

# Attribute Operators for Color Images: Color Harmonization based on Maximal Harmonic Segmentation

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Abstract: Attribute openings and thinnings are morphological connected operators that remove structures from images according to a given criterion. These operators were successfully extended from binary to grayscale images, but such extension to color images is not straightforward. Color attribute operators have been proposed by a combination of color gradients and thresholding decomposition. In this approach, not only structural criteria may be applied, but also criteria based on color features and statistics. This work proposes, in a segmentation framework, a criterion based on color histogram divergence from a harmonic model. This criterion permits a segmentation in maximal harmonic regions. An application indicated that the harmonic segmentation permitted a hue correction that would not cause false colors to appear in regions already harmonic.

## 1 INTRODUCTION

The Mathematical Morphology (MM) provides a toolbox for developing image filters (Najman and Talbot, 2013; Serra and Vincent, 1992). Some of these morphological filters are grouped in a category, the connected operators, that has the characteristic of simplifying the image by merging flat zones (regions that have the same gray value). And as such, they have the property of reducing the number of regions without introducing new borders (Heijmans, 1999).

The attribute filter (Breen and Jones, 1996) belongs to this category, a particular connected operator that removes components of the image according to a criterion. Among the most usual criteria it is worth of mentioning the area, height and volume measurement (Vachier and Meyer, 1995; Vachier, 1995). They are known to have been applied to solving problems such as segmentation of medical images, image compression and structural analysis of ore (Breen and Jones, 1996).

Recently, an extension of attribute operators to color images have been proposed and two criteria based on color homogeneity have been applied for improvement color segmentation (Sousa Filho and Flores, 2017). This paper proposes, in the color attribute framework, a harmonic gradient (Ou and Luo, 2006a) and another color criterion based on the harmonic divergence (Baveye et al., 2013).

This criterion has the property of identifying max-

imal harmonic regions in an image. Based on this property, an application is devised for the improvement of color harmonization (Cohen-or et al., 2006). In this application, these regions are corrected separately which in turn provides that a concentration of color in a specific region of the image will not bias the harmonic correction as a whole.

## 2 PRELIMINARY CONCEPTS

Let  $E \subset \mathbb{Z} \times \mathbb{Z}$  be a rectangular finite subset of points. A *binary image* may be denoted by a subset  $X \subseteq E$ . Let the power set  $\mathcal{P}(E)$  be the set of all binary images.

Let the discrete interval of real numbers  $K = [lk, uk]$  be a totally ordered set. Denote by  $Fun[E, K]$  the set of all functions  $f : E \rightarrow K$ . A *graylevel image* is one of these functions.

Let  $Fun[E, \mathbf{C}]$  be the set of all functions  $\mathbf{f} : E \rightarrow \mathbf{C}$ , where  $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_n\}$ ,  $n \geq 1$  and  $\mathbf{c}_i \in \mathbb{R} : lk_i \leq \mathbf{c}_i \leq uk_i$ .  $Fun[E, \mathbf{C}]$  denotes the set of all *color images*.

Let  $A$  and  $B$  be two images, as described in the last three paragraphs. An *image operator (operator, for simplicity)* is a mapping  $\psi : A \rightarrow B$ .

Let  $B_c$  be the structuring element that defines a connectivity (Najman and Talbot, 2013). A *connected component* of  $E$  is a subset  $X \subseteq E$  such that,  $\forall x, y \in X$ , there is a path  $P(x, y) = (p_0, p_1, \dots, p_t)$ ,  $p_i \in X$ , such that  $p_0 = x$ ,  $p_t = y$  and  $\forall i \in [0, t - 1], \exists b \in B_c : p_i + b = p_{i+1}$ .

## 2.1 Color Gradient

Literature presents several ways to compute color gradients (Busin et al., 2008; Lucchese and Mitra, 2001). They are usually designed taking into account the analysis of each color component image under a certain color space model. For instance, color gradients may be designed under RGB (Busin et al., 2008; Evans and Liu, 2006), HSL (Rittner et al., 2010) or  $L^*a^*b^*$  (Ruzon and Tomasi, 2001).

