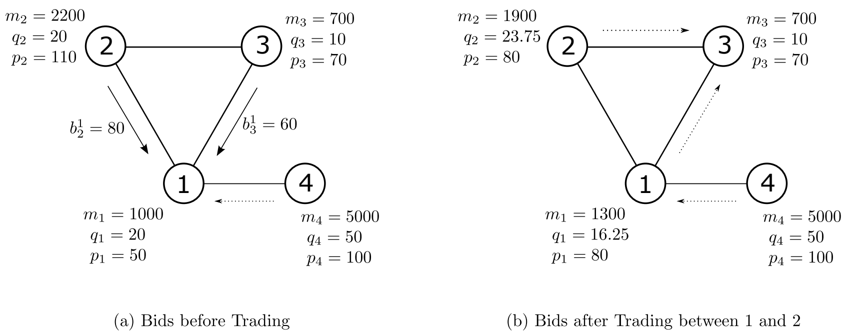

Figure 1: An illustration of protocol **FirstPrice** (Intermediate bidding).

Since the prices of nodes 1 and 2 become equal, we have $\frac{1000+80x}{20-x} = \frac{2200-80x}{20+x}$. *This gives* $x = 3.75$ *and hence* $q_1 = 20 - x = 16.25$, $q_2 = 20 + x = 23.75$, $m_1 = 1000 + 80x = 1300$, *and* $m_2 = 2200 - 80x = 1900$.

After the trade as above, the prices become $(p_1, p_2, p_3, p_4) = (80, 80, 70, 100)$ *as shown in Figure 1(b). Here, agents* a_1 *and* a_2 *can make bids to node 3, and agent* a_4 *can make a bid to node 1. If* b_4^1 *and* b_1^3 *cuncurrently occur at node 1, agent* a_1 *gives priority to* b_4^1 *and delays* b_1^3 *because of avoiding confusion (see the "priority rule").* □

We concern about whether the prices of goods eventually reach an equilibrium price even if they are initially distinct. The following lemma states an asynchronous issue.

Lemma 1. *Even if there is a slow operating agent, the protocols correctly work.*

Proof. Suppose that there is a too slow operating agent i and other agents operates much faster than i. We have to consider two cases.

1. The removal of node i separates the network into two or more components.
2. The removal of node i does not separate the network.

Suppose the move of i, a pair of buy and sell operations of i, is slow enough to stabilize the price in each component.

In the first case, let C_j and C_k be two components of them and p_j and p_k be their prices, where $p_j < p_i < p_k$, respectively. Let $diff_h(j,k)$ the price difference between p_j and p_k after the h-th move of i. After the first move of agent i, some goods move from C_j to i and then from i to C_k. Likewise, some money move from i to C_j and then from C_k to i. Thus, $diff_0(j,k) > diff_1(j,k)$ holds. This can be inductively proved.

In the second case, only price p_i is different from others. Thus, the moves of i eventually stabilize the price. □

In (J.Kiniwa and K.Kikuta, 2011b), we examined a sufficient condition for price stabilization in **FirstPrice**. Suppose that agents a_i and a_j make bids to node $h \in N_i \cap N_j$. We say that *bids have the same order as values* if $v_i^h \le v_j^h$ implies $b_i^h \le b_j^h$ for the goods of node h. The following theorem further shows that an additional condition leads to the price stabilization.

Theorem 1. *(J.Kiniwa and K.Kikuta, 2011b) Suppose bids keep the same order as values. If any contract price lies between buyer's price and seller's price, price stabilization occurs.* □

2.3 Expected Number of Bidders

In our network model, each agent makes a bid to the minimal priced node in the neighborhood. Since the prices vary from time to time, the minimal priced node also changes. So, we consider the expected number of bidding nodes.

Assumption 2. *We assume every agent can know the maximum / minimum price, and we assume the value* v_i *is equal to the price* p_i *at node* i*. The values are uniformly distributed over* $(0,1)$. □

Next, Assumption 3 is necessary for computing expected number of bidders.

