Critical Parameter Consensus for Efficient Distributed Bundle Adjustment

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Abstract: We present a critical parameter consensus framework to improve the efficiency of Distributed Bundle Adjustment (DBA). Existing DBA methods are based solely on either camera consensus or point consensus, often resulting in excessive local computation time or large data transmission overhead. To address this issue, we jointly partition points and cameras, and perform the consensus on both overlapping cameras and points. Our joint block partitioning method first initializes a non-overlapping block partition, maximizing local problem constraints and ensuring a uniform partition. Then overlapping cameras and points are added in a greedy manner to maximize the partition score that quantifies the efficiency of DBA for local blocks. Experimental results on public datasets show that we can achieve better computational efficiency without loss of accuracy.

1 INTRODUCTION

Structure-from-Motion (SfM) relies on Bundle Adjustment (BA) to minimize re-projection errors to optimize the final camera poses and scene point positions (Triggs et al., 2000; Agarwal et al., 2011; Frahm et al., 2010) in multi-view 3D reconstruction pipeline. Time and memory requirements for BA increase dramatically as the size of the BA problem grows. On the other hand, with the convenience of Internet image acquisition and the popularity of aerial equipment such as UAV, the scale of reconstruction increases continually. The high demand for resources by BA solves makes it a major computational bottleneck for the multiview 3D reconstruction.

Distributed Bundle Adjustment (DBA) (Eriksson et al., 2016; Zhang et al., 2017; Ramamurthy et al., 2017) is an iterative computing framework that utilizes out-of-core parallelism. The original BA problem is divided into multiple subproblems with overlappings and distributed to local computing nodes. In each iteration, each node solves a much smaller BA problem. Different local overlap parameters are fused in the primary node and are then returned to the worker nodes. The iterative process converges to a local optimal solution of the original problem. Subproblems require much less computation time and memory. The DBA's memory requirements are determined by subproblems and the total computation time t is given by,

$$t = (t_s + t_o) \times N_{\text{it}} \tag{1}$$

In addition to the time t_s required for solving subproblems, the data transfer of the overlapping parameters results in transmission overhead t_o, and N_{it} is the number of iterations.

Block partitioning methods relate the total computation time t in three aspects: the scale of the largest subproblem, the number of overlapping parameters, and the local constraints, which correspond to t_s, t_o, N_{it}, respectively. The classical LM method for solving the BA problem exhibits a quadratic convergence rate. However, the consensus framework on which the DBA framework is based does not converge faster than linear. Therefore, to accelerate overall convergence, it is necessary to improve the local problem constraint and use local optimization as much as possible to obtain better results. Based on these three aspects, the correlation between partitioning and computational efficiency can be quantified, and a partitioning metric function is proposed.

Under the block partitioning metric, the solution of the optimal partitioning is actually a constrained graph-partitioning problem. The constraint is that all observation points are covered in at least one sub-

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block. Each parameter may be in any block, and so there are exponential possible solutions. We propose to perform block partitioning by a joint division of points and cameras, and so the consensus will use a portion of points and cameras that are critical overlapping parameters.

2 RELATED WORK

2.1 DBA

DBA divides the original high complexity BA problem into smaller subproblems. Optimizing subproblems with no overlap can improve the accuracy of local reconstruction (Zhu et al., 2014) but the solution is not globally optimal. The consensus-based framework was first proposed by Eriksson et al. (Eriksson et al., 2016), which is constructed as a Douglas-Rachford splitting problem and converges to the local minimum of the original BA problem. Zhang et al. (Zhang et al., 2017) formulated the DBA problem based on the alternating direction method of multipliers (ADMM) and further refined the proof of convergence. However, different block partitioning can result in varied performance. Our method is also based on the consensus framework and we focus on improving the efficiency of DBA by exploring how to partition the partitioning of subproblems.

