

Fine-grained 3D Face Reconstruction from a Single Image using Illumination Priors

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Abstract: 3D face reconstruction has a wide range of applications, but it is still a challenging problem, especially when dealing with a single image. Inspired by recent works in face illumination estimation and face animation from video, we propose a novel method for 3D face reconstruction with geometric details from a single image by using three steps. First, a coarse 3D face is generated in morphable model space by landmarks alignment. Afterwards, using the face illumination priors and surface normals generated from the coarse 3D face, we estimate both illumination condition and facial texture, making it possible for the final step that refines geometric details through shape-from-shading method. Experiments prove that our method outperforms state-of-the-art method in terms of accuracy and geometric preservation.

1 INTRODUCTION

3D face reconstruction is useful for a variety of applications, such as facial animation (Cao et al., 2014) and recognition (Zhu et al., 2015). Although the 3D model can be reconstructed through multi-images or special sensors, it still remains challenges to reconstruct a 3D face from a single 2D image due to lack of illumination and depth information.

In recent years, different methods have been proposed for 3D face reconstruction from a single image. Of these methods, the most common way is to use a 3D Morphable Model (3DMM, (Blanz and Vetter, 1999)) to estimate its parameters via landmarks fitting so that the model matches the input image. Besides, Shape-from-shading (SFS) method (Kemelmacher-Shlizerman and Basri, 2011) could also be introduced to solve this reconstruction problem through recovering depth field from the shading variation of the input image. In addition, Convolutional Neural Network (CNN) has been employed to recover 3D face directly via volumetric regression (Jackson et al., 2017).

Although existing methods are capable of recovering fine 3D face from a single image, they also have some limitations. Since the 3DMM is a parametric model of low-dimensional representation, it cannot represent high-dimensional face information, that is, facial features such as wrinkles cannot be recovered. The problem of missing facial geometric details also exists in the CNN volumetric regression method due



Figure 1: Our 3D face reconstruction from a single image. Given an input image (left), we recover a 3D face with fine geometric details (right, second column). The input image is used as texture for the reconstructed face and making the result intuitional (right, first column).

to its small number of model points. SFS method is able to recover fine geometric details from images, but it requires prior information about facial texture and illumination to solve such ill-posed problem. Reconstruction result would be far from the target face on the overall shape if no prior knowledge is provided.

In this paper, we develop an intergrated method based on 3DMM and SFS to recover a 3D face model with geometric details (see Figure 1). Our method consists of following three steps:

- First, we estimate a coarse 3D face that represents

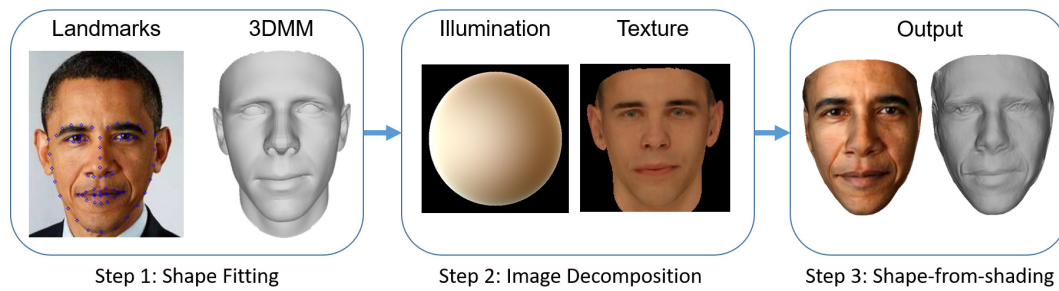


Figure 2: Pipeline of our method. We first estimate pose, shape and expression of the target face in 3DMM space. Afterwards, lighting and texture parameters are estimated using both SH lighting prior and facial texture model. At last, we develop a SFS method to refine facial geometric details.

the overall shape of the target face by fitting landmarks between the 3DMM-based model and the input image. We choose Basel Face Model (BFM, (Paysan et al., 2009)), a dataset with large range in identity and expression, as our morphable model.

