Machine Learning Approach to the Synthesis of Identification Procedures for Modern Photon–Counting Sensors

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Abstract: The article presents the results of developing a machine learning approach to the problem of object identification (recognition) in images (data) recorded by photo-counting sensors. Such images are significantly different from the traditional ones, taken with conventional sensors in the process of time exposure and spatial averaging of the incident radiation. The result of radiation registration by photo-counting sensors (image) is rather a continuous stream of data, whose time frame is characterized by a relatively small number of photocounts. The latter leads to a low signal-to-noise ratio, low contrast and fuzzy shapes of the objects. For this reason, the well-known methods, designed for traditional image recognition, are not effective enough in this case and new recognition approaches, oriented to a low-count images, are required. In this paper we propose such an approach. It is based on the machine learning paradigm and designed for identifying (low count) objects given by point-sets. Consistently using a discrete set of coordinates of photocounts rather than a continuous image reconstructed, we formalize the problem in question as the problem of the best fitting of this set of counts, considered as the realization of a certain point process, to the statistical description of one of the previously registered point processes, which we call precedents. It is shown, that applying the Poisson point process model for formalizing the registration process in photo-counting sensors, it is possible to reduce the problem of object identification to the problem of maximizing the tested point--set likelihood with respect to the classes of modelling object distributions up to shape size and position. It is also demonstrated that these procedures can be brought to an algorithmic realization, analogous in structure to the popular EM algorithms. At the end of the paper we, for the sake of illustration, present some results of applying the developed algorithms to the identification of objects in a small artificial data base of low-count images.

1 INTRODUCTION

In recent decades there has been obvious progress in developing electromagnetic radiation (EMR) sensors (not only for visible light). The main trend of this progress – a decrease in the pixel pitch of the sensors – manifested itself already in the late 1960s with the creation of CCD cameras. In the early 1990s it was firmly established with the invention of CMOS solid-state image sensors. In addition to increasing the spatial resolution of the sensors, this trend also implies: an increase in the recording rate as well as in the dynamic range of sensors, their miniaturization, a reduction in the energy consumption, etc.

Because of this tendency, when the pitch of the pixels decreases, the detection of radiation acquires a more pronounced quantum nature, and in the limiting case it becomes the detection of individual photons (photoelectrons). It is a remarkable fact that this limiting case has already been achieved to some extent by using several technologies for manufacturing the so-called photon-counting sensors (Fossum et al., 2017). As an example, we can point out electron-multiplying charge-coupled devices (EMCCD) (Robbins, 2011), single-photon avalanche diodes (SPAD) (Dutton et al., 2016) and avalanche photodiodes in Geiger counter mode (GMAPD) (Aull et al., 2015).

In a sense, such digital photon-counting sensors can be considered as electronic analogue of classical photographic plates with their high image quality standard (Remez et al, 2016). At the same time, along with the achievement of a high quality

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comparable with classic images, photon-counting sensors also have other advantages. Indeed, as at registration of each photon the released photo-electron can be immediately transferred into microprocessor, there is no need to wait until the sufficient number of electrons is collected to form the image for a subsequent post-processing. So, there is no need for a long-time exposure (Chen and Perona, 2016).

This peculiarity substantially changes the basic concepts used in the theory and practice of image processing. For example, in classical applications of computer vision, the processing of incoming information is carried out in several steps. In the first step, a stream of photocounts is accumulated, then, based on them, the image is restored and in the last step the image is transferred to specialized procedures for extracting certain features, parameters of the objects. On the contrary, when computer vision is realized with the help of photon-counting sensors, it is possible, as noted above, to synchronize the data recording process and the process of feature extracting directly from the stream of photocounts. So, in the case of photon-counting sensors, the need for an intermediate image reconstruction disappears and a new concept of "vision without an image" comes into being (Chen and Perona, 2016)

