Branching Poisson Process Modelling for Reliability Analysis of Repairable Mechanical System

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Abstract: In a series of mechanical system maintenance it is possible to observe failures, sometimes intermittent, which cause series of unsuccessful repair attempts before the correct fault is detected and the repair is effective. Unsuccessful repairs performed for the same failure are not negligible and must be taken into account for the reliability analysis. To solve this type of problem, the model proposed is the branching Poisson process (BPP). This process is the representation of a primary failure that triggers one or more subsidiary failures. A summary and an adaptation for the resolution of the branching Poisson with failures time of repairable mechanical system were highlighted including some clarification regarding the steps of resolution implemented.

1 INTRODUCTION

The reliability is the probability of a system performing its purpose correctly without defects according to the given conditions within a certain time (Barlow and Proschan, 1965). It plays an important role in accomplishing the steps leading to improved maintenance. The understanding of its concepts is essential in order to model the system state and to find an acceptable maintenance scenario. A mechanical system is said repairable if it is possible to restore its primitive qualities when it fails (Ascher and Feingold, 1984). When the failure system occurs at a specific age, maintenance is carried out to restore its initial qualities. These maintenance actions can affect the overall behavior of the system and involve its operation due to variable maintenance resources such as human errors, parts quality and preventive action performance (Procaccia et al., 2011).

Four stochastic processes are commonly cited to analyze the reliability of repairable system: the renewal process (RP), the homogeneous Poisson process (HPP), the non-homogeneous Poisson process (NHPP) and the branching Poisson process (BPP) (Ascher and Feingold, 1984; Garmabaki et al., 2015). The renewal process is an arrival process whose intermediate intervals (times of failure) are positive, independent and identically distributed data (Barlow and Proschan, 1965). The random feature of the renewal process supposes that maintenance has restored the initial primitive qualities to the system, so that it can be considered new and the assumption "as good as new" is applicable. With this process, it is then possible to use conventional statistical techniques to evaluate reliability functions. The HPP is a particular case of the renewal process in which the failure times are i.d.d. whose interarrival distribution is closely related to the exponential distribution (Tobias and Trindade, 2012). The NHPP applies when the "as bad as old" assumption is considered and the reliability has not been improved since the last failure (Barlow and Hunter, 1960). The repair action is just enough to make the system operational again and the failure intensity function remains the same or worse as last maintenance before, which, over time, will degrade the integrity of the system. The BPP is implemented when the failure times are identically distributed but are not independent. This article is focused on this specific Poisson process using a case study. This process is poorly documented in the literature on reliability analysis : it is often referred like a milestone in the process of analyzing failure data, but hardly applied since it is very common that failure data are independent.

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2 MATHEMATICAL FORMULATION

This section summarizes the theory and the model adaptation of branching Poisson process from references (Lewis, 1964a; Lewis, 1964b; Cox and Lewis, 1966; Rigdon and Basu, 2000). In addition, some clarifications are provided in relation with the established model.

2.1 Branching Poisson Theory

The process is characterized by random variables $Z_1, Z_2, \ldots, Z_k$ defined as the times between the primary failures (i.e. time to the k^{th} primary event) and the random variables $Y_1, Y_1+Y_2, \ldots, Y_1+\ldots+Y_{s+1}$ defined as the times between the subsidiary failures and triggered by primary failures (i.e. time to the s^{th} subsidiary event). If the term $H(t)$ is the expected number of subsidiary failures in the finite renewal process, then for a time interval $[0,t]$, the contribution of the k^{th} event in the subsidiary process for the expected number of events is:

$$\mathbb{E}\left[N^{(k)}(t)\right] = \int_0^t H(t-z) f_k(z) dz \tag{1}$$

where $f_k(t)$ denote the probability density function of primary events. In the time interval $[0,t]$, the expected number of failures $\mathbb{E}\left[N^{(0)}(t)\right]$ of the complete process is the sum of the primary failures number and the cumulative sum of the subsidiary failures number from the k^{th} event.

