Performance Evaluation of Universities and Colleges based on Method of Principal Component Analysis and Data Envelopment Analysis

Yan Xia, Xinlin Wu and Hui Feng
Shanghai Joint Laboratory for Discipline Evaluation, Shanghai Education Evaluation Institute, Shanghai, China

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Abstract: The implementation of performance evaluation on higher education is beneficial to optimize resource allocation and to promote sustainable development of higher education. It is challenging how to establish a scientific model of performance evaluation on universities and colleges objectively. This paper proposes a method of performance evaluation based on Data Envelopment Analysis with dimension reduction of performance evaluation indicators based on Principal Component Analysis. An automatic system is developed, implementing the method and analysing data from universities and colleges in Shanghai. It provides advice and guidance for performance evaluation, and establishes foundation for higher education development strategy.

1 INTRODUCTION

With the reform of public management system, the expansion of demand for education resources, and the continuous growth of education financial input in China, the government and the society pay more attention to the performance of higher education. Performance evaluation of higher education is the core part of performance management system (Sarrico et al., 2010). With the evaluation result as basis of decision making of higher education management, it can improve the utility efficiency of funds and optimize resource allocation. Thus it promotes development of construction and optimizes discipline distribution in universities and colleges (Wang and Feng, 2012). It has become a hotspot in higher education field how to implement systematic and scientific performance evaluation to promote the development of universities and colleges. Currently there are some problems in performance evaluation of universities and colleges. Firstly, the evaluation indicator system is complicated, in which some indicators have implicit dependency on others. Secondly, the evaluation process is easily influenced by subjective factors. Thirdly, there are uniform evaluation criteria for different types of universities and colleges. Therefore the evaluation result isn’t so inaccurate due to the above reasons.

This paper proposes an optimized performance evaluation method based on Principal Component Analysis (PCA) and Data Envelopment Analysis (DEA). PCA is used to remove dependent indicators so as to simplify the performance evaluation indicator system by reducing dimension. DEA is used to establish model of performance evaluation. Then it analyses data from universities and colleges by the method comprehensively and systematically. It enriches the content of performance evaluation of universities and colleges. It provides advice and guidance for scientific development strategy in universities and colleges.

2 RELATED WORK

Performance evaluation of universities and colleges is carried out in about 1980s, the result of which is considered as significant evidence of resource allocation and management mode improving. Higher Education Funding Council for England (HEFCE), Scottish Higher Education Funding Council (SHEFC), Higher Education Funding Council for Wales (HEFCW) and the Department of

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Employment and Learning, Northern Ireland etc organize Research Assessment Exercise (RAE), responsible for evaluating the quality of research for higher education institutes in UK and allocating funds (Kitagawa and Lightowler, 2013). Association of universities in the Netherlands (VSNU) is responsible for the process of external evaluation of universities (Bosch and Christine, 2000). Australian University Quality Agency (AUQA) and Australia Higher Education Evaluation Committee do performance evaluation for universities from various perspectives (CWA, 2003). The performance evaluation of higher education in China officially began from 21st Century. *Performance Evaluation Report of Universities in China* was published by National Institute of Education Sciences. Performance evaluation of 72 universities which are led by Ministry of Education of China was conducted in the report (NIES, 2009). The kinds of performance evaluation above mainly use traditional expert evaluation method and statistical analysis method combining with input-output model. However the evaluation indicator system is complicated with implicit dependent indicators. The evaluation result is easy to be influenced by the subjectivity etc during the evaluation process (Afsharian and Emrouznejad, 2018) .

In order to solve the existing problems in performance evaluation of universities and colleges, this paper proposes a new evaluation method based on PCA and DEA. An automatic system is developed, implementing the method and analysing data from universities and colleges.

3 PERFORMANCE EVALUATION METHOD BASED ON DEA AND PCA

The performance evaluation method of universities and colleges is based on DEA and PCA. The theory of DEA and PCA is introduced firstly. The method is then described in details in application.

3.1 Model of PCA

PCA is a multivariate statistical analysis method that converts multiple indicators into fewer comprehensive independent ones (Warmuth and Kuzmin, 2008). It is widely used in the fields of economics and management science (Abdi and Williams, 2010). In multi-indicator systems, there are always dependencies among indicators, which reveal overlapping information. PCA takes the idea of dimension reduction to simplify the situation (Liu et al., 2017). Several principal independent indicators are chosen to represent the whole components, which contains information as much as possible in the system. The basic steps of PCA are as follows.