In this paper, in the context of such a new concept we present the results of a study in problems of identification (recognition) of objects (targets) based on the information obtained directly from the stream of photocounts. Consistently using a discrete set of coordinates of photocounts rather than a continuous image reconstructed, we formalize the problem in question as the problem of the best fitting of this set of counts, considered as the realization of a certain point process, to the statistical description of one of the previously registered point processes, which we call precedents. To solve this problem, we developed an original approach based on the methods of statistical (machine) learning (Hastie et al., 2009). In the framework of the proposed approach, the problem under consideration is treated as a task of the maximum likelihood matching of test counts with the probability distribution of counts for one of the available precedents, which was formed earlier in the training process. An important advantage of the proposed approach is that it can be used to develop effective computational algorithms. One of such identification algorithms, structurally similar to the algorithms of the well-known EM family, is presented in Section 4.

2 STATISTICAL BASES OF PHOTON-COUNTING IDENTIFICATION

For the proposed approach to the identification of objects to be mathematically justified, it is necessary to formalize the process of registering radiation with photon-counting sensors. In other words, it is necessary to choose such a mathematical model, that most adequately relates the set of registered photocounts $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ to the intensity $I(\vec{x})$ of the radiation incident on the sensitive surface Ω of the sensor. Here the $\vec{x} \in \Omega$ denotes the continuous coordinates of the point in some coordinate system in the plane of registration, $\vec{x}_i$ are the coordinates of the pixels corresponding to the registered photocounts.

In the ideal case, when the size of the pixels can be considered arbitrarily small − in the so-called continuous model of the photodetection process (Goodman, 2015), the coordinates of the counts $\vec{x}_i$ are a set of random vectors (of random number n), even for a given, nonrandom intensity $I(\vec{x})$. The most general mathematical model for such phenomenon is a random point process (Streit, 2010). From the semiclassical theory of photodetection, it is well-known that among such processes the Poisson point process (PPP) simulates the mechanisms of photocounts generation in the most adequate manner (Mandel and Wolf, 1995). A remarkable feature of PPP is that a complete statistical description of counts can be determined by the only function − the count density $\lambda(\vec{x})$ − all finite-dimensional probability distributions of count coordinates are expressed in terms of it (Streit, 2010):

$$\rho(n, \vec{x}_1, \ldots, \vec{x}_n) = \\ = \frac{1}{n!}\prod_{i=1}^{n} \lambda(\vec{x}_i) \times \exp\left\{-\int_{\Omega} \lambda(\vec{x}) d\vec{x}\right\} \quad (1)$$

Quantum theory specifies that the density $\lambda(\vec{x})$ in (1) is related by a simple expression to the intensity $I(\vec{x})$ of the incident radiation, namely, $\lambda(\vec{x}) = \eta T I(\vec{x})/h\bar{\nu}$, where η is the coefficient of quantum efficiency of the detector material, T is a frame readout time, $\bar{\nu}$ is the central frequency of the incident radiation and h is the Planck constant (Goodman, 2015).

The above relation, expressing $\lambda(\vec{x})$ through $I(\vec{x})$, together with the distributions (1), defines the model relating the input $I(\vec{x})$ of an ideal (continuous) detector to the statistics of its output −

PPP count coordinates $\{\vec{x}_i\}$. For real sensors with pixels of finite dimensions, the corresponding relation is obtained by integrating (1) over all pixels, taking into account the exact numbers of photocounts $\{n_i\}$ registered by each of them. Note that the model of the real sensor will be close to the ideal model (1) in those cases when the probabilities of two or more photocounts ($n_i > 1$) are small compared to the probability of one. It is easy to show that the necessary condition for that is $\lambda(\vec{x})S < 1$, where S is the area of an individual pixel. Thus, when registering weak radiation $I(\vec{x}) \to 0$, or in high frame rate imaging $T \to 0$, or in the case of photon-counting sensors $S \to 0$, statistical description (1) is an adequate model for the photon-counting sensors.