$$\begin{aligned} \mathbb{E}\left[N^{(0)}(t)\right] &= \mathbb{E}\left[N(t)\right] + \sum_{k=1}^{\infty} \mathbb{E}\left[N^{(k)}(t)\right] \\ &= M_z(t) + \int_0^t H(t-z)\left[\sum_{k=1}^{\infty} f_k(z)\right] dz \end{aligned} \tag{2}$$

where $M_z(t)$ is the expected number of primary events in [0, t]. By definition:

$$m_z(t) = \frac{dM_z(t)}{dt} = \sum_{k=1}^{\infty} f_k(t) \tag{3}$$

Then, Equation (2) becomes:

$$\mathbb{E}\left[N^{(0)}(t)\right] = M_z(t) + \int_0^t H(t-z) m_z(t) dz \tag{4}$$

If the primary process is considered a Poisson process, then the probability density function is equal $f_z(t) = \lambda \exp(-\lambda t)$ and the expected number of events of the primary process becomes $M_z(t) = \lambda t$. The expected number of events $M(t)$ of the complete process is thus obtained:

$$M(t) = \mathbb{E}\left[N^{(0)}(t)\right] = \lambda t + \lambda \int_0^t H(t-z) dz \tag{5}$$

The rate of occurrence of failures function or the intensity function $m(t)$ of the complete process is defined by:

$$m(t) = \frac{dM(t)}{dt} = \lambda\left[1 + H(t)\right] \tag{6}$$

If the number of subsidiary S failures is known, the expected number of events $\mathbb{E}\left[H(t)|S\right]$ should tend to S since all subsidiary failures are required to occur (Rigdon and Basu, 2000). Applying the limit to $H(t)$:

$$\lim_{t\to\infty} H(t) = \mathbb{E}(S) \tag{7}$$

From Equation (6), the intensity function for the branching Poissson process is:

$$\lim_{t\to\infty} m(t) = \lim_{t\to\infty} \lambda\left[1 + H(t)\right] = \lambda\left[1 + \mathbb{E}(S)\right] \tag{8}$$

2.2 Branching Poisson Modelling

The principle of the branching Poisson modelling is to represent the main process from occurrence of the primary failures and the process from occurrence of the subsidiary failures, which is dependent on the main process. Some probability functions are applicable to modeling the first order of branching Poisson like the exponential and the gamma distributions. The objective is to compare the empirical reliability function R_{n_0} with the theoretical data modeled by the reliability function $R_T(t)$, which defines the branching Poisson process. Validation of the model adopted are done by comparing the reliability functions on a logarithmic scale (Lewis, 1964a). The logarithmic definition of empirical reliability function R_{n_0} of i for the i^{th} failure is:

$$\ln R_{n_0}(i) = \ln\left(1 - \frac{i}{n_0+1}\right) \tag{9}$$

where n_o is the number of failures. According to the branching Poisson process, the reliability function $R_T(t)$ of the time between failure t_i is given by:

$$\begin{aligned} &R_T(t) = \frac{\left[1 + aR_Y(t)\right]}{(1+a)} \cdot E_1 \\ &\text{where} \\ &E_1 = \exp\left(-\lambda t - \lambda a \int_0^t R_Y(u) du\right) \end{aligned} \tag{10}$$

where $R_Y(t)$ is the reliability function of the subsidiary process (exponential or gamma) and the parameter $a = \mathbb{E}(S)$ is the expected number of subsidiary failures. The probability density function $f_T(t)$ of the branching Poisson process is defined by:

$$f_T(t) = \frac{\left[\lambda + a f_Y(t) + 2\lambda a R_Y(t) + \lambda a^2 R_Y^2(t)\right]}{(1+a)} \cdot E_1 \tag{11}$$