1. Standardize sample data.

Let $X = (x_{ij})_{n*p}\ i \in \{1, n\}, j \in \{1, p\}$ be the sample data matrix. n is the sample size. p is the number of indicators. Standardize matrix X as $Y = (y_{ij})_{n*p}$.

$$y_{ij} = \frac{x_{ij} - \frac{1}{n}\sum_{i=1}^{n} x_{ij}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2}} \tag{1}$$

$$\bar{x}_j = \frac{1}{n}\sum_{k=1}^{m} x_{kj} \tag{2}$$

2. Set dependency matrix of indicators.

Let $R = (r_{ij})_{p*p}\ i, j \in \{1, p\}$, r_{ij} is the dependency coefficient of indicators i and j, which expresses the correlation between them.

$$r_{ij} = \frac{\sum_{k=1}^{n}\left[(x_{ki} - \bar{x}_i) * (x_{kj} - \bar{x}_j)\right]}{\sqrt{\sum_{k=1}^{n}(x_{ki} - \bar{x}_i)^2} * \sqrt{\sum_{k=1}^{n}\left(x_{kj} - \bar{x}_j\right)^2}} \tag{3}$$

3. Find the eigenvalue and eigenvector of matrix R, and get the principal component expression.

According to eigen equation $|\lambda E - R| = 0$, p eigenvalues are obtained, $\lambda_g\ g \in \{1, p\}$, which are arranged as $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_p \geq 0$. λ_g is the variance of principal indicator, which indicates its importance in evaluation indicator system. Each eigenvalue corresponds to an eigenvector $L_g = \left(l_{g1}, l_{g2}, \ldots, L_{gp}\right)\ g \in \{1, p\}$. Principal indicators are converted to principal component expression, F_g.

$$F_g = L_g * Y^T \tag{4}$$

4. Find the variance contribution rate to determine the number of principal components.

Due to the dependency of indicators, k (k<p) principal components are chosen to do performance evaluation. If the accumulation variance contribution rate, VCR, is greater or equal to 95%, almost all the information of indicator system is contained in these principal components.

$$VCR = \frac{\sum_{g=1}^{k} \lambda_g}{\sum_{g=1}^{p} \lambda_g} \geq 95\% \quad (5)$$

3.2 Model of DEA

DEA is a system evaluation method based on relative efficiency (Ramanathan, 2003)(Chen and Zhu, 2018). It overcomes limits of existing methods (Bouwmans et al., 2015). The universities and colleges which are participated in the performance evaluation are considered as decision making unit (DMU). The operation process can be considered as converting the input resource to output (Avkiran, 2001). Therefore the performance of universities and colleges can be evaluated by the input and output. After simplifying the evaluation indication system by dimension reduction of PCA, the basic steps of DEA are as follows.

1. Set the input and output indicators with principal components.

Let $\mathrm{Input_j} = (x_{1j}, x_{2j} \ldots x_{sj})$, $\mathrm{Onput_j} = (y_{1j}, y_{2j,} \ldots y_{tj})$ $j \in \{1, n\}$. They are the input and output of jth university or college.

2. Calculate the comprehensive efficiency, θ_j, and technical efficiency, δ_j, by C2R and VRS model of DEA.

$$\begin{cases} \min \theta_j \\ s.t. \sum input_j * \lambda_j + s_j = \theta_j * input_j \\ \sum output_j * \lambda_j - z_j = \theta_j * output_j \\ \lambda_j \geq 0, \\ s_j \geq 0,\ z_j \geq 0 \end{cases} \quad (6)$$

$$\begin{cases} \min \delta_j \\ s.t. \sum input_j * \lambda_j + s_j = \delta_j * input_j \\ \sum output_j * \lambda_j - z_j = \delta_j * output_j \\ \sum \lambda_j = 1 \\ \lambda_j \geq 0, \\ s_j \geq 0,\ z_j \geq 0 \end{cases} \quad (7)$$

s_j is the input redundancy, while z_j is the output deficiency.

3. Determine the performance of universities and colleges.

Definition 1

If $\theta_j = 1$, it is weak efficiency of DMU_j for C^2R model.

If $\theta_j = 1$ && $s_j = z_j = 0$, it is efficiency of DMU_j for C^2R model.

Definition 2

If $\delta_j = 1$, it is weak efficiency of DMU_j for VRS model.