- Afterwards, we analyze the Intrinsic Image (Land and McCann, 1971) properties and face illumination prior knowledge. using the facial texture model and aforementioned solved overall shape, we iteratively estimate the illumination condition and face texture parameters of the target image, providing prior information for the SFS step.
- Finally, we utilize both the illumination and texture parameters to carry out shape refinement based on the shading variation of the input image. A height-field face model that captures the fine geometric details and preserves the overall shape of the input image is eventually generated.

Our approach combines the advantages of the 3DMM-based method with the SFS method, while avoiding their respective disadvantages. The 3DMM method provides good overall face shape, and our innovative introduction of illumination priors makes the SFS refinement more reliable. Experiments shows that our method outperforms other method in terms of accuracy and geometric preservation.

2 RELATED WORK

3DMM. Human faces have many common features, which makes it possible to characterize 3D face model using low-dimensional parameters. The most known parametric face model is 3DMM (Banz and Vetter, 1999), which is a PCA-based statistical model of facial shape and texture. 3DMM has been used in a wide range of fields, such as 3D face reconstruction (Roth et al., 2016), face recognition (Zhu et al., 2015) and make-up suggestion (Scherbaum et al., 2011). In the field of 3D reconstruction, one of the benefits of

using 3DMM is that it constrains the solution only to the possible face space, which simplifies the problem of 3D face reconstruction. Since 3DMM is derived from a limited 3D face dataset and focus on the principle components, its solutions cannot fully characterize all faces and always lack fine geometric details.

Intrinsic Image and Lighting. Intrinsic Image Decomposition (IID, (Land and McCann, 1971)) is a problem to decompose an image into its shading and reflectance components. For decomposition of face image, different prior information could be used to facilitate the accuracy of decomposition (Li et al., 2014). One of the latest face priors is the face illumination prior (Egger et al., 2018). The authors propose a illumination estimation technique and apply it to face images under various illumination conditions, resulting in a huge illumination dataset. The prior is a probability distribution of natural illumination conditions and is modeled using first-three-bands Spherical Harmonics (SH, (Ramamoorthi and Hanrahan, 2001)). In this paper, we utilize such prior to estimate intrinsic components of the face image.

Shape-from-shading. Shape-from-shading (SFS, (Zhang et al., 1999; Durou et al., 2008)) is a traditional problem that recovers 3D shape from images using shading variation. The SFS problem is extremely ill-posed, which needs the knowledge of the reflectance and illumination information first to recover target geometry. Since such information is often unable to be achieved, corresponding priori assumptions have been made for specific problems. For example, assuming that the reflectance of the object is uniform and the light sources are all distant. As for 3D face reconstruction, some people improve the robustness of SFS by employing a separate reference face (Kemelmacher-Shlizerman and Basri, 2011). In this paper, we incorporate the prior knowledge about facial geometry, illumination condition and texture solved in previous steps to achieve reliable solution.

3 OVERVIEW

We provide an overview of our paper in this section. The pipeline of our approach mainly consists of three steps, as illustrated in Figure 2.

Our paper is organized as follows. Section 2 describes related work. Sections 4, 5 and 6 discuss the three main steps of our reconstruction method, respectively. Section 7 describes the experimental evaluations, and conclusions are drawn in Section 8.

To be more specific, in Section 4 we iteratively estimate the pose, identity and expression parameters of the target face, resulting in a coarse 3D face model. In section 5, we decompose the input image and extract its texture and lighting parameters, by employing illumination priors. In section 6, SFS process is carried out to generate a fine geometric model.

4 SHAPE FITTING

3DMM is a linear combination of principle components of face dataset. It could be represented as a mesh with the same connectivity, and its vertex coordinates $\mathbf{V} \in \mathbb{R}^{3n_v}$ are computed as

$$\mathbf{V}(\alpha, \beta) = \mu + \mathbf{U}_{\text{id}}\alpha + \mathbf{U}_{\text{exp}}\beta, \quad (1)$$

where n_v is the number of vertices, and $\mu \in \mathbb{R}^{3n_v}$ is the mean face vector. \mathbf{U}_{id} is principle components matrix of face identity whose size is $3n_v \times k$, and α is the identity parameter of 3DMM with length of k . Similarly, \mathbf{U}_{exp} and β are principle components matrix and parameter of facial expression respectively.