Let  $\mathbf{f} \in Fun[E, \mathbf{C}]$  be a color image under the  $L^*a^*b^*$  color space model (Busin et al., 2008). Let  $D(\mathbf{a}, \mathbf{b})$  be a measure of distance between two colors  $\mathbf{a}, \mathbf{b} \in \mathbf{C}$ . The *color gradient*  $\nabla_{B_c} : Fun[E, \mathbf{C}] \rightarrow Fun[E, \mathbb{R}_+]$  is given by,  $\forall x \in E$ ,

$$\nabla_{B_c}(\mathbf{f})(x) = \bigvee_{b_1, b_2 \in B_c} D(\mathbf{f}(x + b_1), \mathbf{f}(x + b_2)). \quad (1)$$

In order to apply the thresholding decomposition, the color gradient needs to be converted to an image  $\bar{\nabla}(\mathbf{f}) \in Fun[E, K]$ , as follows:

1. Normalize  $\nabla_{B_c}(\mathbf{f})$  from the interval  $[\min\{\nabla_{B_c}(\mathbf{f})\}, \dots, \max\{\nabla_{B_c}(\mathbf{f})\}]$  to  $[0, \dots, k]$  (rounding it down);
2. Image  $\bar{\nabla}(\mathbf{f}) \in Fun[E, K]$  is given by the complement of the normalized gradient. This is done because objects are represented by valleys in the color gradient, and the negation of such valleys makes them sliceable by the thresholding decomposition.

## 2.2 Thresholding Decomposition

Let  $\mathbf{thr} : Fun[E, K] \times K \rightarrow \mathcal{P}(E)$  be the *thresholding function*, given by,  $\forall f \in Fun[E, K], \forall t \in K$ ,

$$\mathbf{thr}(f, t) = \{x \in E : f(x) \geq t\}. \quad (2)$$

Let  $f_{bin} : \mathcal{P}(E) \rightarrow Fun[E, k]$  be the mapping that gives a numerical representation of a binary image  $X \subseteq E$ , such that,  $\forall x \in E$ ,

$$f_{bin}(X)(x) = \begin{cases} 1, & \text{if } x \in X, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Let  $f \in Fun[E, K]$ . The *thresholding decomposition* is given by,

$$f = \sum_{t=1}^k f_{bin}(\mathbf{thr}(f, t)). \quad (4)$$

In other words, a grayscale image  $f$  may be decomposed as a stack of binary images, each one provided by the thresholding of  $f$  by a distinct graylevel  $k \in K$ .

The “addition” of all binary images in that stack returns image  $f$ .

Thresholding decomposition is a way to extend some operators - designed for binary images - to the grayscale context (Breen and Jones, 1996). Formally, the extension of  $\psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$  to  $\Psi : Fun[E, K] \rightarrow Fun[E, K]$  is given by,

$$\Psi(f) = \sum_{t=1}^k f_{bin}(\psi(\mathbf{thr}(f, t))). \quad (5)$$

## 2.3 Attribute Operators

Attribute operators are applied to remove all structures that do not fit into a given criterion (Breen and Jones, 1996). Such criterion is usually tied to a measurement and a comparison to a numerical parameter  $p$ . For instance, an image structure must be removed if its area is not greater than  $p$  - this is the criterion for area opening.

Let  $X \in \mathcal{P}(E)$  be a binary image. Let  $C \subseteq X$  be a connected component. The trivial operator  $\Gamma_T : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$  evaluates if  $C$  satisfies a criterion  $T$  (Breen and Jones, 1996):

$$\Gamma_T(C) = \begin{cases} C, & \text{if } C \text{ satisfies criterion } T, \\ \emptyset, & \text{otherwise,} \end{cases} \quad (6)$$

where  $\Gamma_T(\emptyset) = \emptyset$ .

The attribute operator is given by, for all  $X \in \mathcal{P}(E)$ ,

$$\Gamma^T(X) = \bigcup_{C \subseteq X} \Gamma_T(C), \quad (7)$$

where  $C$  is a connected component of  $X$ . Note that this is the binary case - the extension of Eq. 7 to grayscale case is given by thresholding decomposition (see Eq. 5).

For detailed information about attribute openings and thinnings, see (Breen and Jones, 1996). Notice that only structural criteria are mentioned in that paper.

## 2.4 Color Attribute Operators

Color attribute operators (Sousa Filho and Flores, 2017) allows the choice and application of criteria based on color information for attribute filtering of color images. Analysis of local morphological structures from a color gradient provides regions where information is collected from the color input image. And, such morphological structures are filtered in function of the collected color information.

