Shape Recognition in High-level Image Representations: Data Preparation and Framework of Recognition Method

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Abstract: The automatic shape recognition is an important task in various image processing applications, including medical problems. Choosing the right image representation is key to the recognition process. In the paper, we focused on high-level image representation (using line segments), thanks to which the amount of data necessary for processing in subsequent stages is significantly reduced. We present the framework of recognition method with the use of graph grammars.

1 INTRODUCTION

Since image processing and image analysis are key in decision supporting systems and process automatisation, there is a need to develop processing techniques. As a complex multistage process consisting of segmentation, transformation, extraction of features, and pattern classification, it is necessary to choose the best technique for each of them, which results in the success of the final recognition.

First of all, it is important to preapare data – choosing the right representation of the image, depending on the task. The algorithms can be based on a digital image in an unprocessed version called matrix (pixel) representation (low-level representation) or any other high-level representation, e.g., using line segments or edges. The last of these (high-level representation) is particularly adequate to problems in which the structure of recognized objects is important. In the paper we discuss data preparation process and framework of recognition method.

As important as the selection of image representation is the use of an appropriate recognition method. Among the methods of image recognition, three approaches can be distinguished: statistical pattern recognition (Chen, 1973; Devijver and Kittler, 1982; Fukunaga, 1972; Fukunaga, 1990; Kurzynski, 1997; Schurmann, 1996; Vapnik, 1998; Webb and Copsey, 2011), syntactic pattern recognition (Bunke and Sanfeliu, 2000; Fu, 1982; Gonzales and Thomason, 1978; Miclet, 1986; Pavlidis, 1977; Skomorowski, 2013) and neural pattern recognition (Dunne, 2007; Omidvar and Dayhoff, 1997; Pao, 1989; Schurmann, 1996).

Solving problems in which the image structure is important requires a syntactic approach. The image is decomposed into primary components (simple elements) whose mutual relations build its structure. The syntactic approach includes string, tree and graph methods. Images are represented respectively by means of string, tree and graph grammars. Image recognition consists in performing a parsing (syntactic analysis) that determines whether graph is acceptable, correct for a defined grammar.

Based on the above observations, we proposed a framework for shape recognition in images in high-level representations using graph grammars. The choice of graph grammars is due to their greater descriptive power – in comparison with string or tree grammars. The descriptive power of graph grammars gives more options in solving complex problems, but at the same time is a challenge in the process of analyzing them.

2 DATA PREPARATION: HIGH-LEVEL IMAGE REPRESENTATIONS

As mentioned in the Introduction, image analysis requires the interpretation of a huge amount of low-level data – pixels. Proper data preparation is key.

Two stages can be distinguished in the process of image preparation for recognition: preprocessing and

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data reduction. Preprocessing methods are used to eliminate noise and unnecessary information, including changing the colour model, mathematical operations on images, improving contrast, removing artefacts, and identifying areas of interest (Malina and Smiatacz, 2008).

Reducing the amount of data can be achieved using the methods of detecting lines, edges and segmentation (Ballard and Brown, 1982). It allows finding consistent, in terms of specific criteria, areas – constituting objects or their fragments. In principle, three groups of methods are distinguished: methods of region growing, clustering and detection of boundaries.

So far, a wide variety of methods have been developed to detect edges and lines (Forsyth and Ponce, 2003; Gonzalez and Woods, 2008; Pratt, 1991; Shapiro and Stockman, 2001; Sonka et al., 2007). The edge detection methods are the first step in the methods of detecting lines. Many of the edge detection methods use convolution masks and differential operators. Among the known edge detection methods, one should mention the Roberts operator, the Sobel operator, the Prewitz operator, the Laplacian operators, Kirsch and Robinson masks. More advanced methods include the following algorithms: Marr-Hildreth (1980) (Marr and Hildreth, 1980), Canny (1986) (Canny, 1986), Boie-Cox (1986/87) (Boie and Cox, 1987), Shen-Castan (1992) (Shen and Castan, 1992), Frei-Chen (1970) (Frei and Chen, 1977). One of the well-known methods is the Hough transform (Gonzalez and Woods, 2008; Sonka et al., 2007). In 2013, Krylov and Nelson proposed a method to detect line segments and curvilinear structures, (Krylov and Nelson, 2014; Krylov et al., 2013). Another line segment detector LSD was developed in 2014 by Grompone von Goi (Grompone von Goi, 2014).

Based on detected complex structures such as lines, shapes, it is possible to change the representation of the image from low-level to high-level representation.