For this reason, the set of finite-dimensional distributions (1) could be chosen as the basis for the statistical synthesis of recognition / identification methods for photon-counting sensors. However, one more step can be made in this direction if we slightly extend the formulation of the problem. Namely, let us take as the identity relation of objects the similarity of the form of their radiation intensities regardless of the total brightness of each of them (see discussion of this formulation in (Antsiperov, 2018.)). From the statistical point of view this means, that instead of the joint distributions (1) the conditional distributions of the count coordinates $\{\vec{x}_i\}$ for a given total number n should be chosen as the basis:

$$\rho(\vec{x}_1, \dots, \vec{x}_n | n) = \prod_{i=1}^{n} \rho(\vec{x}_i)\,,$$
$$\rho(\vec{x}) = \frac{\lambda(\vec{x})}{\int_\Omega \lambda(\vec{x}) d\vec{x}} = \frac{I(\vec{x})}{\int_\Omega I(\vec{x}) d\vec{x}}\,. \quad (2)$$

The distributions (2) express the known property of the PPP − the conditional joint distribution of the counts decomposes into a product of identical distributions of each of them (Streit, 2010). In other words, for a given n, the counts $\{\vec{x}_i\}$ are a set of independent identically distributed (iid) random vectors. Such samples are the source data for most statistical approaches, which makes the given formalization of the problem very attractive. Moreover, the count distribution $\rho(\vec{x})$ (2) coincides directly with the (normalized) intensity $I(\vec{x})$ and does not depend on either the detector material η, or the carrier frequency $\bar{\nu}$ of the radiation, or the registration time T and pixel area S. In view of these remarkable properties of the distributions (2), they have a universal character and for this reason they were chosen as the statistical basis of our approach.

3 PRECEDENT DESCRIPTION BY GAUSSIAN MIXTURES

As noted in the introduction, the proposed approach to objects / targets identification is oriented toward machine learning methods (Hastie et al, 2009). Therefore, the available precedents are initially also represented by registered (in the training step) sets of photocounts, their statistics is also given by distributions (2). However, the use of the direct registration data of precedent in identification procedures is not rational, both because of the low efficiency of resulting methods, and because of wasteful use of computing resources (large amounts of data, respectively, large search times for target precedents, etc.). The obvious way here is to use the direct registered data to form some compact descriptions of precedent radiation intensities $I(\vec{x})$ and then to use these descriptions to identify the tested sets of photocounts.

Any restoration of $I(\vec{x})$ from the registered data $\{\vec{x}_i\}$ is an inverse problem to (2), therefore it belongs to the class of the so called ill-posed problems. The standard approach here is to approximate the intensity, in our case the normalized intensity $\rho(\vec{x})$, by model distribution $p(\vec{x} \mid \vec{\theta})$, belonging to some parametric family of distributions with a relatively small number of parameters $\vec{\theta} = \{\theta_1, \dots, \theta_m\}$. A flexible tool for modelling multivariate distributions of a rather arbitrary type is a family of Gaussian mixture models (GMM) (Mengersen, et al, 2011), that was chosen as the basis for intensity approximations in the approach under discussion. In this connection, it is assumed that the intensity of each (k−th) precedent can be approximated by the sum (mixture) of N_k two-dimensional Gaussian (normal) distributions:

$$p_k(\vec{x}|\vec{\theta}) =$$
$$= \sum_{j=1}^{N_k} p_{k,j} \frac{1}{2\pi} \sqrt{\det\{A_{k,j}\}} \exp\left\{-\frac{1}{2} Q_{k,j}(\vec{x})\right\}, \quad (3)$$
$$Q_{k,j}(\vec{x}) = \left(\vec{x} - \vec{m}_{k,j}\right)^T A_{k,j} \left(\vec{x} - \vec{m}_{k,j}\right),$$

where parameters $\vec{\theta}$ include both the number N_k of mixture components and the set of N_k triples $\{(p_{k,j}, \vec{m}_{k,j}, A_{k,j})\}$, that are probabilities $p_{k,j}$ of belonging count $\vec{x}$ to the component j, whose mean and the matrix of the quadratic form $Q_{k,j}(\vec{x})$ are respectively $\vec{m}_{k,j}$ and $A_{k,j}$.