In this section, we align 3D landmarks on the 3DMM with corresponding 2D landmarks from the input image. Since the 3DMM shares the same connectivity under different parameters, its indices of 3D landmarks would be fixed during reconstruction of different images. Given an input image, we detect the face and find out its corresponding landmarks using the method in Dlib C++ library (King, 2009). Parameters of pose, identity and expression are estimated iteratively in this section.

4.1 Pose Estimation

We suppose that the projective model is a weak perspective projection along the Z direction, so the projection just scale the X and Y coordinates of the face object after it has been rotated and made translations. Therefore, we can formulate the following energy function to align the projection of 3D landmark vertices with the detected 2D landmarks.

$$f(s, R, T, \alpha, \beta) = \sum_{i=1}^m \|sR * \mathbf{V}_{c_i}(\alpha, \beta) + T - W_i\|_2^2 \quad (2)$$

Here W_i is the i -th of the m landmarks on 2D image. \mathbf{V}_{c_i} is the 3D landmark vertex that are corresponding to W_i , where c_i is the i -th fixed index of the 3D landmarks in 3DMM. s is a scalar acting as the weak perspective projection matrix. R is the first two rows of the rotate matrix with a size of 2×3 , which ommits the effect of Z direction. T is a 2×1 vector representing image translations.

We first estimate the pose parameters by fixing the parameters of face identity and expression. Thus the problem is reduced to

$$s, R, T = \arg \min_{s, R, T} f(s, R, T, \alpha, \beta), \quad (3)$$

which could be efficiently solved by SVD. Especially, We set $\alpha = \mathbf{0}$ and $\beta = \mathbf{0}$ at the first iteration.

4.2 Identity and Expression Estimation

Once the camera pose is solved, we turns to optimize face identity parameters with pose and expression parameters fixed. We consider to add regularization term in order to get rid of abnormal result. Therefore the optimization turns to be

$$\alpha = \arg \min_{\alpha} f(s, R, T, \alpha, \beta) + \gamma_1 \sum_{i=1}^k \left(\frac{\alpha_i}{\sigma_i} \right)^2, \quad (4)$$

where σ_i is the corresponding singular values of the identity components. This is a linear least-squares problem and can be efficiently solved. Afterwards, we fix the pose and identity parameters, and optimize the expression parameters in the same way.

$$\beta = \arg \min_{\beta} f(s, R, T, \alpha, \beta) + \lambda_2 \sum_{i=1}^{k'} \left(\frac{\beta_i}{\sigma'_i} \right)^2, \quad (5)$$

where σ'_i is the corresponding singular values of the expression components.

Since we just set $\alpha = \mathbf{0}$ and $\beta = \mathbf{0}$ when estimate pose at first, parameters $\{s, R, T\}$ may not be accurate enough. Besides, identity and expression solutions based on the previous pose parameters may also deviate from the real result. Hence, we solve the pose-identity-expression problem iteratively, until the energy function converges. Finally, a parametric coarse model is generated.

5 IMAGE DECOMPOSITION

In this section, we firstly backproject the image to the aligned coarse 3D model, and then decompose the face image in model space. Both SH lighting priors and 3DMM texture model are employed to estimate corresponding parameters.

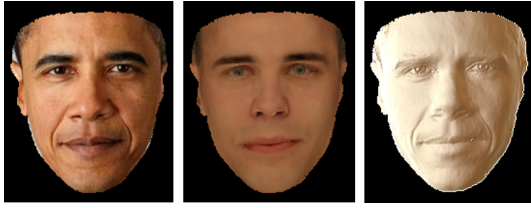


Figure 3: Intrinsic Image Decomposition of an example face image (left). It is decomposed into two components, which are albedo (middle) and shading (right) respectively. Shading could provide important cues for facial geometric refinement.