where $f_Y(t)$ is the probability density function of the subsidiary process. For a time $t \gg \mathbb{E}(Y)$, $R_Y(t) \to 0$ and $\int_0^t R_Y(u)du \to \mathbb{E}(Y)$ (Karyagina et al., 1998). By substituting $b = \mathbb{E}(Z)/\mathbb{E}(Y) = 1/\{\lambda\mathbb{E}(Y)\}$ and applying the logarithm to the reliability function $R_T(t)$ from equation (10), the next equation is formed:

$$\ln R_T(t) \approx \ln(1+a) - \frac{a}{b} - \lambda t \tag{12}$$

The term λ is estimated as the slope of the straight line on the tail of the distribution $\ln R_{n_0}(i)$ (if the slope is obvious for the sample). The tail of the distribution is defined from the point of truncation, i.e. where there is a decrease in the occurrence of subsidiary failures data. This truncation point is arbitrary and is determined with the values generated with Equation (9). The term a is according to the intensity function from Equation (8):

$$a = \frac{\mathbb{E}(Z)}{\mathbb{E}(T)} - 1 \tag{13}$$

where $\mathbb{E}(T)$ is the expected value of the interarrival times t and $\mathbb{E}(Z) = 1/\lambda$ is the expected value of the times adjusted to the tail of the distribution $\ln R_{n_0}(i)$. If the term a is small ($a < 1$), then the resulting curve will be very close to the logarithm of the homogeneous Poisson distribution (Karyagina et al., 1998). The term b of Equation (12) is the intercept of line fitted to the tail of the distribution $\ln R_{n_0}(i)$. The expected value $\mathbb{E}(Y)$ for the subsidiary distribution $\{Y_i\}$ is given by:

$$\mathbb{E}(Y) = \frac{\mathbb{E}(Z)}{b} \tag{14}$$

The failure rate $z_T(t)$ of the branching Poisson process is the ratio of the probability density function on the reliability function.

$$z_T(t) = \frac{f_T(t)}{R_T(t)} \tag{15}$$

The probability density function $f_Y(t)$ for the exponential function is:

$$f_Y(t) = \beta \exp(-\beta t) \tag{16}$$

where $\beta = 1/\mathbb{E}(Y)$ is the intensity parameter. Determined from the integral of $f_Y(t)$, the reliability function of the subsidiary process $R_Y(t)$ for the exponential function is:

$$R_Y(t) = \exp(-\beta t) \tag{17}$$

From Equations (10) and (17), the logarithm of the reliability function for an exponential function $R_T(t)$ is given by :

$$\begin{aligned} &\ln R_T(t) = \ln\left[\frac{[1 + a\exp(-\beta t)]}{(1+a)} \cdot E_2\right] \\ &\text{where} \\ &E_2 = \exp\left[-\lambda t - \lambda a\left(\frac{1-\exp(-\beta t)}{\beta}\right)\right] \end{aligned} \tag{18}$$

If the choice of the distribution for $\{Y_i\}$ is a gamma function, then the probability density function $f_Y(t)$ is (Hogg and Craig, 1978):

$$f_Y(t) = \frac{\beta^k t^{k-1} \exp(-\beta t)}{\Gamma(k)} \tag{19}$$

where Γ is the gamma function, $\beta = k/\mathbb{E}(Y)$ is the intensity parameter and k is the shape parameter. The cumulative distribution function $F_Y(t)$ of gamma function is defined by (Abramowitz and Stegun, 1972):

$$F_Y(t) = \frac{1}{\Gamma(k)} \int_0^t \left[(\beta t)^{(k-1)} \exp(-\beta t)\right] dt \tag{20}$$

If the parameter k is a strictly positive integer, then the cumulative distribution function follows the Erlang distribution (Papoulis, 1991):

$$F_Y(t) = 1 - \exp(-\beta t) \sum_{\upsilon=0}^{k-1} \left[\frac{(\beta t)^\upsilon}{\upsilon!}\right] \tag{21}$$

The reliability $R_Y(t)$ for the gamma distribution is:

$$R_Y(t) = 1 - F_Y(t) = \exp(-\beta t) \sum_{\upsilon=0}^{k-1} \left[\frac{(\beta t)^\upsilon}{\upsilon!}\right] \tag{22}$$