The choice of the GMM (3) in addition to the convenience of modelling / analysis is also convenient because within the framework of machine learning there are effective algorithms to find maximum likelihood (ML) parameters of the mixture. The group of such algorithms includes popular EM-algorithms (Gupta, 2010). For the version of the EM-algorithm used in our approach in the training step to form the precedent description, the number of components N_k was chosen manually, other parameters were recursively calculated according to the following scheme:

```
E-step:
   for i=1 to n
      for j=1 to Nk
```

$$Q_{k,j}^{(m)}(\vec{x}_i) = \left(\vec{x}_i - \vec{m}_{k,j}^{(m)}\right)^T A_{k,j}^{(m)} \left(\vec{x}_i - \vec{m}_{k,j}^{(m)}\right) \ ;$$

$$V_{j|i}^{(m+1)} = \frac{1}{\Sigma_V} p_{k,j}^{(m)} \sqrt{\det\left\{A_{k,j}^{(m)}\right\}} \exp\left\{-\frac{1}{2} Q_{k,j}^{(m)}(\vec{x}_i)\right\};$$

$$\Sigma_V = \sum_{j=1}^{N_k} p_{k,j}^{(m)} \sqrt{\det\left\{A_{k,j}^{(m)}\right\}} \exp\left\{-\frac{1}{2} Q_{k,j}^{(m)}(\vec{x}_i)\right\};$$

```
      end;
   end;
M-step:
   for j=1 to Nk
```

$$\begin{aligned} p_{k,j}^{(m+1)} &= \tfrac{1}{n} \textstyle\sum_{i=1}^{n} V_{j|i}^{(m+1)}, \\ \vec{m}_{k,j}^{(m+1)} &= \tfrac{1}{n p_{k,j}^{(m+1)}} \textstyle\sum_{i=1}^{n} V_{j|i}^{(m+1)} \vec{x}_i \ ; \\ \left[A_{k,j}^{(m+1)}\right]^{-1} &= \tfrac{1}{n p_{k,j}^{(m+1)}} \textstyle\sum_{i=1}^{n} V_{j|i}^{(m+1)} \times \\ &\times \left(\vec{x}_i - \vec{m}_{k,j}^{(m+1)}\right)\left(\vec{x}_i - \vec{m}_{k,j}^{(m+1)}\right)^T \end{aligned} \tag{4}$$

```
   end;
```

4 IDENTIFICATION PROCEDURE FOR GAUSSIAN MIXTURES

Having established the form of the precedent descriptions (3) and the procedure of such description calculations (4), as a final step in describing the approach proposed, it is necessary to define the tested data identification procedure for these descriptions. The standard step in this direction is the appropriate choice of a quantitative measure of correspondence, similarity between the tested PPP counts $\{\vec{x}_i\}$ and precedent descriptions from the generated database (DB). It is reasonable that the candidate for the role of such a measure of similarity is, in view of (2) and (3), the (logarithmic) likelihood function:

$$L_k(\{\vec{x}_i\}) = \ln\left[\textstyle\prod_{i=1}^{n} p_k\left(\vec{x}_i | \vec{\theta}^{(*)}\right)\right] \tag{5}$$

where $\{\vec{x}_i\}$ are the coordinates of the counts of the PPP tested, and $\vec{\theta}^{(*)}$ are the parameters of the k–precedent description, obtained with the help of (4). Theoretically, calculating measure $L_k(\{\vec{x}_i\})$ for all k–precedents, we could use the maximum likelihood (ML) approach as the procedure of identification. Namely, considering $L_k(\{\vec{x}_i\})$ for a given realization $\{\vec{x}_i\}$ as a function of k, we can identify the tested PPP with the precedent $k^{(*)}$, which provides the maximum value for the similarity measure (5):