By default, the image is represented by pixels, and more precisely by a rectangular pixel grid. Such representation is not natural for a human who perceives more complex objects – lines, shapes, spots. In addition, image analysis in the low-level – pixel representation requires the analysis of a very large number of data. Hence, it is justified to distinguish in the picture significant structures, objects – coherent in terms of certain criteria, e.g . level of grey, colour, texture. This process is very important because it allows further image processing at a level higher than pixel (Umbaugh, 2011) (figure 1). High-level representations can be based on e.g.:

- line segments or edge segments (Grompone von Goi, 2014; Krylov and Nelson, 2014; Lazarek and Szczepaniak, 2014; Lazarek et al., 2014),
- superpixels (Achanta et al., 2012),
- contours – „active partitions” (Pryczek et al., 2010; Tomczyk et al., 2012),
- OB (ang. Object Bank) (Li et al., 2010; Li et al., 2014).

The higher the image representation level, the greater the knowledge about it, and the number of data necessary for processing decreases.

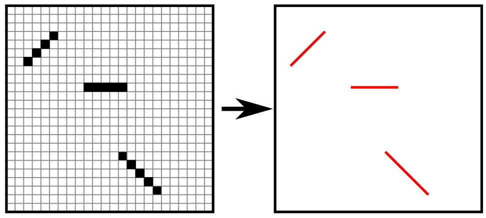

Figure 1: Visualization of the change of representation of the image from low-level (pixels) to high-level representation (line segments).

Usage of context information is presented in figure 1 – analysis of adjacent pixels allows detection of more complex structures. Instead of describing the image using the values associated with each pixel independently, it is possible to use the description using e.g., the coordinates of the ends of the segments, which increases the knowledge about the image being processed.

3 HIGH-LEVEL IMAGE REPRESENTATION – EXAMPLE: MAMMOGRAM IMAGE

The selection of the right representation is closely related to the problem being solved. The concept of the use of high-level image representation is presented on the example of mammogram data preparation for the task of recognizing spiculated changes.

3.1 High-level Image Representation Using Line Segments

Spiculated changes are characteristic masses that can be observed on mammography images. Spiculated change consists of a bright centre and star-shaped extensions of spicules spreading from it. An example of a spiculated change is shown in figure 2. An occurrence of spiculated changes is a strong premise of breast cancer (Kopans, 2007).

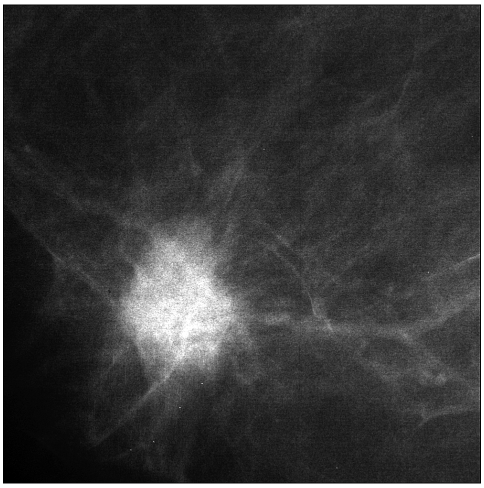

Figure 2: Spiculated change in mammogram.

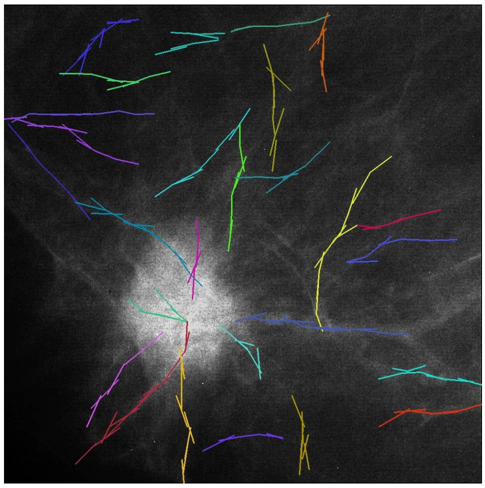

Figure 3: Spiculated change with detected line segments (with the use of the Krylov-Nelson method).

By analyzing figure 2 it can be observed that bright bands radiate from the bright centre – the spicules, that can be represented by line segments. Therefore, for the purpose of analysis of mammographic images for the presence of spiculated lesions on them, it was proposed to change the representation of the image from low-level (pixels) to the representation of a higher level (line segments). In order to detect the segments of the line, the Krylov-Nelson method was used. The image in such a representation can be used to further analysis, enabling the detection and recognition of spiculated changes. Figure 3 presents a fragment of a mammogram image with detected line segments that create its new representation (high-level).