Assumption 3. *Agent* i *knows any node* u *within distance 3 from* i*.* □

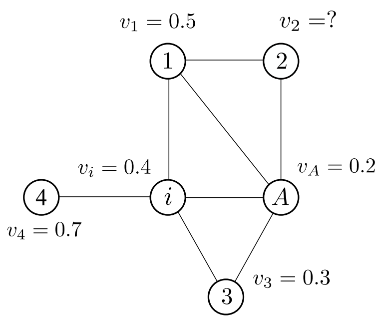

Figure 2: Certain agent (node 3) vs. uncertain agent (node 2) for i with respect to A.

Figure 2 illustrates an example in which node A sells goods and some nodes in N_A make bids to A. Let $v_u (= p_u)$ be the value of agent u, and the price p_u for each u is not explicitly depicted. Suppose agent i wants to make a bid to A. Since agent i is not adjacent to agent 2, he does not know agent 2's value, however, knows the existence of node 2 and N_2 (by Assumption 3). Since agent 1 is adjacent to agent 2, agent i does not know agent 1's behavior. Thus the uncertain decisions of agent 1 and agent 2 depend on $N_1^+ \backslash N_i = \{2\}$ and $N_2^+ \backslash N_i = \{2\}$, respectively. Further notice the decision of agent 1 depends on agent 2's value, that is, agent 1 makes a bid to A if $v_2 > v_A$. We say that such agent 1 is **dependent** on 2 with respect to A. Agent i surely knows agent 3 makes a bid to A because $v_A < v_3$. We call such agent 3 a **certain agent** for i with respect to A. Let $P(\rho_u)$ (resp. $P(\sigma_u)$) be the probability that agent u makes (resp. does not make) a bid to A, and let k_u be the number of $|N_u^+ \backslash N_i|$. Thus, the probabilities that make bids to A are

$$\begin{aligned}(P(\rho_1), P(\rho_2), P(\rho_3)) &= ((1-v_A)^{k_1}, (1-v_A)^{k_2}, 1) \\ &= (0.8, 0.8, 1),\end{aligned}$$

where $P(\rho_1\rho_2) = P(\rho_2)$ and $P(\rho_1\sigma_2) = P(\sigma_1\rho_2) = 0$. We consider probability $P_i(j)$ that there are j bidding nodes to A when agent i makes a bid to A. Then, the probability that four agents make bids to A is

$$P_i(4) = P(\rho_i\rho_1\rho_2\rho_3) = P(\rho_i\rho_2\rho_3) = 1 \cdot 0.8 \cdot 1.$$

The probability that three agents make bids to A is

$$P_i(3) = P(\rho_i\rho_1\sigma_2\rho_3) + P(\rho_i\sigma_1\rho_2\rho_3) = 0.$$

The probability that two agents make bids to A is

$$P_i(2) = P(\rho_i\sigma_1\sigma_2\rho_3) = P(\rho_i\sigma_2\rho_3) = 1 \cdot 0.2 \cdot 1.$$

More formally, the probability that v_A is the smallest value in $|N_u^+ \backslash N_i|$ is $P(\rho_u) = (1-v_A)^{k_u}$. Conversely, the probability that v_A is not the smallest value is $P(\sigma_v) = 1-(1-v_A)^{k_v}$. For independent agents u_s and u_t, we can represent

$$P_i(j) = \sum_{P(\rho_i)=1} \left(\prod_{\substack{1\le s\le j \\ j+1\le t\le N_A}} P(\rho_{u_s})P(\sigma_{u_t}) \right).$$

If $k_u = |N_u^+ \backslash N_i| = 0$, agent i knows every neighboring value of u, and thus understands whether agent u makes a bid to A. Otherwise, agent i cannot know some neighboring values of u, and thus estimates the possibility of u's bidding. We call such agent u an **uncertain agent** for i with respect to A.