$$k^{(*)} = \arg\max_{k} L_k(\{\vec{x}_i\}) \tag{6}$$

However, from a practical point of view, such an identification procedure is unrealistic. The problem here is that in the hypothetical DB the same object with different locations, scales and foreshortenings would have different precedent descriptions. The solution to the problem is the identification of the tested PPP data $\{\vec{x}_i\}$ not with a specific precedent description, but with a whole class of similar descriptions, each of which is obtained from the single one by some group of transformations. For example, if we restrict ourselves to the group of affine transformations $\vec{x} \to \left(\vec{x} + \vec{t}\right)/s$ (translate to vector $\vec{t}$ and scale in s times) and give some a priori distribution $\rho_{apr}\left(\vec{t}, s\right)$, then identification can still be carried out on the basis of the ML approach (6), but as a measure of similarity we should use $\bar{L}_k(\{\vec{x}_i\})$ – the logarithm of the averaging over $\rho_{apr}\left(\vec{t}, s\right)$ of all descriptions $p_k\left(s\vec{x} - \vec{t} | \vec{\theta}^{(*)}\right)s^2$, obtained from $p_k\left(\vec{x} | \vec{\theta}^{(*)}\right)$ by the group actions:

$$\begin{aligned} \bar{L}_k(\{\vec{x}_i\}) = \ln\Big[\textstyle\int\int \rho_{apr}\left(\vec{t}, s\right) \times \\ \times \textstyle\prod_{i=1}^{n} P_k\left(\vec{x}_i | \vec{t}, s\right) d\vec{t} ds\Big] , \end{aligned} \tag{7}$$

$$\begin{aligned} P_k\left(\vec{x} | \vec{t}, s\right) &= p_k\left(s\vec{x} - \vec{t} | \vec{\theta}^{(*)}\right)s^2 = \\ &= \textstyle\sum_{j=1}^{N_k} p_{k,j}^{(*)} \frac{1}{2\pi} \sqrt{\det\left\{A_{k,j}^{(*)}\right\}} \times \\ &\times \exp\left\{-\tfrac{1}{2} Q_{k,j}^{(*)}\left(s\vec{x} - \vec{t}\right)\right\} s^2 \, . \end{aligned}$$

Using the definition (7) and utilizing the variational Bayesian approach, we have synthesized the following EM-like algorithm (Antsiperov, 2019) for calculating the identification procedure similarity measure $\bar{L}_k(\{\vec{x}_i\})$ (7):

```
E-step:
   for i=1 to n
      for j=1 to Nk
```

$$Q_{k,j}^{(m)}(\vec{x}_i) = \left(\bar{S}^{(m)}\,\vec{x}_i - \vec{T}^{(m)} - \vec{m}_{k,j}^{(*)}\right)^T \times \times A_{k,j}^{(*)}\left(\bar{S}^{(m)}\,\vec{x}_i - \vec{T}^{(m)} - \vec{m}_{k,j}^{(*)}\right);$$

$$V_{j|i}^{(m+1)} = \frac{1}{\Sigma_V} p_{k,j}^{(*)} \sqrt{\det\left\{A_{k,j}^{(*)}\right\}} \exp\left\{-\frac{1}{2} Q_{k,j}^{(m+1)}(\vec{x}_i)\right\};$$

$$\Sigma_V = \sum_{j=1}^{N_k} p_{k,j}^{(*)} \sqrt{\det\left\{A_{k,j}^{(*)}\right\}} \exp\left\{-\frac{1}{2} Q_{k,j}^{(m+1)}(\vec{x}_i)\right\};$$

```
    end;
  end;

M-step:
  for j=1 to Nk
```

$$\pi_j^{(m+1)} = \frac{1}{n}\sum_{i=1}^{n} V_{j|i}^{(m+1)};$$

$$\vec{X}_j^{(m+1)} = \frac{1}{n\pi_j^{(m+1)}}\sum_{i=1}^{n} V_{j|i}^{(m+1)}\vec{x}_i;$$

$$R_j^{(m+1)} = \frac{1}{n\pi_j^{(m+1)}}\sum_{i=1}^{n} V_{j|i}^{(m+1)} \times \times \left(\vec{x}_i - \vec{X}_j^{(m+1)}\right)\left(\vec{x}_i - \vec{X}_j^{(m+1)}\right)^T; \tag{8}$$

$$\sigma^{(m+1)} = \sqrt{\frac{2}{Tr\left[\sum_{j=1}^{N_k}\pi_j^{(m+1)}A_{k,j}^{(*)}R_j^{(m+1)}\right]}};$$