Let  $\bar{\Gamma}_T : \mathcal{P}(E) \times Fun[E, \mathbf{C}] \rightarrow \mathcal{P}(E)$  denote the *color trivial operator*. It is similar to Eq. 6, but  $\bar{\Gamma}_T(C, \mathbf{f})$

may also take into account local color information given by  $\{\mathbf{f}(x) : x \in C\}$ , if a criterion  $T$  involves color features or statistics.

Connected component  $C$  is enough for assessment when criterion  $T$  is structural, like in graylevel version (Breen and Jones, 1996). For criteria based on color features or statistics, we cite average color error (Zhang et al., 2003; Borsotti et al., 1998), color harmony (Ou and Luo, 2006b) and entropy (Duda et al., 2001). In this paper, we apply the color harmony criterion (see Sec. 3).

The computation of the *color attribute operator*  $\bar{\Gamma}^T : Fun[E, C] \rightarrow Fun[E, K]$  is described in the algorithm as follows:

- 1: **procedure** COLORATTRIBUTEOPERATOR(image  $\mathbf{f}$ , criterion  $T$ )
- 2:  $g \leftarrow \bar{V}(\mathbf{f})$
- 3: decompose  $g$  into a set of slices  $G_i = \mathbf{thr}(g, i)$ ,  $\forall i \in [1..k]$
- 4: **for all**  $G_i$  **do**
- 5:  $F_i \leftarrow \bigcup_{C \subseteq G_i} \bar{\Gamma}_T(C, \mathbf{f})$
- 6: **end for**
- 7:  $\bar{\Gamma}^T(\mathbf{f}) \leftarrow \sum_{i=1}^k f_{bin}(F_i)$
- 8: **return**  $\bar{\Gamma}^T(\mathbf{f})$
- 9: **end procedure**

Note that, except for the application of the color trivial operator  $\bar{\Gamma}_T(C, \mathbf{f})$ , the color attribute operator is computed like the grayscale version (Breen and Jones, 1996).

The use of the filtered image  $\bar{\Gamma}^T(\mathbf{f})$  depends on the application of the method. For instance, one can use the *residue* of  $\bar{\Gamma}^T(\mathbf{f})$ . Another approach is to apply the watershed operator (Najman and Talbot, 2013; Beucher and Meyer, 1992) to the negation of  $\bar{\Gamma}^T(\mathbf{f})$  in order to compute a hierarchical segmentation (Meyer, 2001) of  $\mathbf{f}$ . This approach is adopted in this work.

## 2.5 Color Harmony

Generally, color harmony can be understood as a color combination that causes a pleasing effect to the observer. Subjective, the definition of harmony can have a different interpretation in according to the context in which it is applied as in product design, in painting or in photo enhancement.

Among scientists there is no harmonic theory largely accepted yet. Across the centuries many works tried to explain the psychological effects caused by the colors. Some works tried to explain through a ordered arrangement of colors (Ostwald, 1969; Munsell, 1969; Itten, 1961) while others approach the individual relations between colors (Von Goethe, 1840; Chevreul, 1967; Moon and Spencer, 1944).

### 2.5.1 Two-color Harmony

In the line of the harmony theories that explore these color relations, it is worth mentioning a function (Ou and Luo, 2006a) that can quantify this relation.

Let  $h_{ab}$ ,  $C_{ab}^*$ ,  $L_{sum}$ ,  $\Delta_L$ ,  $\Delta H_{ab}^*$ ,  $\Delta C_{ab}^*$  being respectively the hue, the chroma, the sum of lightness, the absolute difference of lightness and the difference of hue and chroma values defined in the  $L^*a^*b^*$  color space. Two-color harmony  $CH$  is defined as:

$$CH = H_C + H_L + H_H, \quad (8)$$

where  $H_C$  is the chromatic effect,  $H_L$  is the lightness effect and  $H_H$  is the hue effect, defined empirically in his paper.

### 2.5.2 Harmonic Template Determination

The classic Kullback-Leibler divergence is an relative entropy between two probabilities distributions and can be interpreted as the information loss when using a probabilities distribution  $Q$  to approximate a distribution  $P$ . In the discrete case it is defined as:

$$D_{KL}(P||Q) = \sum_i P(i) \ln \frac{P(i)}{Q(i)}. \quad (9)$$

One application of this divergence is when  $P$  is a distribution obtained from real data and  $Q$  represents a model. The model  $Q$  that minimizes this divergence is the one that best fits distribution  $P$ .