We describe the fundamental equation for intrinsic image decomposition as follow.

$$\mathbf{I}(x, y) = \mathbf{A}(x, y)\mathbf{S}(x, y), \quad (6)$$

where $\mathbf{I}(x, y)$ is the input RGB vector for pixel (x, y) . $\mathbf{A}(x, y)$ and $\mathbf{S}(x, y)$ are corresponding reflectance (or says albedo) and shading vectors. For each channel, this formulation takes element-wise multiplication. A decomposition example is shown in Figure 3.

5.1 Spherical Harmonic Lighting

It is usually assumed that human face surfaces are Lambertian and all light sources are distant. Hence, 3-order SH lighting would be sufficient for approximating shading effects (Ramamoorthi and Hanrahan, 2001). Based on these two assumptions, we employ SH lighting to describe illumination condition.

$$S^c(x, y) = \sum_{k=1}^9 L_k^c Y_k(\mathbf{n}(x, y)), \quad (7)$$

where $S^c(x, y)$ denotes shading effects of one single channel c (c denotes either r , g , or b) at pixel (x, y) . L_k^c is the corresponding SH lighting coefficient, with $1 \leq k \leq 9$. The SH coefficient L_k^c is estimated for R,G,B color channels separately in order to account for color illumination. $Y_k(\mathbf{n}(x, y))$ denotes spherical harmonics (SH) basis composed of surface normal $\mathbf{n}(x, y) = (n_x, n_y, n_z)^T$.

$$\mathbf{Y}(\mathbf{n}) = (1, n_y, n_z, n_x, n_x n_y, n_y n_z, 3n_z^2 - 1, n_x n_z, n_x^2 - n_y^2)^T. \quad (8)$$

Therefore, we could rewrite the equation (7) in a new form as follow.

$$\mathbf{S} = \mathbf{Y}(\mathbf{n}) \cdot \mathbf{L} \quad (9)$$

where \mathbf{S} is a $m \times 3$ matrix representing shading effect in all m vertices, and $\mathbf{Y}(\mathbf{n})$ is a $m \times 9$ matrix, and \mathbf{L} is a 9×3 matrix.

5.2 Lighting and Texture Estimation

We employ 3DMM texture model to represent the face albedo. The formulation of texture model is similar to that of shape model described in equation(1).

$$\mathbf{A}(\delta) = \mu_{\text{tex}} + \mathbf{U}_{\text{tex}}\delta, \quad (10)$$

where μ_{tex} is mean texture vector whose length is $3n$, \mathbf{U}_{tex} is principle components whose size is $3n \times k$, δ is the texture parameter of 3DMM whose length is k .

In order to estimate lighting and texture parameters, we form the data term of face image decomposition as follow.

$$f_{\text{sh}}(\mathbf{L}, \delta) = \|\mathbf{A}(\delta)\mathbf{Y}(\mathbf{n})\mathbf{L} - \mathbf{I}\|_2^2 \quad (11)$$

We innovatively employ a face illumination dataset (Egger et al., 2018) to generate illumination priors. This dataset consists of a wide range of illumination conditions (see Figure 4). Therefore, we add a Gaussian-based regularization term to constrain SH lighting parameters in a most probable range.

$$f_{\text{L}}(\mathbf{L}) = (\mathbf{L} - \mu_{\text{L}})^T \mathbf{C}_{\text{L}}^{-1} (\mathbf{L} - \mu_{\text{L}}), \quad (12)$$

where μ_{L} is the average of the illumination coefficients of the dataset, and \mathbf{C}_{L} is the corresponding covariance matrix.

Besides, similar to that of identity estimation, we use the same regularization term for texture parameter δ . Therefore, we could draw the full energy function for lighting and texture estimation.

$$\mathbf{L}, \delta = \arg \min_{\mathbf{L}, \delta} f_{\text{sh}}(\mathbf{L}, \delta) + \gamma_3 f_{\text{L}}(\mathbf{L}) + \gamma_4 \sum_{i=1}^k \left(\frac{\delta_i}{\sigma_i} \right)^2 \quad (13)$$

We set $\delta = \mathbf{0}$ at first, and estimate lighting and texture parameters iteratively. This is a linear least-squares problem and could be solved efficiently.