$$\vec{X}^{(m+1)} = \left[\hat{A}^{(*)}\right]^{-1}\sum_{j=1}^{N_k}\pi_j^{(m+1)}A_{k,j}^{(*)}\vec{X}_j^{(m+1)};$$

$$\vec{M}^{(m+1)} = \left[\hat{A}^{(*)}\right]^{-1}\sum_{j=1}^{N_k}\pi_j^{(m+1)}A_{k,j}^{(*)}\vec{m}_{k,j}^{(*)};$$

where $\hat{A}^{(*)} = \sum_{j=1}^{N_k}\pi_j^{(m+1)}A_{k,j}^{(*)}$;

$$\Sigma = \frac{\sum_{j=1}^{N_k}\left(\vec{m}_{k,j}^{(*)} - \vec{M}^{(m+1)}\right)^T \pi_j^{(m+1)} A_{k,j}^{(*)}\left(\vec{m}_{k,j}^{(*)} - \vec{M}^{(m+1)}\right)}{\sum_{j=1}^{N_k}\left(\vec{X}_j^{(m+1)} - \vec{X}^{(m+1)}\right)^T \pi_j^{(m+1)} A_{k,j}^{(*)}\left(\vec{X}_j^{(m+1)} - \vec{X}^{(m+1)}\right)};$$

$$\varepsilon = \frac{\left[\sigma^{(m+1)}\right]^{-2}}{\sum_{j=1}^{N_k}\left(\vec{X}_j^{(m+1)} - \vec{X}^{(m+1)}\right)^T \pi_j^{(m+1)} A_{k,j}^{(*)}\left(\vec{X}_j^{(m+1)} - \vec{X}^{(m+1)}\right)};$$

$$\mu_\sigma = \frac{\varepsilon}{1+\varepsilon}\,; \quad \mu_\Sigma = \frac{1}{1+\varepsilon}\,;$$

$$\bar{S}^{(m+1)} = \mu_\sigma \sigma^{(m+1)} + \mu_\Sigma \Sigma^{(m+1)}\,;$$

$$\vec{T}^{(m+1)} = \bar{S}^{(m+1)}\vec{X}^{(m+1)} - \vec{M}^{(m+1)}\,;$$

```
  end;
```

Note that the essence of the synthesized algorithm (8) is that it recursively calculates the ML values $\vec{T}^{(*)}$ and $S^{(*)}$ of the parameters $\vec{t}$ and s, which specify the most likelihood fitting of the registered $\{\vec{x}_i\}$ for the description $P_k\left(\vec{x}\middle|\vec{T}^{(*)}, \bar{S}^{(*)}\right)$ (7) from the class defined by the description of $p_k\left(\vec{x}\middle|\vec{\theta}^{(*)}\right)$ (3). Correspondingly, it can be shown that for some natural assumptions (see (Antsiperov, 2019)) the similarity measure $\bar{L}_k(\{\vec{x}_i\})$ (7) converges to $L_k(\{\vec{x}_i\})$ (5) with the parameters $\vec{m}_{k,j}^{(**)} = \left(\vec{m}_{k,j}^{(*)} + \vec{T}^{(*)}\right)/\bar{S}^{(*)}$ and $A_{k,j}^{(**)} = \bar{S}^{(*)2}A_{k,j}^{(*)}$:

$$\bar{L}_k(\{\vec{x}_i\}) = \ln\left[\prod_{i=1}^{n} P_k\left(\vec{x}_i\middle|\vec{T}^{(*)}, \bar{S}^{(*)}\right)\right] \cong \cong L_k(\{\vec{x}_i\}) \tag{9}$$

5 SIMPLE EXPERIMENTS ON IDENTIFICATION OF OBJECTS ON ARTIFICIAL IMAGES

In this paper, for the purpose of illustrating the proposed identification method, the results of a computational experiment for a small, artificially formed database are presented. Descriptions of precedents in this database were obtained in processing registration data of the objects with the simple structure. The latter are chosen as binary images from the well-known base "MPEG-7 Core Experiment CE-Shape-1" (Latecki et al., 2000). This image base is usually used to compare the shape recognition algorithms and includes 70 categories of objects with 20 images in each category.