From Equations (10) and (22), the model adapted to a logarithm reliability function for a gamma function $R_T(t)$ is given by:

$$\ln R_T(t) = \ln\left[\frac{\left[1 + a\exp(-\beta t)\sum_{\upsilon=0}^{k-1}\left[\frac{(\beta t)^{\upsilon}}{\upsilon!}\right]\right]}{(1+a)} \cdot E_3\right]$$

where

$$E_3 = \exp\left[-\lambda t - ab\left\{1 - \exp\left(-\frac{kt}{\mathbb{E}(Y)}\right)\cdot E_4\right\}\right]$$

$$E_4 = \sum_{\upsilon=0}^{k-1}\left[\left(\frac{t}{\mathbb{E}(Y)}\right)^{\upsilon}\frac{k^{\upsilon-1}(k-\upsilon)}{\upsilon!}\right] \tag{23}$$

3 RELIABILITY ANALYSIS PROCESS

In the reliability analysis process for reparable system, the first step is to verify if the independence hypothesis of the failure data is respected. If the hypothesis validation is not confirmed then classical statistical techniques cannot be applied and it is necessary to use the non-homogeneous Poisson process (Ascher and Feingold, 1984). Independent data implies that there is no trend: each failure is independent of the previous or the next failure. Identical distributions indicate that data come from the same probability distribution. If the process is free of trends, the application of the dependency test will specify whether the data follow a renewal process or a branching Poisson process. The detection and the anlalysis of dependency can be realized with the correlation coefficient (Lewis, 1964a). If there is acceptance of the hypothesis which accepts the existence of correlation between data, it is possible to model the failures by the branching Poisson process. The process to analyze the reliability of repairable system with applicable tests is presented in Figure 1.

3.1 Trend and Dependence Tests

After the collection, sorting and classification of the chronological failures data, the trend evolution can be examined with hypothesis testing methods. Since some trend tests have a greater sensitivity to the number of events, it is preferable to apply different investigations to validate the same assumption of resolution. In this study, the Laplace test and Military Handbook test (MIL-HDBK-189) are effective hypothesis methods to validate the null hypothesis of homogenous Poisson process (HPP), in order to verify non-trend behavior. The Laplace test compares the mean value of the observed data with the midpoint of the interval and a trend is observed

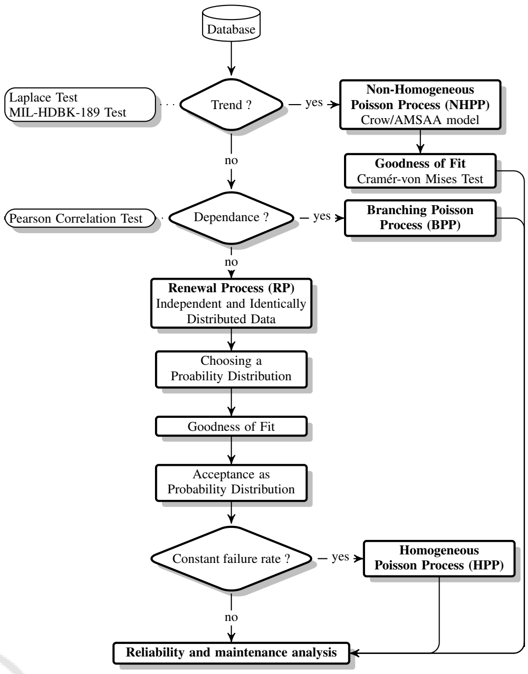

Figure 1: Reliability analysis process (adapted from Ascher and Feingold, 1984).