2.2 Block Partitioning

Block partitioning methods in the multi-view 3D reconstruction pipeline can basically be divided into two categories. The first one is point-based clustering. Based on the density, accuracy, and spatial proximity of points, the adjacency graph (Zhu et al., 2014) can be established. Then points are iteratively divided into many parts. However, a large number of points can make division a difficult task. Another category includes image-based clustering methods. Kushal et al. (Kushal and Agarwal, 2012) partitioned cameras based on the Canonical Views algorithm (Simon et al., 2007). A camera arrangement matrix can be obtained according to clustering results, which is then used as a pre-conditioner to ensure the stability of the normal equation. The cluster method involves no overlapping parameters. However, for a valid block partitioning in DBA, overlapping parameters are required for consensus across different blocks. In the pre-processing step of dense reconstruction, Furukawa et al. (Furukawa et al., 2010) hierarchically divided images to ensure that the number of local images does not exceed a predefined value. In addition, they added overlapping images to the cluster to ensure that scene points are well reconstructed. Although the local problem size constraint can be satisfied, the number of local problems is not known before computation. In the case of the approximate geometry of the known scene, image clustering and scene segmentation can be combined with (Zhang et al., 2015). For reconstruction methods that are based on depth map, each image finds adjacent images (Mostegel et al., 2018; Mostegel et al., 2016) and naturally forms a partition. The number of such partitions is equal to the number of images and the result of the partitioning can have a high degree of overlap.

At present, there are few studies on the block partitioning method for DBA. Block partitioning can have a significant impact on efficiency because it determines the data transmission overhead and local computation time, and can also result in varied number of iterations. Existing methods perform point clustering and add all visible images of points to local blocks (Zhang et al., 2017), or add all visible points of images to local blocks after image clustering. (Eriksson et al., 2016). The former introduces a large number of overlapping points. Since the number of points is generally much larger than the number of images, it will increase the data transmission overhead. Nearly all points will present in each block when images see a large portion of scene points. The latter increases the local optimization time since the computation time of the local optimization is mainly determined by the number of cameras.

3 CONSENSUS FRAMEWORK

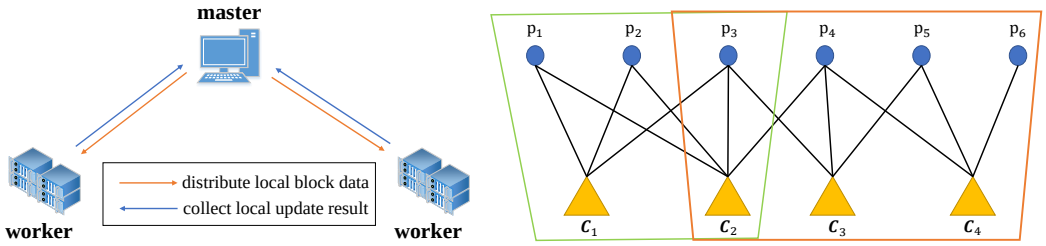
3.1 ADMM Algorithm

Consensus framework for DBA can be constructed based on DR Splitting or ADMM, which are mathematically equivalent (Giselsson and Boyd, 2014). The introduction below is based on ADMM. Consider an optimization problem for univariate and multi-objectives,

$$\min_x \ f(x) = \Sigma_{i=1}^{N} f_i(x) \tag{2}$$

where $x \in \mathbf{R}^n$, f_i is the ith objective term. Solving the problem in a distributed system requires that the objective terms can be processed separately. By introducing the local variable $x_i \in \mathbf{R}^n$ and the global variable z, this problem can be rewritten as a constrained problem, written as

$$\begin{aligned} &\min_x \ \Sigma_{i=1}^{N} f_i(x_i) \\ &\text{s.t.} \, x_i = z, i = 1, \ldots, N \end{aligned} \tag{3}$$



(a) (b)

Figure 1: (a) Our consensus framework consists of a master node and K worker nodes (K=2 in the figure). In each iteration, worker nodes optimizes local problems and passes results to the master node, which is transferred back to the working node after the master node's consensus computation. (b) An example of block partitioning. Each point is visible by multiple cameras. The line between the point and the visible camera represents an observation. Block partition divides the visibility graph into multiple subgraphs, which have overlapping cameras or 3D points for consensus.

The constraint is that all local variables are consistent, and this problem is also known as a *global consensus problem*.