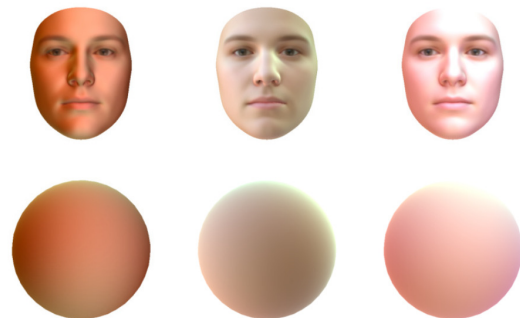


Figure 4: Samples from illumination dataset (Egger et al., 2018) which represent a wide range of different real-world illumination conditions. The samples are rendered with the mean face of the Basel Face Model (top row) and a sphere with the average face albedo (bottom row).

6 SHAPE-FROM-SHADING

In this section, we finally reconstruct a height field surface model that has fine geometric details over the face region of the input image. Using the known parameters of lighting and texture solved in previous steps, we optimize a refined normal map that represents geometric details of the target face. Afterwards, the refined normal map is integrated to recover a height field surface as the final model.

6.1 Height Field Integration

Surface $z(x, y)$ of the target face is pixelwise corresponding to the input image $\mathbf{I}(x, y)$, and its surface normal could be represented by two variables as

$$\mathbf{n}(x, y) = \frac{(p, q, -1)^T}{\sqrt{p^2 + q^2 + 1}} \quad (14)$$

where

$$\begin{aligned} p(x, y) &= z(x+1, y) - z(x, y) \\ q(x, y) &= z(x, y+1) - z(x, y). \end{aligned} \quad (15)$$

Obviously, the surface $z(x, y)$ could be integrated by

$$z = \arg \min_z \sum \left(\frac{\partial z}{\partial x} - p \right)^2 + \left(\frac{\partial z}{\partial y} - q \right)^2 \quad (16)$$

Once the surface normal map is refined, the height field model could be easily solved by such a linear least-squares optimization.

6.2 Surface Normal Refinement

Before integrating to surface height field, we should optimize depth field gradient p and q by minimizing following energy function.

$$f_{\text{sfs}}(p, q) = f_{\text{data}} + \lambda_1 f_{\text{grad}} + \lambda_2 f_{\text{close}} + \lambda_3 f_{\text{smo}} + \lambda_4 f_{\text{int}} \quad (17)$$

For convenience, we denote the intensity differences between recovered result and input image as

$$\mathbf{D}(p, q) = \mathbf{A}(x, y) \mathbf{Y}(\mathbf{n}(x, y)) \mathbf{L} - \mathbf{I}(x, y). \quad (18)$$

First of all, using the lighting and texture parameters obtained during previous step, we can generate a 2D image from the normal map according to Eq.(6). It is naturally to force rendered image to be close to the input image, which indicates the following energy function.

$$f_{\text{data}}(p, q) = \sum_{(x, y) \in \mathbf{I}} \|\mathbf{D}(p, q)\|_2^2 \quad (19)$$

However, if we only consider the intensity differences, we may get unreliable results due to some extreme lighting conditions such as highlights. Therefore, we also minimize the difference in intensity gradients between the input image and the reconstructed

one, resulting in following energy function.

$$f_{\text{grad}}(p, q) = \sum_{(x, y) \in \mathbf{I}} \left\| \frac{\partial \mathbf{D}}{\partial x} \right\|_2^2 + \left\| \frac{\partial \mathbf{D}}{\partial y} \right\|_2^2 \quad (20)$$

Taking f_{data} and f_{grad} into consideration is not sufficient for good results. Hence we employ three additional regularization terms for the surface normal and height field. Firstly, since the coarse model generated in the 3DMM step captures overall shape of the target face, we minimize the difference between normal map and the surface normals \mathbf{n}^0 from the coarse model.

$$f_{\text{close}}(p, q) = \sum_{(x, y) \in \mathbf{I}} \|\mathbf{n}(x, y) - \mathbf{n}^0(x, y)\|_2^2 \quad (21)$$

Secondly, we create a Laplacian constraint to emphasize smoothness of the surface normal map.