If $\delta_j = 1$ && $s_j = z_j = 0$, it is efficiency of DMU_j for VRS model.

Definition 3

Set $\eta_j = \theta_j / \delta_j$ as the performance efficiency rate. According to (6) and (7), $\theta_j \leq \delta_j$, $0 < \eta_j \leq 1$

Definition 4

If $\theta_j = \delta_j$, performance efficiency rate of DMU_j is efficient.

If $\theta_j < \delta_j$ && $\sum \lambda_j > 1$, performance efficiency rate of DMU_j decreases.

If $\theta_j < \delta_j$ && $\sum \lambda_j > 1$, performance efficiency rate of DMU_j increases.

Use the calculation of (6) and (7) to do performance evaluation of universities and colleges.

4 APPLICATION OF THE METHOD ON PERFORMANCE EVALUATION OF UNIVERSITES

4.1 Evaluation Indicator System of University and College Performance

This paper uses the optimized evaluation indicators from Performance Evaluation Indicator System for reference, which is promulgated by National Institute of Education Sciences (NIES, 2009). The content of Performance Evaluation Indicator System is shown in Table 1. The Evaluation Indicator System is composed of 2 parts, input indicators and output indicators. Input indicators consist in 3 primary indicators, including Human Resource, Financial Resource, and Material Resource. Output indicators consist in 4 primary indicators, including Personnel Cultivation, Scientific Research, Social Service, Development and Characteristics. Each primary indicator is composed of several secondary indications, 14 secondary indicators in all. Each secondary indicator contains some observation points with different weight, which can be considered as tertiary indicators. For example, Personnel Cultivation, one of the primary indicators, contains 2 secondary indicators. There are 15 observation points in Cultivation Quality, one of the secondary indicators, such as Survey result of student satisfaction, Number of teaching achievement award and so on. There are 68 observation points in all.

Table 1: Performance Evaluation Indicator System.

Parts	Primary	Secondary	Observation Point
Input	Human Resource	Teacher Structure	Number of full-time teachers and researchers
			Number of part-time teachers in enterprises
			Outstanding teachers
			Ratio of full-time teachers with oversea learning experiences
			Ratio of doctoral degree in full-time teachers and researchers
			Number of professor and associate professor
			…
		Student Structure	Average score in college entrance examination
			Ratio of master students who graduate from first class universities
			Ratio of master students who graduate from excellent universities
			…
	Financial Resource	Financial Income	Amount of government funds (RMB)
			Amount of business expenses (RMB)
			…
		Tuition and other Income	Amount of appropriation for education (RMB)
			…
	Material Resource	Teaching Area	Covering area
			…
		Teaching Resource	Number or volume of books
			Value of fixed assets
			Amount of experimental facilities
			…
Output	Personnel Cultivation	Cultivation Scale	Number of students
			Number of graduate students
			…
		Cultivation Quality	Survey result of student satisfaction
			Number of teaching achievement award
			Number of foreign students with academic background
			Rate of employment signature
			Survey result of employer satisfaction
			…
	Scientific Research	Research Funds	Amount of science and technology funds (RMB)
			…
		Research Achievement	Number of monograph
			Number of academic papers published domestically and internationally
			Number of science and technology projects
			Number of patent authorizations
			…
	Social Service	Research	Number of transformation of achievement
			Number of contract of technology transfer
			Income of technology transfer (RMB)
			Number of consulting report
			…
		Teaching	Teaching resources open to society
			…
	Development Characteristics	Financial Management	…
		Innovation and Improvement	…

4.2 Data Selection

In order to ensure authenticity, reliability and authority, all the data related to the Performance Evaluation Indicator System are from reports of the educational administrative department, reports of universities and colleges. Performance evaluation is implemented on 61 universities and colleges in Shanghai. According to the regular pattern of higher education, the output is hysteretic to input. Data of continuous five years are collected, and the average value is taken as the attribute value of the indication.

4.3 Dimension Reduction of Evaluation Indicator System by PCA

SPSS Statistic 24.0 is used to do PCA to reduce dimension of evaluation indicator system.

4.3.1 Principal Component Selection of Indicator System

The dependency matrix of input and output indicators is analysed by PCA firstly with accumulation variance contribution rate, VCR, greater or equal to 955%. The dependency matrix of input indicators is shown in table 2. The eigenvalue and accumulation variance contribution rate are shown in table 3. The input indicators can be transferred to 28 principal components.