The above constrained problem can be further transformed into an unconstrained one through the augmented Lagrangian method.

$$L_\rho(\mathbf{x},y,z) = \sum_{i=1}^{N}(f_i(x_i) + y_i^T(x_i - z) + \frac{\rho}{2}\|x_i - z\|_2^2) \tag{4}$$

where $\mathbf{x} = x_1,\ldots,x_N$, $\rho > 0$ is the penalty parameter. When $\rho = 0$, the above formula becomes the standard Lagrangian form, and y_i is the estimation of the Lagrange multiplier.

The ADMM algorithm iteratively optimizes $x_1,\ldots,x_N$, y, z. In the $t+1$ iteration,

$$x_k^{t+1} = \arg\min_{x_k} L_\rho(x_k, y, z^t) \tag{5}$$

$$w^{t+1} = \arg\min_{w} L(x_k^{t+1}, w, y_k^t) = \sum_{k=1}^{K} x_k^{t+1} \tag{6}$$

$$y_k^{t+1} = y_k^t + \rho(x_k^{t+1} - w^{t+1}) \tag{7}$$

The iterations proceed until the difference between the local variable and the global variable (the primal residual) as well as the amount of change of the global variable (the dual residual) are smaller than a given threshold value.

3.2 Consensus for DBA

The original BA problem corresponds to $f(x)$ in ADMM in DBA. BA jointly optimizes camera parameters $C = \{C_1, C_2, ..., C_m\}$ and sparse 3D scene point coordinates by minimizing reprojection errors $P = \{P_1, P_2, ..., P_n\}$:

$$\arg\min_{C,P} f(C,P) = \arg\min_{C,P} \sum_{i=1}^{m}\sum_{j=1}^{n}(v_{ip}\Pi(C_i,P_j) - x_{ij})^2 \tag{8}$$

Where $v_{ij} \in \{0,1\}$ is the visibility between the camera C_i and the 3D point P_j and . The projection operator $\Pi(\cdot)$ can project the given 3D point X_j to the camera C_i. The projected point must coincide with observations o_{ij}.

The objective function is the sum of multiple reprojection errors and can be easily represented as multiple objective terms. The parameter (C,P) to be optimized corresponds to x in the formula 2. Suppose that there is a block partitioning $\mathcal{P} = \{(C_k,P_k)\}$ and (C_i,P_i) corresponds to x_i, so that

$$f(C,P) = \Sigma_{k=1}^{N} f_k(C_k,P_k) \tag{9}$$

Algorithm 1: DBA: Processing by worker node k.

```
repeat
    wait
until receive block data in P_k from master node
initialize y_k = 0
repeat
    update y_k as Equation 7
    update x_k as Equation 5
    send overlap parameters in x_k to master node
    repeat
        wait
    until
    receive the updated z_k^{t+1} from master node
until termination
```

The above ADMM-based consensus framework can be implemented in a star-topology distributed system with one master node and multiple worker nodes, as shown in Figure 1(a). The processings in the master node and worker nodes are shown in the Algorithm 2 and Algorithm 1, respectively.

Because the projection operator is non-linear, $f(x)$ is a non-convex function. According to the theorem in (Zhang et al., 2017), $f(x)$ must additionally satisfy the $\nabla f(x)$ local Lipshitz continuous condition.

Algorithm 2: DBA: Processing by master node.

compute block splitting $\mathcal{P}$
for all block $\mathcal{P}_k \in \mathcal{P}$ **do**
    send block data in $\mathcal{P}_k$ to worker node k
end for
initialize iteration count $t \leftarrow 0$
repeat
    repeat
        wait
    until
    receive overlapping parameter updates from all workers
    compute w^{t+1} in parameter consensus as Equation 6
    broadcast w_k^{t+1} to all worker nodes
    $t \leftarrow t+1$
until termination
retrieve non-overlapping parameters from all workers.

Specifically, 1) The gradient of the distortion function must be Lipshitz continuous 2) Parameterization of rotation requires Lipshitz continuous. Therefore, the angle-rotation axis representation can be used in the projection operator, while the over-parameterized representation of such rotation of the quaternion cannot. 3) The scene depth is greater than $d_{min} > 0$.