The automatic detection of spiculated changes, despite significant research in this area, remains a challenge (Jiang et al., 2008). This is due to the fact that spiculated changes are often very subtle and are characterized by a large variety in appearance (Jiang et al., 2008), the number of spicules depending on the case may vary enormously.

3.2 Selection of a Significant Part of the Image – ROI

In order to prepare data for the recognition process, it is necessary to extract interesting fragments from an image – which may contain a spiculated change.

In the considered task, the choice of the ROI (area of interest) can be realized in two ways – automatic or supervised.

Regardless of the way ROI is selected, the square area is extracted and then the coordinates of its centre (intersection diagonals) are determined – (c_x, c_y). In figure 4, the ROI containing the spiculated change is selected.

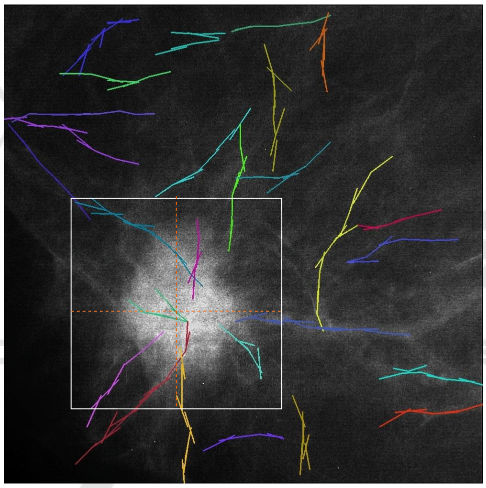

Figure 4: The area of interest – ROI (with highlighted subareas) including the spiculated change visible in the mammogram image. Initial ROI position selected in a supervised way.

After determining the ROI in the high-level image representation, it can be seen that the ROI frame intersects the found structures – groups of line segments, marked with the same colour as in figure 4. Omitting the line segments belonging to the separated structures but remaining outside the ROI would result in a loss of both the number and the quality of the data. To solve this problem, an original solution was proposed to create a ”flexible” ROI frame that adapts its shape to the groups of founded line segments. According to the author’s knowledge, the proposed method of expanding the area of interest (ROI) has not been described in the literature.

In figure 4 the selected area (ROI) is divided into four sub-areas. It can be observed that some of detected groups of line segments (marked with the same colour) cross the boundaries of the area of interest, or are simultaneously in two adjacent sub-areas:

- a green group of line segments lying partly in the right upper ROI sub-area, and partly outside it,
- a blue group of line segments lying partly in the right, lower ROI sub-area, and partly outside it,
- yellow group of line segments lying partly in the right, lower ROI sub-area, and partly outside it,
- a red group of line segments lying partially in the left, lower sub-area of the ROI, partly in the right, lower sub-area of the ROI and - partly outside the ROI area,
- a purple group of line segments lying partially in the left, lower ROI sub-area, and partly outside it,
- a blue group of line segments lying partially in the left upper ROI sub-area, and partly outside it.

Omitting these fragments of groups of line segments that lie outside the boundaries of the selected ROI would result in the loss of a significant part of the information, and thus affect the effectiveness of the diagnosis. Similarly to ignoring information about the belonging of detected line segments to larger groups, i.e. only a geometric interpretation of the position of line segments, which does not take into account the line segments belonging to complex structures (located in several sub-areas).

To avoid losing information, an innovative solution was proposed to match the ROI's area to the information in the image. Each of the four sub-areas expands independently of the others, adapting to the information in the image. Systems of groups of line segments are included in those sub-areas in which they are in the majority. Three main steps may be listed:

- segments are initially assigned to sub-area, in which lays at least one of its ends;
- segments are reorganized – segments are assigned to sub-area, in which the majority of segments from the same group lay;
- new bounding boxes for sub-areas are created – the smallest rectangle which contains all segments from sub-area is drawn.

The extended original ROI frame shown in figure 4 is shown in figure 5.

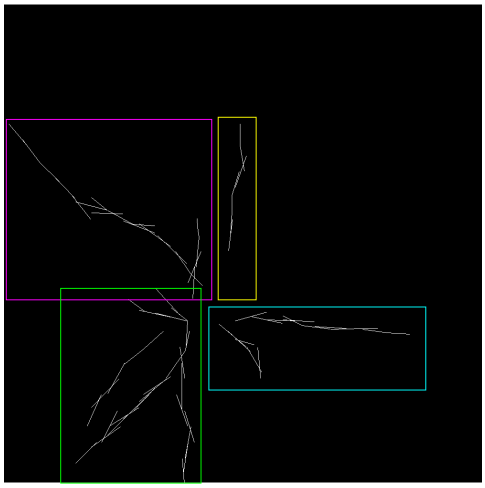

Figure 5: Flexible ROI frame - built of four segments.