It is known that Bayesian-Nash equilibrium occurs when each agent i makes a bid $(j-1)/j \cdot v_i$ when there are j bidders (V.Krishna, 2002). We consider the expectation of $(j-1)/j$, called an expected rate. The expected bidding rate, denoted by R_i^I, of the Bayesian-Nash equilibrium is

$$R_i^I = \sum_{j=|C_i^A|+1}^{|N_A|} \frac{j-1}{j} P_i(j),$$

where C_i^A be the set of certain bidders for i which make bids to A. (J.Kiniwa et al., 2017a) has also shown that the optimal bidding rate of our second-price protocol is

$$R_i^{II} = 1.$$

Thus, the optimal bidding price for **FirstPrice** and **SecondPrice** is $b_i^A = R_i^I v_i$ and $b_i^A = v_i$, respectively.

Theorem 2. *(J.Kiniwa et al., 2017a) In arbitrary networks, price stabilization is guaranteed by our second-price protocol. In contrast, it is not always guaranteed by our first-price protocol.* □

We call the first-price protocol (resp. second-price protocol) with its optimal bidding rate **FirstOptBid** (resp. **SecondOptBid**). Since the price stabilization is not always guaranteed by the **FirstOptBid** in any network (J.Kiniwa et al., 2017a), we introduce the following alternative protocols instead of **FirstOptBid**,

- Intermediate bidding (with first-price) protocol, and
- Pseudo first-price (with optimal bidding) protocol.

In the former protocol, each buyer makes an intermediate bid between the buyer's price and the seller's price, and pays his bidding price, as illustrated in Figure 1. In the latter protocol, the highest priced agent makes an optimal bid and always wins his contract with a seller. Though some agent may not follow the auction rule, we just consider it from the viewpoint of circulation of money. The price stabilization is guaranteed by both methods.

3 VELOCITY OF MONEY

In the synchronous model (J.Kiniwa et al., 2017b), we already have the following result.

Theorem 3. (J.Kiniwa et al., 2017b) *Let T be the total volume of transactions, interpreted as the quantity of goods, and let M be the total amount of money. In any synchronous system, the equilibrium price P_e in a connected network G is presented by*

$$P_e = \frac{M}{T}.$$

□

Notice that this equality coincides with *Fisher's quantity equation* $MV_m = P_eT$ when $V_m = 1$.

To extend this to general V_m, we have to consider an asynchronous system in which every operation occurs at any time. The *velocity of money* is defined as the mean distance money is passed from one holder to the next in a round.

Let $flow_i$ be a variable which represents cumulatively paid money at node i. The velocity of money is obtained as in the following theorem.

Theorem 4. *Suppose that each node i has paid money of $flow_i$ in a round. Then, the velocity V_m of money is*

$$V_m = \frac{\sum_i flow_i}{M}.$$

Proof. In the Fisher's equation $MV_m = P_e T$, the right-hand side $P_e T$ means the total amount of selling goods in the system. It is equal to the total amount of paid money in the system. Thus, we can measure it by using the variable $flow_i$ for every node $i \in V$. Then we have $P_e T = \sum_i flow_i$. Therefore, the theorem follows. □

Figure 3 illustrates the idea of velocity of money. For simplicity, suppose agent 1 holds money M at time 0. Then, it passes through several nodes and each agent i records his spent money in $flow_i$. At the end of the round, suppose all the money M reaches nodes distance k from node 1. Then, the total sum of $flow_i$ consists of the sums of them with respect to distance $0, 1, \ldots, k$. Thus the distance k during one round, the velocity of money, is derived as in Theorem 4.