4 BLOCK PARTITIONING FOR EFFICIENT BA

4.1 Objective of Block Partitioning

In the DBA consensus framework, the block partitioning results will be used to construct multi-objective functions, which can include any combination of reprojection error summations. The points and camera parameters involved in a single objective function are in the same block. Figure 1(b) illustrates an example of block partitioning in which the triangle represents the camera and the circle represents the three-dimensional point. If there is a line between the point and the camera, there is an observation.

Suppose a block partitioning method divides the original objective function into K objective functions. Their sum must be equal to the original target and each observation must appear in at least one block. When the same observation occurs in multiple blocks, the objective function must be reweighted.

$$\mathcal{P} = \{(C_k, P_k, O_k)\} s.t. \cup_{k=1}^{K} O_k = O \tag{10}$$

where $1 \leq k \leq K, C_k \subset C, P_k \subset P, O_k$ is a collection of observations corresponding to the camera and points in the block. In the BA problem, points and cameras are related to each other. A simple partition with no overlap will discard some observations, resulting in the loss of the original information. In order to include all observations, there must be overlapping parameters. Therefore, block partitioning is a partitioning problem with overlap. To take full advantage of hardware resources, we set the number of partitions to a fixed value, which can be determined based on, for example, the number of cores.

Any block partitions that satisfy the constrain can make the consensus converge in the end. However, different partitions will affect their computational efficiency. We score block partitions based on three factors that determine the efficiency of DBA.

- **Largest Subproblem Size**. A larger block contains more parameters and requires more time for local optimization. Different BA solvers can possess different time complexity. For example, in the BA solver without Schur complement, the number of points is much larger than in the cameras, which determines the computation time. Cameras weigh much more if the camera parameters are first solved using Schur complement. Here we consider both the points and the cameras. The LSS score is determined by the average of the local point and the camera's proportion of all points and cameras.

$$\text{LSS} = 1 - (|C_i|/m + |P_i|/n)/2 \tag{11}$$

- **Data Transfer Overhead.** The total amount of data transmission can be computed based on parameterization. We normalize this score based on the maximum size of possible overlapping parameters, and so the final DTO score is given by

$$\text{DTO} = 1 - (a_c \left|\tilde{C}\right| + a_p \left|\tilde{P}\right| - a_t)/((K-1)a_t) \tag{12}$$

- **Local Coverage.** Predicting how well a scene is reconstructed is actually determined by several aspects, such as visibility, angles, and distances. Here, we simplify the problem by measuring only the visibility as follows,

$$\text{LC} = \sum_{k=1}^{K} (\sum_{i=1}^{|C_k|} V_{i,:}^k / V_{i',:} + \sum_{j=1}^{|P_k|} V_{:,j}^k / V_{:,j'}) / 2K \tag{13}$$

To compare different methods more conveniently, a numerical value for the classification method is required. We use a weighted average method to combine the three metrics. The three values are not directly comparable, and their importance will also vary

depending on the network condition, computation resource, etc.

$$F_{bs} = (w_1\text{LSS} + w_2\text{DTO} + w_3\text{LC})/(w_1 + w_2 + w_3). \quad (14)$$

In the next section, we introduce our block partitioning method.

4.2 Partitioning Method

Block partitioning is actually a graph partitioning problem on a bipartite graph, with each edge in at least in one partition. Our goal is to determine a block partition with a good partition score and improve the efficiency of DBA.

For a given number of divisions K, each point or camera may be in any of the divisions, and so there are $((m+n)^K)$ possible partitions in total. The proposed block partitioning method consists of two steps. First, the constraint that all observations must be covered is relaxed and we find an initial non-overlapping partition. Then overlapping parameters are added to satisfy the constraints.