3.3 Graph Construction

Due to the structural nature of spiculated changes, we proposed to recognize them using graph grammars. The framework of the method is presented in the next subsection.

To recognize spiculated changes using graph grammars, it is necessary to construct a graph representing the area of interest. Graph consists of five vertices – one of them is a previously designated point that is the centre of the original ROI, the other four are associated with four sub-areas of the extended ROI. The vertex in the centre of ROI is assigned the label s, while the labels for the remaining vertices are assigned in – depending on the arrangement of line segments located in the given sub-area of the ROI. The drawing 6 illustrates the concept. The red line segments represent the dominant direction of the line segments in a given sub-area (black), are determined on the basis of the median of directions of all segments located in a given sub-area. The selected segments are assigned the appropriate labels in accordance with the principles presented in figure 7. Subsequent vertices are indexed according to the principles presented in

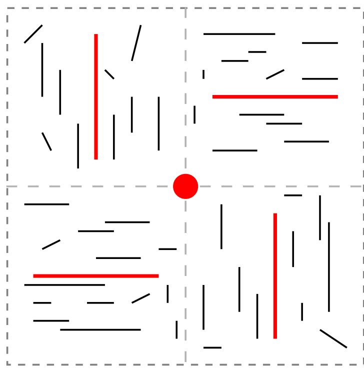

Figure 6: ROI with 4 sub-areas with detected line segments (black) and red line segments – being a symbolic representation of the dominant line segments direction in a given segment.

figure 8. The next step is the proper connection of the vertices – the edge comes out from the vertex with the smaller index and goes to the vertex with the larger index. Next, a label describing relations between vertices is assigned to each edge. A two-element set of edge labels was defined – $\Gamma = \{r, t\}$:

- r – is assigned to the edge from a central vertex to each non-central vertex,
- t – is assigned to the edges connecting non-central vertices.

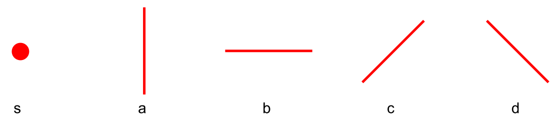

Figure 7: Visualization of a set of labels for dominant line segments $\Delta = \{s, a, b, c, d\}$.

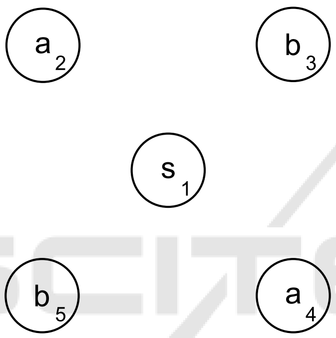

Figure 8: Graph representation of the pattern shown in figure 6 with assigned vertex labels and vertex indexes.

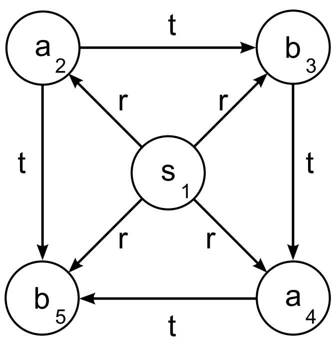

Figure 9: Graph representation of the pattern shown in figure 6 with assigned vertex labels, vertex indexes, and edge labels.

Graph representation of the pattern shown in figure 5 (mammogram) is presented in figure 10.

Patterns presented in figure 9 and 10 may be recognized with dedicated graph grammar. Whats consists one of the stages of graph grammar based shape recognition method described in the next section.

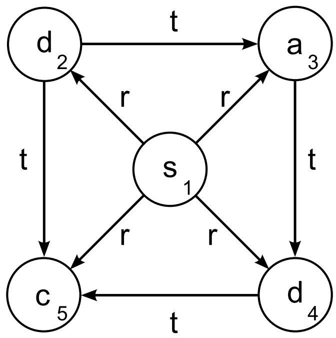

Figure 10: Graph representation of the pattern shown in figure 5 (mammogram) with assigned vertex labels, vertex indexes, and edge labels.

4 GRAPH GRAMMAR BASED SHAPE RECOGNITION METHOD

The method of semantic image analysis in high-level representation (e.g., line segments) with the use of graph grammars enables the detection and recognition of a selected class of objects. A universal processing sequence has been proposed that can be adapted to the task of detection and recognition of any class of objects that can be described using dedicated graph grammars. The image processing scheme is shown in figure 11.