From the MPEG-7 database were arbitrarily chosen as precedents images of 5 different categories - "device1-7", "device5-16", "heart-14", "shoe-8", "turtle-9". For unification, they were reduced to a size of 500 × 500 pixels and centred in the frames. Since the form of the object's intensity is simple - uniform over the object, the simulation of registration data of precedents was trivial – 1000 of the first uniformly distributed across the frame random points that fell into the object were selected.

The example of the precedent object and corresponding registration data (1000 samples) are presented in Figure 1.

To form a description of the selected precedents in the form of Gaussian mixtures with $N_k = 6$ components (for all precedents), the EM-algorithm discussed above, in Section 3, was applied. Figure 2 shows the results of initialization and application of the EM-algorithm for the formation of the "device1-7" precedent description (see Figure 1). The components of the initial and resulting description are represented by centres and ellipses of constant level ~ e^{-2} from the absolute maximum of the entire Gaussian mixture.

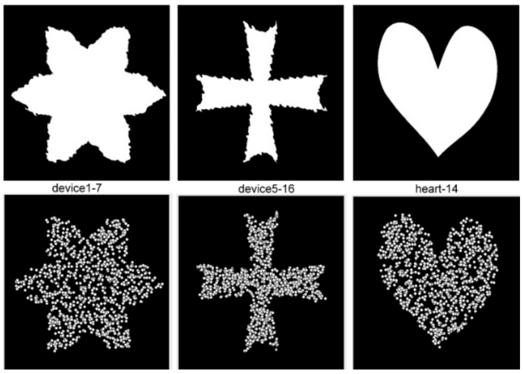

Figure 1: Precedents from the "MPEG-7" database (Latecki et al., 2000) represented by 500 × 500 binary images (top) and their registration model data as 1000 samples, uniformly distributed over their shapes (bottom).

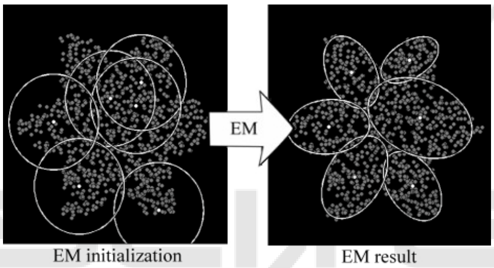

Figure 2: The results of initialization (on the left) and the application (on the right) of the EM – algorithm for the formation of the "device1-7" precedent description. The components of the corresponding mixtures are represented by their centres (points) and lines (ellipses) of a constant level, approximately 0.1 times the maximum value of the entire distribution.

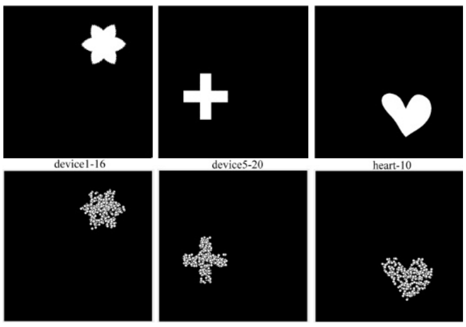

Figure 3: The tested objects from the database "MPEG-7" (Latecki et al, 2000) are selected from the same categories as the precedents in Figure 1 but shifted and reduced randomly (top) and model data of their registration in the form of uniformly distributed 300 counts (bottom).

As the tested objects, the images "device1-16", "device5-20", "heart-10", "shoe-11", "turtle-7" were selected from the same 5 categories as the precedents. All of them were reduced to the same size of 300 × 300 points (with the scale = 0.6) and randomly disposed from the centre of the frames 800 × 800. The simulation of registration data was a selection of 300 generated random points, uniformly distributed within the shapes of the objects. Tested objects and the corresponding registration data (300 samples) are presented in Figure 3.