4.2.1 Biclustering-based Initialization

Biclustering (Dhillon, 2001) is a method of simultaneously clustering rows and columns of a matrix at the same time. It is very similar to Normalized Cut (Shi and Malik, 2000) and maximizes the inter-block spacing and intra-block correlation while ensuring uniform division. The difference is that bicluster can be used directly for different adjacency matrices for rows and columns. We compute the number of observations in a block as

$$vis(C_k, P_k) = \sum_{c_i \in C_k, p_j \in P_k} v_{ij} \quad (15)$$

Then the association between a block and other blocks can be expressed as *cut*

$$cut(C_k, P_k) = vis(C_k, {P_k}^c) + vis({C_k}^c, P_k), \quad (16)$$

the degree of association within the block is *within*,

$$within(C_k, P_k) = 2vis(C_k, P_k) + cut(C_k, P_k) \quad (17)$$

Then the goal of the final optimization is

$$f_{bc}(P) = \sum_{k=1}^{K} cut(C_k, P_k)/within(C_k, P_k) \quad (18)$$

This problem can be solved by the spectral method. By solving the SVD decomposition of the visibility matrix, each parameter is represented as a K dimension vector. The sparsity of the visibility graph makes the SVD decomposition easy to compute. Then a K-means algorithm clusters the vectors and the parameters into K blocks.

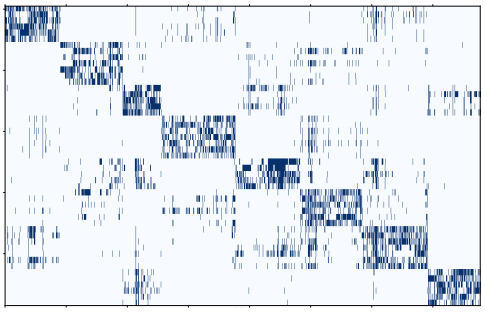


Figure 2: Initialization of block partitioning based on biclustering. Rows, columns, and elements represent cameras, 3D points, and observations, respectively. We reorganized the 3D points and camera index values based on their labels. Points and cameras in the same block will form a block in the visible view.

After biclustering, we reorganize them based on the point index of the point and camera. When the corresponding camera and point appear in the same block, it is also assigned to this block. As can be seen from Figure 3, aggregation occurs in the observations in the different blocks. Intuitively, our partitioning method hopes to ensure that the size of the block is as uniform as possible, thereby reducing the largest block, and reducing the local optimization time of the largest block, thereby optimizing the local calculation time. It is also necessary to reduce the number of observation points that are not on the diagonal block as much as possible. For a point that is not on the diagonal block, the coverage of the point and the camera will be reduced. In the block partition metric function, it is expressed as a reduction in the objective function. We have initialized a partition with no overlap, and there is a need for overlap in the DBA so that consensus can be achieved.

4.2.2 Critical Parameter Voting

The initial non-overlapping parameter partitioning does not cover all observations. The set of all observations O is equal to the union of the uncovered observations O^- and the set of observed observations O^+, $O^- \cup O^+ = O$. For $o_{ij} \in O^-$, L_i represents the block where C_i is located, and M_j represents the block of P_j. In a valid block partition, $O^- = \emptyset$. Each parameter in the initialization has its own block, so $L_i \neq \emptyset$ and $P_j \neq \emptyset$. We use the triplet $op = (k, i, b)$ to represent an operation that adds a parameter to the block, where k is the index of the block, $1 <= k <= K$. Flag b specifies the operation adds a camera or a point and i represents the index of the point or the camera. Then for each o_{ij}, the set of candidate operations corresponds to is S_{ij}. The set of all candidate operations is S. There will be overlaps in the set of candidate operations for

different observations.

In this step, our goal is to obtain a set of operations that cover all the observations and maximize the partitioning function. This is a combinatorial optimization problem. We propose a greedy strategy to calculate the sequence of operations, $Q = \mathrm{op}_1, \mathrm{op}_2, ..., \mathrm{op}_q$. Each operation picks the best action, making it the largest variable for the split metric. Each add operation changes the block partition and its evaluation, and so an add operation can be evaluated based on the amount of score change.

$$f_{\mathrm{v}}(op) = \Delta f_{\mathrm{bc}} \tag{19}$$

The scores of all candidate operations must to be calculated when the parameters are added. There are three main aspects involved in evaluating a single operation. Add a camera or point to a block, and the transmission overhead increase is fixed, For a local maximum block, it changes only when it corresponds to the current largest block. The change in coverage of a point depends on the number of points added. Although the cost of a single operation is easy to calculate, the number of points and cameras can be very large because of large-scale problems. It is computationally infeasible to score all candidate operations each time an operation is added. Therefore, we first vote on these operations by adding the number of observations to obtain a smaller candidate set S. In addition, the addition will change the existing block partition and will also affect the score of the next add operation. Therefore, we compute the first K batch operations each time, which improves efficiency. So our final voting algorithm is shown in the Algorithm 3.