$$f_{\text{smo}}(p, q) = \|\Delta \mathbf{n}\|_2^2 \quad (22)$$

At last, due to the property of integrity from gradient p and q to surface height z , we force the gradient p, q to satisfies the following formula.

$$p(x, y) + q(x+1, y) - p(x, y+1) - q(x, y) = 0 \quad (23)$$

Hence we propose the last energy function about height field integrability.

$$f_{\text{int}} = \sum_{(x, y) \in \mathbf{I}} [p(x, y) + q(x+1, y) - p(x, y+1) - q(x, y)]^2 \quad (24)$$

Now we combine all these five energy functions to form a complete energy function as (17). p, q could be optimized by

$$p, q = \arg \min_{p, q} f_{\text{sfs}}(p, q). \quad (25)$$

After weights $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are specified, we solve this nonlinear least-squares problem using the L-BFGS algorithm.

7 EXPERIMENT

In this section, we present several experimental results, and compare with other method to demonstrate the reliability of our approach.

We develop our code in C++ and the reconstruction algorithm is run on a PC with an Intel Core i7-3770 3.40 GHz CPU and 16 GB RAM. The weights in optimization problems (4), (5), (13), (17) are set as follows: $\gamma_1 = \gamma_2 = 5.0 \times 10^3, \gamma_3 = 0.01, \gamma_4 = 1.0 \times 10^4; \lambda_1 = 8.0, \lambda_2 = 0.3, \lambda_3 = 0.3, \lambda_4 = 0.5$; We adopt the L-BFGS solver (Liu and Nocedal, 1989) to solve the nonlinear optimization problem in the SFS

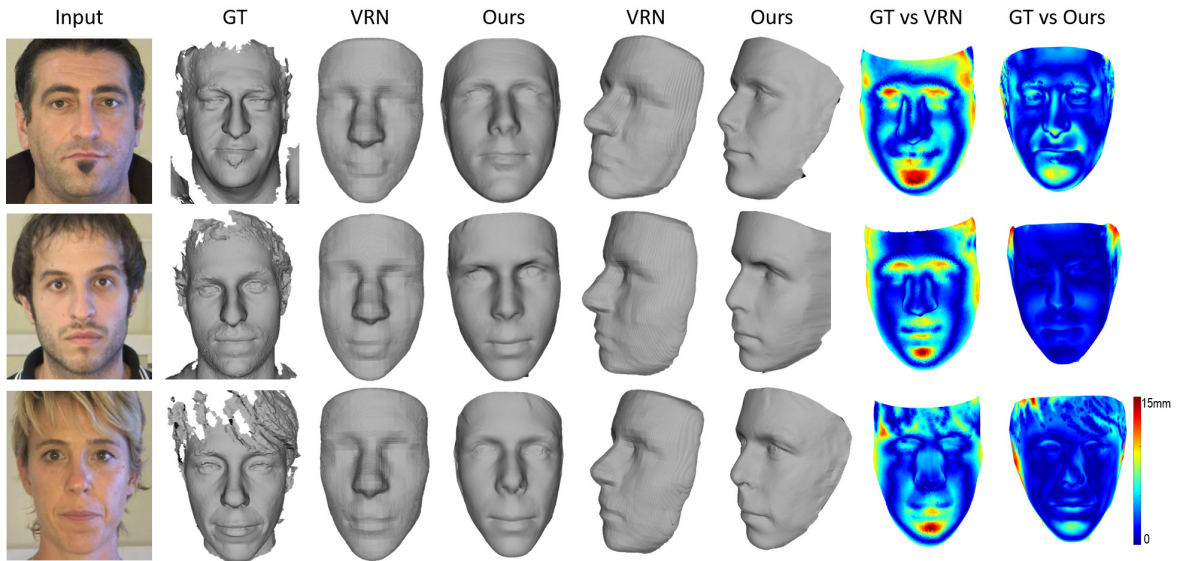


Figure 5: 3D Face reconstruction of three subjects from the MICC dataset. For each input image, we show the ground-truth (GT) and the results using our method and VRN method in two viewpoints. We also show the error map (according to 3DRMSE) for these two methods.

process (25). In addition, we implement all derivative functions by ourselves in order to speed up the optimization. Our algorithm has no limit on the size of the input image, but usually we take an image with a size of around 480×480 as input, which would cost about 30 milliseconds, 100 milliseconds, and 10 seconds respectively in corresponding steps.