For each of the tested objects an array of similarity measure values was calculated for each of the precedent. Essentially, as noted above, the similarity measures calculated on the basis of the proposed procedure are (logarithmic) likelihood functions $L_k(\{\vec{x}_i\})$ (9) assuming that $\{\vec{x}_i\}$ are the result of registering the samples of the k–precedent affine transformation with maximum likelihood parameters $\vec{T}^{(*)}$ and $S^{(*)}$ (8). Recalculated with $\vec{T}^{(*)}$ and $S^{(*)}$ (their own for each precedent) descriptions $P_k(\vec{x}|\vec{t},s)$ (7) for $k =$ "device1-7", "device5-16", "heart-14" and the values of the corresponding $\bar{L}_k(\{\vec{x}_i\}) - \text{LnLf}$ (9) for the registration data of the tested object "device5-20" are presented in Figure 4.

The full results of the calculation based on the proposed identification procedure, including all tested objects are presented in Table 1.

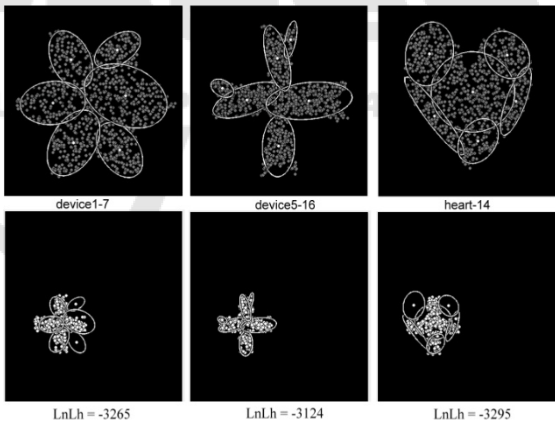

Figure 4: Descriptions of precedents from the database (top) and their recalculated using the found maximum-likelihood parameters $\vec{T}^{(*)}$ and $S^{(*)}$ (8) descriptions (bottom) for the registration data of the tested object "device5-16" (see Figure 3). Below for each precedent the logarithm of the mean likelihoods denoted as "lnLh" are represented.

Note that the identification based on the criterion (6) for the set of all precedents (categories) in all cases of tested objects (in rows of Table 1) is correct. By the way, there is no such regularity among the values of the log-likelihood function for a given precedent. For example, in the case of precedent "heart-14" (in the corresponding column of Table 1), the probabilities of data for objects "device1-16", "shoe-11", "turtle-7" are higher than the probability of data for an object "heart-10" of the same category as the precedent. However, in the remaining columns, the expected behavior still occurs. The marked asymmetries can be related to the inequality of data volumes for precedents (1000) and for the tested objects (300).

Table 1: Values of the similarity measure (log likelihood) for tested objects of different categories (300 samples data).

Tested objects ↓	**Precedents**				
	device1-7	device5-16	heart-14	shoe-8	turtle-9
device1-16	-3153	-3458	-3228	-3296	-3287
device5-20	-3265	-3124	-3295	-3381	-3271
heart-10	-3362	-3451	-3250	-3437	-3332
shoe-11	-3231	-3272	-3199	-3024	-3246
turtle-7	-3173	-3330	-3165	-3242	-3062

Of course, for an objective, statistically reliable assessment of the proposed approach, it is necessary to test it on much larger databases, however, the results obtained already suggest optimistic predictions regarding its potential.

6 CONCLUSIONS

It is shown in the paper, that the formalization of the process of registering radiation by photon-counting sensors by the model of Poisson point processes (Streit, 2010) is most adequate from the physical (quantum) point of view (Goodman, 2015) and extremely fruitful for the statistical approaches (Hastie, et al, 2009). On this basis, using the principles of machine learning (Mengersen, et al, 2011), we succeeded in developing an effective approach to the synthesis of procedures for identification (recognition) of objects belonging to a well-proven family of EM-algorithms. Numerical simulation (Antsiperov, 2019) showed that the synthesized identification procedure has a high convergence rate: for the complexity of describing precedents of Gaussian mixtures with $N_k \sim 10$ components recursive calculations (8), converge in less than 10 iterations.

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