The dependency matrix of output indicators, accumulation variance contribution rate and the output principal components can be obtained in the same way.

Table 2: The dependency matrix of input indicators.

Input$_1$	Input$_2$	Input$_3$	…	Input$_{67}$	Input$_{68}$
0.7401	0.5120	0.9541	…	0.6739	0.0658
0.7093	0.5109	0.9502	…	0.4348	0.1202
0.5508	0.5037	1.0000	…	0.1008	0.2303
0.4519	0.5047	0.9595	…	0.0692	0.1159
0.9117	0.6738	0.9531	…	0.1403	0.0529
…	…	…	…	…	…
…	…	…	…	…	…
0.6062	0.6703	0.9464	…	0.2194	1.0000
0.4003	0.6649	0.9338	…	0.5000	0.1073
0.5119	0.5144	0.9826	…	0.0020	0.6924
0.4939	0.6275	0.8968	…	0.2300	0.1116
1.0000	0.9261	1.0000	…	0.0020	0.0973
0.9164	1.0000	1.0000	…	0.0000	0.0086
0.5989	0.6360	1.0000	…	1.0000	0.0100

Table 3: The eigenvalue and accumulation variance contribution rate.

Component	Eigen value	variance contribution rate	accumulation variance contribution rate
F_1	4.208	35.065%	35.065%
F_2	2.276	18.967%	54.032%
F_3	2.169	18.079%	72.111%
F_4	2.012	17.725%	80.361%
…	…	…	…
…	…	…	…
F_{27}	0.617	7.538%	93.479%
F_{28}	0.548	6.901%	95.012%

4.3.2 Principal Component Expression

Through varimax rotation, the principal component matrix of input indicators is shown in table 4. The principal component expression is shown in (8). The coefficient reflects the influence of original input indicators.

The principal component matrix of output indicators and output principal component expression can be obtained in the same way.

Table 4: The principal component matrix of input indicators.

Input Indicators	Principal Components				
	F_1	F_2	…	F_{27}	F_{28}
Input$_1$	0.247	0.597	…	0.520	0.364
Input$_2$	-0.245	-0.307	…	0.404	0.771
Input$_3$	-0.246	0.323	…	0.570	-0.158
Input$_4$	0.848	0.112	…	0.276	0.239
Input$_5$	0.790	0.474	…	-0.268	0.106
…	…	…	…	…	…
…	…	…	…	…	…
Input$_{62}$	0.951	0.039	…	-0.009	0.113
Input$_{63}$	0.929	0.145	…	-0.064	-0.09
Input$_{64}$	0.669	-0.549	…	0.457	-0.054
Input$_{65}$	0.019	0.600	…	0.184	-0.671
Input$_{66}$	-0.254	0.759	…	0.119	0.232
Input$_{67}$	0.484	-0.095	…	-0.782	0.071
Input$_{68}$	0.411	-0.463	…	0.626	-0.442

$$\begin{bmatrix} F_1 \\ F_2 \\ \cdots \\ \cdots \\ F_{27} \\ F_{28} \end{bmatrix} = \begin{bmatrix} 0.247 & -0.245 & \cdots & 0.411 \\ 0.597 & 0.307 & \cdots & -0.463 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0.520 & -0.404 & \cdots & 0.626 \\ 0.364 & 0.771 & \cdots & -0.442 \end{bmatrix} * \begin{bmatrix} Input_1 \\ Input_2 \\ \cdots \\ \cdots \\ Input_{67} \\ Input_{68} \end{bmatrix} \quad (8)$$

4.4 Performance Evaluation by DEA

MATLAB 2017a and DEAP 2.1 are used to do DEA to evaluate the performance of 61 universities and colleges in Shanghai.

4.4.1 Comprehensive Score of Principal Components of Input and Output Indicators

According to the principal component expression, comprehensive score of principal components of input and output indicators can be calculated. The eigenvalue is set as weight of corresponding principal component. The comprehensive score of input indicators is shown in table 5 for example.

Table 5: The comprehensive score of input indicators.