Algorithm 3: Critical Parameter Voting Algorithm.

Input: visibility graph $\mathcal{V}$, number of blocks K
Output: block splitting $\mathcal{P}$

1: Initialize a non-overlapping block splitting $\mathcal{P}$ with biclustering as in Section 4.2
2: **repeat**
3: Initialize candidate operations set $\mathcal{Q} = \emptyset$
4: **for all** o_{ij} that not in any block **do**
5: $\mathcal{Q}_{ij} \leftarrow$ candidate adding operation for o_{ij}
6: $\mathcal{Q} \leftarrow \mathcal{Q} \cup \mathcal{Q}_{ij}$
7: **end for**
8: Vote all the operations in $\mathcal{Q}$ as Equation 19
9: Sort operations with descending order by voting score
10: Perform $\min(N_{\mathrm{batch}}, |\mathcal{Q}^{+}|)$ operations in $\mathcal{Q}$ with largest voting score
11: Update block splitting $\mathcal{P}$
12: **until** $\mathcal{Q} = \emptyset$

Although the cost of a single operation is easy to calculate, because the number of points and cameras can be very large in large-scale problems, a single operation will have an impact on the ratings of subsequent operations. We calculate a batch operation each time to improve computational efficiency.

5 EXPERIMENT

5.1 Dataset and Experiment Platform

We conducted experiments on the BAL public dataset (Agarwal et al., 2010). The dataset statistics are presented in Table 1. Scenes in the dataset have different sizes and initial reprojection errors. We also recorded the runtime of the traditional BA method for a single iteration.

Table 1: Dataset statistics. Different data sets have different numbers of images, 3D points, 2D observations. The table also shows the iteration time t_{BA}.

dataset	N_c	N_p	N_o	t_{BA}
ladybug_s	49	7k	31k	0.29
trafalgar	257	65k	226k	1.4
ladybug	1723	157k	679k	4.9
venice	1778	994k	5.00M	31
final	13682	4456k	29M	279

The experiments are performed on a star-shaped topology distributed platform of several workstations. One of the workstations is the master node, and the others are working nodes. Each node has an 8-core 3.6 GHz Intel i7-4770k CPU with 16GB memory. The master node and the working node communicate through the LAN, and the maximum transferring speed between two nodes is about 100MB/s.

In the local problem solver, DBA is a meta-algorithm for BA problems. In our consensus framework, we use the iterative Schur algorithm in Google's Ceres Solver to solve the BA subproblem. The iterative method accelerates the solution of the linear equation in the Schur complement method.

Ceres Solver is implemented in C++, and we call its interface directly in Python based on Cython. The block partitioning and the consensus operation on the master node are implemented in Python, and the data transfer is directly based on socket. Setting of the weight in the block partition evaluation must be carried out according to the actual situation. For example, the weight of DTO is related to data transmission rate. The weight of LSS and LC depends on the computing power of the node.

Table 2: Comparison of PC (Eriksson et al., 2016), CC (Zhang et al., 2017),and our CPC on partitioning metrics. The three partitioning metrics are LSS, DTO and LC, which correspond to the maximum block size of the block partition, the number of overlapping parameters, and the proportion of local visible points, respectively.