4.1 Graph Grammars

Graph representations and graph grammars are used in many areas of practical importance, e.g.:

- representation of the logical structure of algorithms (Nagl, 1979),
- defining the semantics of programming languages (Gottler, 1983),
- code optimization (Nagl, 1979),
- modeling and processing of databases (Angles and Gutierrez, 2005; Angles and Gutierrez, 2008; Cheng et al., 2009; Nagl, 1979),
- information processing (Jiang and Bunke, 2017),
- system modeling (Kotulski and Sedziwy, 2011; Kotulski and Szpyrka, 2011; Rafe et al., 2009; Sedziwy et al., 2012; Szpyrka and Kotulski, 2011; Szpyrka et al., 2017),
- defining visual languages (Ehrig et al., 1999; Hermann et al., 2008; Rekers and Schurr, 1997; Zhang et al., 2001).

The important field being of interest in this paper is computer image analysis (Flasinski, 1989; Flasinski, 2007; Flasinski and Myslinski, 2010; Fu, 1974; Gonzales and Thomason, 1978; Lazarek and Szczepaniak, 2016; Lin et al., 2009; Pavlidis, 1977; Rosenfeld, 1976; Shaw, 1969; Tadeusiewicz and Flasinski, 1991).

Graph grammars are commonly used to describe images, but rarely to recognize. The reason for this situation is due to the computational complexity of the syntactic analysis process (Skomorowski, 2013). However, graph grammars are a very interesting tool for creating recognition systems in which object classes are defined explicitly. Due to the known problem related to the computational complexity of the syntactic analysis algorithms, it is particularly important to select the appropriate class of graph grammars to ensure effective syntactic analysis. Grammatical graphs that have these features are ETPL class grammars (k) (embedding transformation preserved production ordered, k-left nodes unambiguous). Their effective syntactic analysis is possible, whose computational complexity is $O(n^2)$ (Flasinski, 1989; Flasinski, 2007; Skomorowski, 2013).

4.2 Method of Semantic Image Analysis in High-level Image Representation

The method of semantic image analysis in high-level representation (e.g., line segments) with the use of graph grammars enables recognition of a selected class of objects. A processing sequence has been proposed that can be adapted to the task of recognition of any class of objects that can be described using dedicated graph grammars.

The method requires two main steps – changing the representation of the image (to high-level) and designing the appropriate graph grammar. Processing schema (figure 11) is as follows:

1. Changing the representation of the image from pixel to the representation of a higher level (e.g., line segments).
2. Detection of ROI – regions of interest.
3. Representation of ROI using a graph.
4. Graph analysis with the use of dedicated graph grammar to recognize the object in the image.

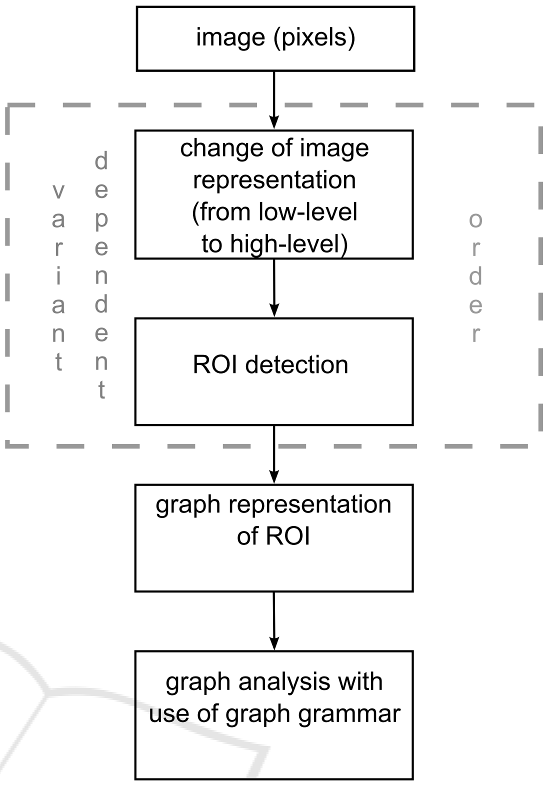

Figure 11: Processing scheme for the method of semantic image analysis in high-level representation with the use of graph grammars.

5 SUMMARY

In the paper, we have shown the importance of preparation data process, which demands to change image representation from low-level to high-level representation. Choosing proper representation strongly depends on recognition purpose. We have depicted that such representation may be used for the creation of object structure and its further recognition with the use of graph grammars, what was explained in the section 4 about the framework of recognition method.

The presented general framework of graph grammar based shape recognition method was successfully used for recognition of spiculated masses in mammographic images where the high-level image representation (line segments) and dedicated graph grammar were applied for recognition purpose (Lazarek, 2017).

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