7.1 Texture and Lighting Result

We first focus on the texture and lighting estimation. Due to the lack of real-world face dataset about texture and lighting condition, we synthesize 2D images from 3DDFA dataset (Zhu et al., 2016). Each image in the dataset corresponds to relevant 3DMM parameters, such as pose, texture, identity, and expression parameters, but except for SH lighting parameters. We select SH lighting parameters from illumination prior dataset (Egger et al., 2018), and apply them on the 3DDFA dataset to render corresponding 2D images.

For each rendered image, we estimate its facial texture and illumination condition. For visualization, we render the illumination parameters onto a sphere with the average face albedo. Then we calculate the Mean Square Error (MSE) between rendered sphere images and their corresponding ground-truths. Figure 6 shows several samples of our lighting results compared with ground-truths. As for facial texture, the MSE could be easily computed since the texture model shares semantic information and could be generated by multiplying principle component matrix with texture parameter. The average MSE of texture and

lighting estimations are shown in Table 1.

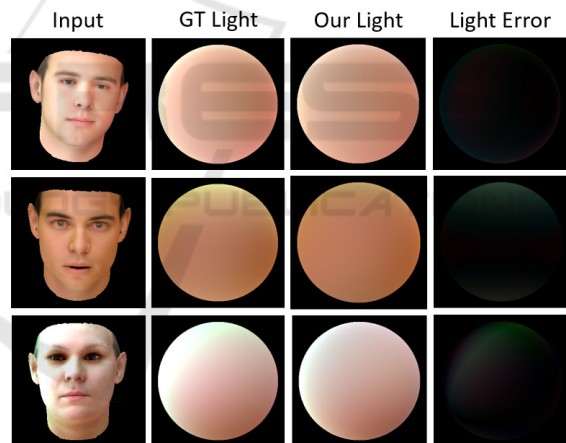


Figure 6: Samples of lighting results by applying our algorithm on synthesized images. Compared with those of ground-truth, our results have low error, which proves the accuracy of illumination estimation and guarantees the reliability of SFS.

Table 1: Average MSE of texture and lighting by applying our algorithm on synthesized dataset derived from 3DDFA.

Estimation	Texture	Lighting
Average MSE	0.0335	0.0238

Compared with traditional methods, we pay more attention to analyzing the accuracy of image decomposition components, which is an important prerequisite for geometric detail recovery. Both visualization results and numerical errors prove that our algorithm is reliable.