This application can be used (Baveye et al., 2013) to find which harmonic template best describes a color image.  $P$  is the weighted Hue histogram of the image computed on the HSV using 360 bins:

$$P(i) = \frac{\sum_{x \setminus H(x)=i} S(x) \times V(x)}{\sum_x S(x) \times V(x)}, \quad (10)$$

$i \in [0, 359]$ .

$Q$  is calculated for each rotation of the eight harmonic templates  $T_m (m \in \{i, V, L, J, I, T, Y, X\})$  defined in the chromatic circle (Matsuda, 1995) as shown in figure 1. Each template is composed of sectors, each sector having a center and a width.

The distribution  $Q$  given a template  $m$  and a rotation  $\alpha$  is given by:

$$Q_m^\alpha(i) = \begin{cases} \sum_{k=1}^n S_m^\alpha(i, k) & \text{if } \sum_{k=1}^n S_m^\alpha(i, k) \neq 0 \\ \frac{0,01}{360} \times \sum_i (\sum_{k=1}^n S_m^\alpha(i, k)) & \text{otherwise,} \end{cases} \quad (11)$$

where  $n$  is the number of sectors defined for the template  $T_m$  and

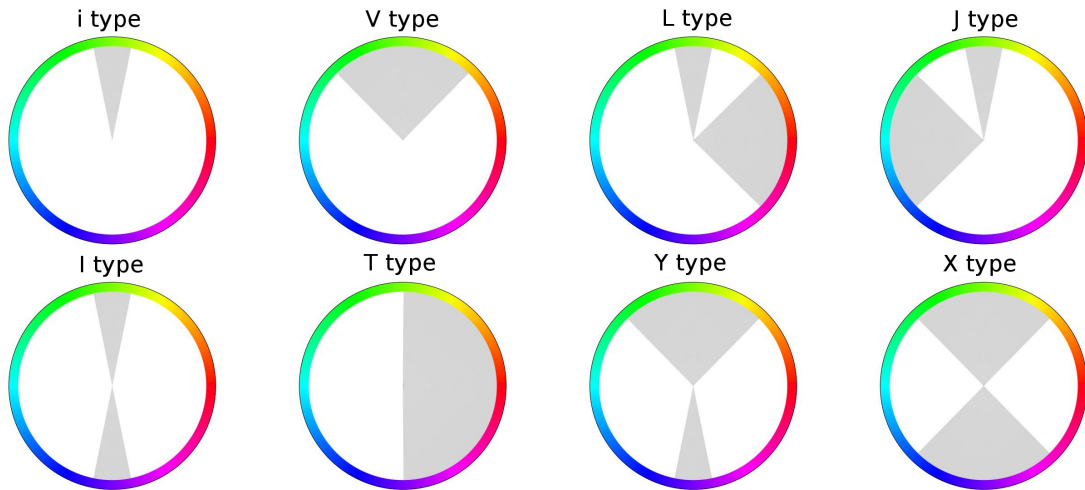


Figure 1: Harmonic models  $T_m$ . The gray area represents a sector of hue values that composes the harmonic model.

$$s_m^{\alpha}(i, k) = \begin{cases} \exp\left(-\frac{1}{1 - \left(\frac{2 \times \|i - \alpha_k\|}{w}\right)^{10}}\right) & \text{if } i \in [\alpha_k - \frac{w}{2}, \alpha_k + \frac{w}{2}] \\ 0 & \text{otherwise,} \end{cases} \quad (12)$$

where  $\|\cdot\|$  is the angular distance in the chromatic circle and  $\alpha_k$  and  $w$  are respectively the center and width of the sector  $k$  of the template  $T_m$ .

### 2.5.3 Color Harmonization

Color harmonization (Cohen-or et al., 2006) is a process of mapping the hue values of a image to match a harmonic model  $T_m$ . This can be done by a function (Baveye et al., 2013) as defined:

$$H'(x) = C(x) + \text{Signal}_{quadrante} \times \frac{w}{2} \times \tanh\left(\frac{2 \times \|H(x) - C(x)\|}{w}\right), \quad (13)$$

where  $C(x)$  and  $w$  represents the central hue vale and the angular width, respectively, of the sector associated with the pixel  $x$  and  $\|\cdot\|$  is the angular distance in the chromatic circle.

To find the the template sector and the signal that one pixel must be mapped, the chromatic circle of the hue histogram is divided in 4 sectors (or 2 for templates with just one sector) separated by the centers of each template sector and by two borders as illustrated in the figure 2.

Each of the 4 sectors define a different combination of a template sector and a signal, where the sectors around a template sector center are a mapping to it.