University and College	Principal Components				
	F_1	F_2	…	F_{27}	F_{28}
DMU_1	2.8621	2.4989	…	0.6527	0.0604
DMU_2	2.3737	1.8620	…	0.6527	0.0555
DMU_3	2.0445	1.0124	…	0.6527	0.0331
DMU_4	2.2334	2.4351	…	0.6527	0.8824
DMU_5	2.7093	2.1054	…	0.8824	0.1672
…	…	…	…	…	…
…	…	…	…	…	…
DMU_{56}	1.8124	1.7335	…	0.8824	0.1937
DMU_{57}	1.4282	1.0487	…	0.8824	0.1722
DMU_{58}	1.1745	1.6277	…	1.1268	0.6217
DMU_{59}	0.9424	1.1940	…	0.8342	0.4670
DMU_{60}	1.9292	0.2176	…	0.0090	0.9522
DMU_{61}	3.8928	1.3893	…	0.6210	0.6037

4.4.2 DEA Operation to Evaluate Performance

Comprehensive score of principal components of input and output indicators are standardized as input parameters in DEA module. Comprehensive efficiency, θ_j, and technical efficiency, δ_j, in (6) and (7) are calculated to evaluate the performance of 61 universities and colleges in Shanghai. The evaluation result is shown in table 6.

Table 6: Performance Result of Universities and Colleges.

University and College	Comprehensive Efficiency	Technical Efficiency	Performance Efficiency Rate	Performance	Rankings
DMU_1	1.0000	1.0000	1.0000	-	1
DMU_2	0.7969	1.0000	0.7969	decrease	8
DMU_3	0.7269	0.7841	0.9271	decrease	19
DMU_4	0.6532	0.7025	0.6254	-	31
DMU_5	0.7453	0.7453	1.0000	increase	10
…	…	…	…	…	…
…	…	…	…	…	…
DMU_{56}	0.4450	0.5237	0.5028	increase	47
DMU_{57}	0.7308	0.9612	0.7603	-	11
DMU_{58}	0.6357	0.7147	0.5896	decrease	26
DMU_{59}	0.4891	0.7532	0.5230	-	38
DMU_{60}	0.8525	0.9758	1.0000	-	5
DMU_{61}	0.6984	0.9892	0.7131	decrease	23

4.5 Performance Evaluation Result Analysis

From the running result of MATLAB 2017a and DEAP 2.1, comprehensive efficiency, θ, technical efficiency, δ, and performance efficiency rate, η, of 61 DMUs are obtained.

Since $\theta = 1$ and $s = z = 0$, universities and colleges of DMU_1, DMU_5, DMU_{60}, etc are efficient by DEA. They obtain better achievement in performance. The ratio of efficient DMU is 50%. It shows that the performance management of higher education in Shanghai is better.

Since $\theta < 1$ and $\delta = 1$, universities and colleges of DMU_2, DMU_4, DMU_{61} etc are insufficient or redundant in input investment while the internal management and resource allocation is rational. After analysing λ, performance efficiency rate of DMU_2 and DMU_{61} are found in the status of decreasing, while performance efficiency rate of DMU_4 are in the status of increasing. Therefore the universities of DMU_2 and DMU_{61} shall reduce input investment and increase output efficiency. The university of DMU_4 shall increase input investment in order to improve the output efficiency.

Since $\theta < 1, \delta < 1, and\ \eta = 1$, universities and colleges of DMU_{38} etc are rational in current scale status. It shall optimize management quality and resource allocation.

Since $\theta < 1, \delta < 1, and\ \eta < 1$, universities and colleges of DMU_{38} etc are inefficient by DEA. It shall optimize in current scale, management scale and resource allocation. After analysing λ, there are redundant input investment in these universities and colleges. Human resource and financial resource shall be optimized.

In general, the performance evaluation method of universities and colleges based on PCA and DEA pays attention to dimension reduction in indicator system and value combination of comprehensive efficiency, technical efficiency and performance efficiency rate, etc. The rankings of the performance of 61 universities and colleges in Shanghai by this method is consistent with the popular university and college rankings in the country.

5 DISCUSSION AND CONCLUSIONS

This paper proposes a new method of performance evaluation based on PCA and DEA. PCA is used to simplify the performance evaluation indicator system by reducing dimension. DEA is implemented to evaluate performance of universities and colleges. 61 universities and colleges in Shanghai are carefully analysed by the method. The study of the method is helpful to reveal improve the utility efficiency of funds and resource allocation. Meanwhile it provides basis for the educational administrative department to develop new optimized strategies for higher education.

In the future, we will take further research on analysing specific principal component with PCA and DEA to deduce performance evaluation result more scientifically.

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