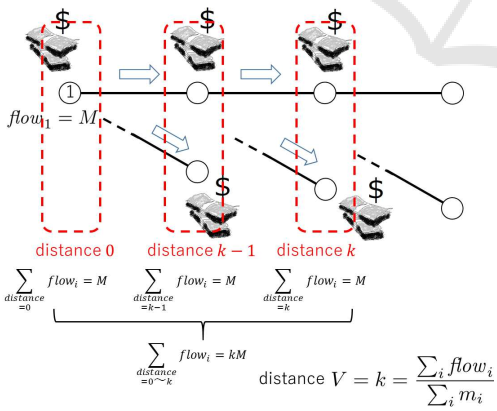

Figure 3: Distance of Money Traveling.

4 SIMULATION

In this section, we execute simulation experiments for the protocols above in path and grid networks. We investigate the influence of the network topologies and other aspects on the velocity of money.

Next, we consider the following three issues in two kinds of networks, a path and a grid. If we execute (a) intermediate bidding protocol, (b) pseudo first-price protocol, and (c) **SecondOptBid**, we investigate how

[1] the number of nodes,

[2] the number of money-injection nodes, and

[3] the rate of concurrency

have great influence on the velocity of money. More precisely, [1] we increase/decrease the number of nodes, [2] we add much money at some selected nodes, called money-injection nodes, and change the number of them, and [3] we change the number of concurrently trading agents.

Table 1 shows the constants used in our experiments. We repeat the experiment up to 50 trials, where a trial ends with an equilibrium, and obtain mean results. Initially, each node has money 100 units, and has goods between 50 and 100 units at random. Then, the equation $(\star)$ determines the price for each node. The total injection of money is 30,000 units. Table 2 shows the parameters used in our experiments. It has the third column named "standard" which means a constant value if another parameter is being varied. For example, the number of nodes is 300 when we vary the number of injection nodes from 1 to 10, or when we vary the rate of concurrency from 0.1 to 0.9, and so on.

Table 1: Constants.

Meaning	Value
Number of Trials	50
Iteration	500
Money per Node	100
Goods per Node	[50, 100]
Total Injection of Money	30,000

Table 2: Parameters.

Meaning	Value	Standard
Number of Nodes	50—500	300
Number of Money -injection Nodes	1—10	1
Rate of Concurrency	0.1—0.9	0.5

4.1 Number of Nodes

Figures 4 and 5 show how the number of nodes has influence on the velocity of money in two networks. All the curves increase as the number of nodes grows because there may be some extremely slow agents in large population. In the path, since almost all nodes have two degrees, not so many conflicts of bidding occur. Thus the payment of two equilibrium bidding protocols is low and that of the intermediate bidding protocol is high. In the grid, the payment of the second-price protocol is higher than others because each bidder i makes a bid $b_i^j = v_i^j$ to $j \in N_i$ and pays the second highest bid.

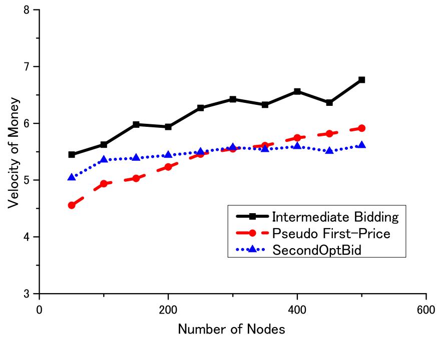

Figure 4: Varying Number of Nodes in Path.

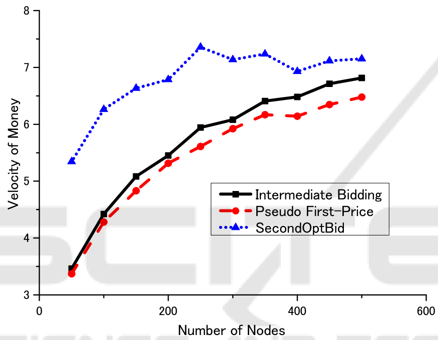

Figure 5: Varying Number of Nodes in Grid.