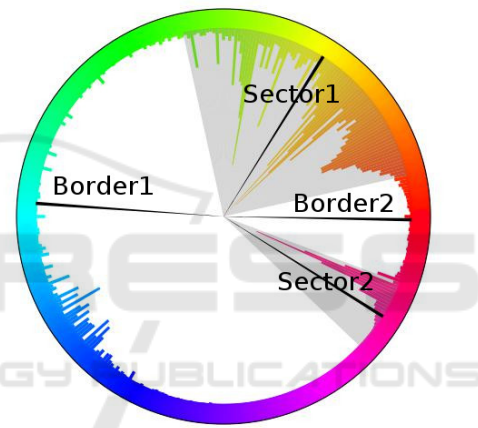


Figure 2: Example of the chromatic circle division.

## 3 CRITERION: HARMONIC DIVERGENCE

We can define a color attribute operator based on the harmonic divergence to analyze harmony within the regions of the image. The trivial operator of the harmonic divergence is given by the minimum divergence between the harmonic distributions and the color histogram of the elements within the region:

$$D(R, f) = \min D_{KL}(P \| Q_m^{\alpha}) \quad (14)$$

The harmonic divergence criterion  $T_D$  is defined by  $T_D = (D(R, f) \leq p)$ . In other words, the *harmonic divergence*  $D(R, f)$  must be *lesser or equal*  $p$ .

## 4 IMPROVEMENTS FOR COLOR HARMONIZATION

Finding the maximal regions of an image is a very interesting propriety and to explore it this work applied it for harmonic correction. The application was implemented using the Mathematical Morphology Library (Beucher, 2014) and tested on images of the Berkeley Segmentation Dataset and Benchmark 500 (Arbelaez et al., 2011).

As a measure of distance for the color gradient, a variation of the harmony between two colors (Ou and Luo, 2006a) was chosen. The use of this distance had the objective of enhancing the borders between harmonic regions. This distance was defined as:

$$D_H = \begin{cases} -CH & \text{if } CH < 0 \\ 0 & \text{otherwise,} \end{cases}$$

An use of this gradient can be seen in the figure 3. It's behavior is similar to others colors gradients, but the border is only enhanced where the color transition is disharmonic.



(a) Sample "23050"

(b) Harmonic gradient

Figure 3: An example of the gradient using the harmonic distance.

The segmentation is then obtained using the watershed on the negation of  $\bar{\Gamma}^T(\mathbf{f})$  when varying the criterion parameter  $p$  until the resulting regions are optimal. This can be automatized by fixing the number of resulting regions proportional to the number of segments of a non filtered watershed or by statistical analysis of the attribute variation.

The figure 4 exemplifies the segmentation result of a filtered image using the color harmony criterion. This method was capable of segmenting harmonious groups independently of size, in this case it was able to isolate a region that was dominated by the color pink from the rest of the image.

Then, for each segment, it was chosen the template and rotation that minimized the intra-region divergence and the hue values where mapped according



Figure 4: Example of harmonic segmentation.

to them. A modification proposed by this paper is to find the borders based in the idea that they divide a bi-modal section of the histogram, in this sense Otsu's method (Otsu, 1975) was used.

The figure 5 shows an result of the application of the mapping inside each of the harmonic segments in comparison with the result of the mapping applied on the whole image. Because of the segmentation, the clothes of the guy and the boy on the right were not mapped to pink. That was not the case of the blue hat that was mapped in the same harmonic region.

## 5 CONCLUSIONS

Besides attribute operators were extended to color images, they became more versatile with the extension of criteria to color information. Color attribute operators where recently proposed (Sousa Filho and Flores, 2017) with two criteria successfully been applied for improvement of image segmentation.

This works presented color harmony as a color criterion, the use of a harmonic distance in the color gradient and the use of Otsu's method in the hue mapping. Also showed the application of the color attribute operator to improve the color harmonization of images as it allowed to segment the image into maximal harmonic regions. In the application, a color correction applied locally on the segments was capable of harmonizing the image without creating artifacts from a global correction.



Figure 5: Comparison of the hue mapping with and without the support of the harmony criterion.

Future works include the proposal of improvements to the implementation of color attribute operators, like the use of a newer representation (Souza et al., 2015; Xu et al., 2017) or an optimization of the opening with the use of Viterbi algorithm (Viterbi and Omura, 1979) and the proposal of new color criteria for attribute color processing.

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