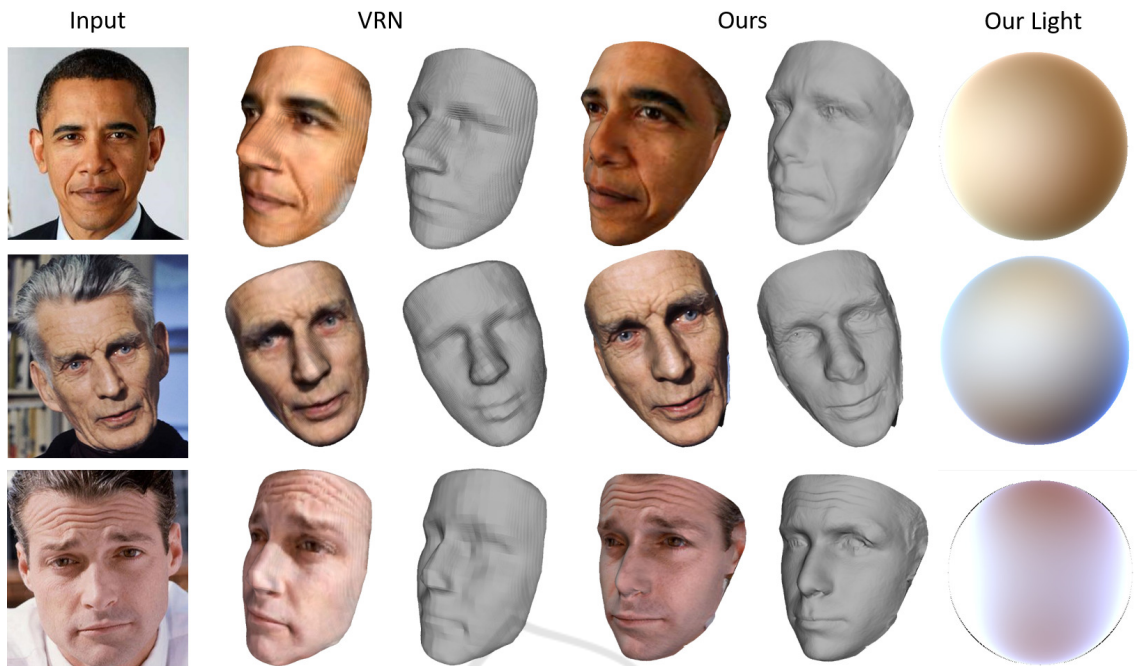


Figure 7: Face reconstruction results from Internet images. We show results (with texture) using VRN method and our method, respectively. In addition, we display our lighting estimations (without facial texture), in the last column.

7.2 3D Reconstruction Result

In order to demonstrate the effectiveness of our algorithm, we evaluate 3D points reconstruction error on a ground-truth dataset. To measure such error, each reconstructed model is aligned with its corresponding ground-truth face using Iterative Closest Point (ICP) method (Rusinkiewicz and Levoy, 2001). After that, we compute the 3D Root Mean Square Error (3DRMSE) between vertices of reconstructed model and their corresponding vertices on ground-truth model by

$$3DRMSE = \sqrt{\frac{\sum_i (\mathbf{Y} - \mathbf{Y}_{gt})^2}{N}}, \quad (26)$$

where \mathbf{Y} is the reconstructed model, \mathbf{Y}_{gt} is the ground-truth, and N is the number of vertices of the reconstructed model.

We compare our method with the VRN method (Jackson et al., 2017) by applying on the MICC dataset (Bagdanov et al., 2011). The MICC dataset contains 53 videos of different subjects and illumination conditions. The ground-truths are generated through a structured-light scanning approach. 42 subjects with suitable image resolution and low noises in point clouds are chosen for our experiment. We compute the 3D reconstruction errors using the 3DRMSE measurement described above. To be detailed, we manually choose the most frontal face image from the videos for each subject, and reconstruct the 3D face

model by taking it as input using VRN method and our method respectively. The mean and standard variation of 3DRMSE are illustrated in Table 2.

Table 2: 3D reconstruction error comparison on the MICC dataset. The mean and standard variation of 3DRMSE.

Method	Mean of 3DRMSE	Standard Variation
VRN	2.737	0.728
Ours	2.224	0.683

Table 2 shows that our reconstruction error is lower than that of VRN method (Jackson et al., 2017), while Figure 5 and Figure 7 show in an intuitive way that our method can recover more geometric details of the face than the VRN method, whether using images from MICC dataset or from Internet. Taking the third row of Figure 7 as an example, wrinkles on the man's forehead are faithfully recovered in shape, rather than in texture. Our lighting estimations from real-world images are also show in the last column of Figure 7. We add facial geometric details to the target face while keeping overall shape not changed. This is due to our combination of 3DMM overall face fitting and SFS detail recovery. It is clear that with the prior of reliable illumination estimation and SFS refinement, our approach have good estimations on single image reconstruction.

8 CONCLUSIONS

In this paper, we develop a novel approach to reconstruct a fine-grained 3D face model from a single image using illumination priors. We generate a coarse model in 3DMM space through landmarks alignment, providing the overall shape for next optimizations. By employing illumination priors and image intrinsic features, spherical harmonic lighting environment and facial texture are accurately estimated. At last, a shape-from-shading method is implemented to obtain a fine-grained 3D face model. The experiments demonstrate that our method can effectively reconstruct 3D face model with fine geometric details from single image.

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