Dataset	PC (Eriksson et al., 2016)			CC (Zhang et al., 2017)			Our CPC		
	LSS	DTO	LC	LSS	DTO	LC	LSS	DTO	LC
ladybug_s	0.284	0.210	0.919	0.250	0.995	0.780	0.292	0.924	0.935
ladybug	0.313	0.305	0.813	0.250	0.981	0.760	0.322	0.837	0.945
trafalgar	0.652	0.761	0.570	0.437	0.988	0.676	0.756	0.985	0.797
venice	0.884	0.953	0.930	0.504	0.969	0.512	0.923	0.995	0.988
final	0.679	0.694	0.832	0.437	0.995	0.647	0.726	0.987	0.923

5.2 Experimental Results

For each data set, the number of partitions is chosen based on the size of the data set. We compare the impact of different partitioning methods on DBA efficiency based on three aspects: local computing time, data transmission overhead and local coverage. Our DBA method is referred as Critical Parameter Consensus (CCP) to distinguish between existing Camera Consensus (CC) method and Point Consensus (PC) method. According to the partition metrics in Section 4, the three aspects correspond to the size of the largest block divided by the block, the amount of overlapping parameter transmission, and the coverage of the partial block. Table 2 presents the comparison of different block partitioning methods on the partitioning metric. Our approach combines point and camera information and optimize LSS and LC during the voting process. It can be seen from the table that our method is superior to other methods in local calculation time and coverage.

In terms of data transmission, the CC method is slightly better than other methods. The number of points is usually much larger than the number of cameras, and so adding only the camera will obviously bring more overlapping parameters. The time taken for data transmission is actually related to the time of data transmission. In an extreme network environment, the CC method can be adopted to reduce the time overhead caused by data transmission. In addition, our method can change the weight according to the actual situation, and changing weight to use more overlapping cameras can achieve similar effects using the CC method.

The values of LSS and DTO are the ratio of the maximum subproblem size and the overlap parameter, respectively, and their effects on transmission overhead and computation time are relatively straightforward. The value of LC affects the speed at which the DBA converges. We compare the convergence rates of different methods through experiments, as shown in Figure 3. Our CPC method exhibits the highest LC value and the fastest convergence. In addition, we can also observe from the table that greater local coverage will also make the final reprojection error smaller.

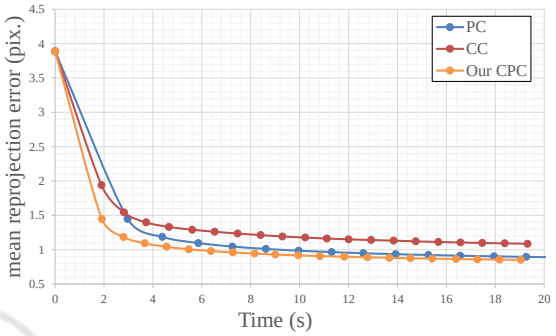


Figure 3: Comparison of the convergence rates of different partitioning methods on the ladybug dataset. The horizontal axis is the time spent and the vertical axis is the average reprojection error.

Using more partitions will make the local problem smaller, and so the local calculation time is small, which increases the number of overlapping parameters and reduce the local coverage. Increasing the number of partitions will reduce the calculation time, but as the number of partitions increases, the efficiency increase will be less significant.

We also recorded the iteration time and corresponding reprojection error of the traditional BA method and different DBA methods. On smaller data set, the actual computation time of a traditional BA is less than the computation time of the DBA. The computational overhead in the DBA is significant when the problem size is small. On a larger data set, DBA demonstrates computational efficiency advantage because the local computation time is much smaller, making up for the disadvantages of data overhead and convergence speed. DBA also uses less memory, and so when a memory bottleneck exists, DBA is a better choice than traditional BA methods.

6 CONCLUSION

In this paper, we have studied how to use both points and cameras information in block partitioning to im-

prove the consensus framework for DBA. Our intuition about this problem is that by properly assigning the parameters in local blocks, the three indicators LSS, DTO and LC that determines the total computation time can be balanced. We propose to biclustter the visibility graph to initialize a non-overlapping partition first, and then select the overlapping parameters that are critical for consensus using a scoring scheme. Experimental results on multiple datasets show that our joint block partitioning based critical parameter consensus method is more efficient than camera consensus and point consensus. The final mean projection error is comparable since our block partitioning also guarantees the visibility information in local blocks.

Future Work. In the current study, the local computation is performed synchronously, which means that all worker nodes must wait for the largest block to complete the computation. We plan to experiment with the asynchronous consensus framework (Zhang and Kwok, 2014) to further improve the performance of DBA. In addition, we hope to use more computing nodes and test larger datasets to further explore the potential of our consensus framework.

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