4.2 Money-injection Nodes

Figures 6 and 7 show how the number of money-injection nodes has influence on the velocity of money in two networks. In the path, since many number of money-injection nodes grow the spread of money in intermediate pseudo first-price protocols, the velocity of money becomes high. On the other hand, the influence of the second-price protocol is not outstanding. This is because the payment from a money-injection node would be equal to the bidding price of its neighboring nodes. And then, the payment would be low in the second-price protocol. Thus, the velocity of money in the second-price protocol is slow. In the grid, the tendency of the former two protocols looks alike.

4.3 Concurrently Trading Nodes

Figures 8 and 9 show how the number of concurrently trading nodes has influence on the velocity of money

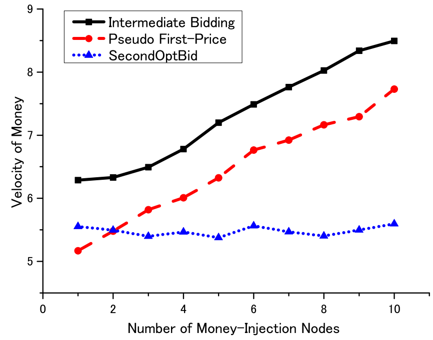

Figure 6: Varying Money-Injection Nodes in Path.

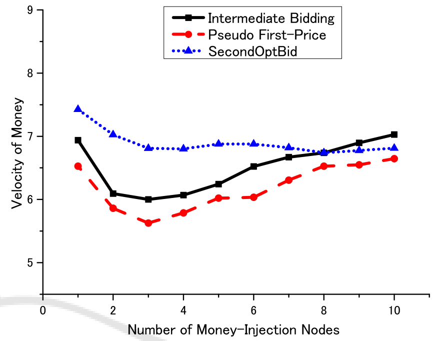

Figure 7: Varying Money-Injection Nodes in Grid.

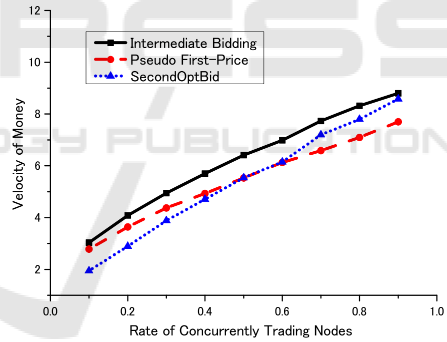

Figure 8: Varying Concurrently Trading Nodes in Path.

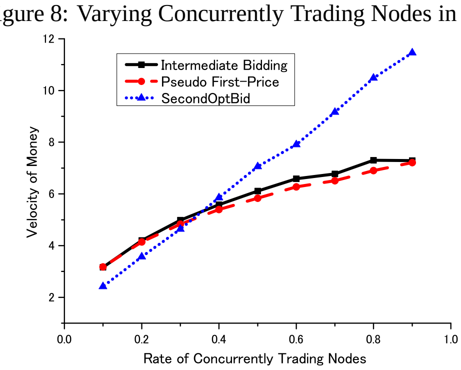

Figure 9: Varying Concurrently Trading Nodes in Grid.

in two networks.In the path, since the conflicts of bidding do not occur so often, the curves grow according as the rate of concurrently trading nodes. In the grid, since winner's payment of the second-price protocol is higher than others, the velocity of money is fast.

5 CONCLUSION

In this paper we extended our synchronous model for the price stabilization to an asynchronous system. Then we have obtained the following two advantages:

- we can consider a general model which is close to an actual system, and
- we can explain the velocity of money in Fisher's quantity equation.

First, we described how to express the velocity of money in Section 3. Then, we executed simulation experiments and revealed several features of the velocity of money in a path and a grid in Section 4.

The velocity of money for the second-price protocol is faster than that for the first-price protocol. Such a property is remarkable in grid networks rather than in path networks. This is because the second-price protocol accepts higher bidding price for many bidders, and then the amount of trade grows at an interval.

Our future work includes developing a practical stabilization model, for example, on-line shopping, and other protocols.

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