in the data when the mean value of the failure time moves away from the central point. The Laplace test statistic criterion for testing H_0 (HPP) against H_1 (NHPP) is based on the normality of the variable Z_L with the significance level α. The test, with the number of failures $\hat{n}$, the occurrence of failures time t_i and the time interval of observation [T_a, T_b], is given by (Kvaløy and Lindqvist, 1998):

$$Z_{L[T_a,T_b]} = \frac{\sum_{i=1}^{\hat{n}} t_i - \frac{1}{2}\hat{n}(T_b + T_a)}{(T_b - T_a)\sqrt{\hat{n}/12}} \tag{24}$$

The MIL-HDBK-189 test compares a null hypothesis test H_0 associated to a HPP against the alternative hypothesis H_1 associated to a NHPP (U.S. Department of Defense, 1981). The test is distributed according to a chi-squared distribution with $2\hat{n}$ degree of freedom with the significance level α. The statistic with the number of failures $\hat{n}$ and the occurrence of failures time t_i, is defined for a time interval of observation [T_a, T_b] as follows (Stephens, 2012):

$$MH = 2\sum_{i=1}^{\hat{n}} \ln\left(\frac{T_b - T_a}{t_i - T_a}\right) \tag{25}$$

When the situation gives opposite hypothesis validation for the trend tests, then the Cramér-von Mises goodness-of-fit test can be used to confirm the assumption for the trend. This test verifying the null hypothesis H_0 associated to a NHPP against the alternative hypothesis H_1 associated to a rejection of NHPP and when the statistic's value C_M^2 is greater than the critical value, then the hypothesis H_0 is rejected at the significance level α chosen. This statistic is expressed in this form (Crow, 1990):

$$C_M^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left(Z_i^{\bar{\beta}} - \frac{(2i-1)}{2M}\right)^2 \quad (26)$$

where $M = n, Z_i = t_i/T$, T is the total time on the test, β is the shape parameter from the NHPP, defined by the power law model (Rigdon and Basu, 2000):

$$\beta = \frac{n-1}{\sum_{i=1}^{n} \ln\left(\frac{T}{t_i}\right)} \quad (27)$$

The dependency between observed data can be found by the correlation test. In this paper, the numerical analysis of the dependence is given with the Pearson correlation coefficient (Cox and Lewis, 1966).

$$r_k = \frac{\mathrm{Cov}(X_i, X_{i+k})}{\mathrm{Var}(X)} \quad (28)$$

where k is the time lag, $\mathrm{Var}(X)$ is the variance of the X, $\mathrm{Cov}(X, X_{i+k})$, $\sigma(X_i)$ and $\sigma(X_{i+k})$ are respectively the covariance and the standard deviations of the quantitative variables X_i et X_{i+k}. The test rejection criterion is based on the null hypothesis H_0 which admits an absence of correlation and the alternative hypothesis H_1 which admits the existence of correlation. The test calculating the variable t_0 and comparing it to the value of the significance level α from the Student's t-distribution (Vaurio, 1999).

4 RESULTS AND DISCUSSION

4.1 Data Collection

The data selected come from a mechanical repairable system of Load-Haul-Dump vehicle, more precisely from the powertrain system (transmission, parking brakes, gear box, drive lines front axle and rear axle), and represented by the recorded time between failures (TBF). These failures can be examined and evaluated for the applicability of the BPP. In some industrial context, it is not possible to take the failure data since the beginning of the procedure: a series of events was taken between two times of the system operation. The initial time interval T_a corresponds to the first observation time of failure. Since the failures are censored by time, the time T_b corresponds to the final recording time. Table 1 shows the failures time and the operating age range.

Table 1: Time between failures.

n_o	t_i (h)	n_o	t_i (h)
1	11 977	16	13 820
2	12 450	17	13 867
3	12 513	18	13 917
4	12 654	19	14 042
5	12 844	20	14 075
6	13 066	21	14 240
7	13 155	22	14 560
8	13 280	23	14 933
9	13 380	24	15 275
10	13 394	25	15 369
11	13 440	26	15 635
12	13 479	27	16 625
13	13 515	28	16 729
14	13 525	29	17 380
15	13 605	30	17 400
		T = T_b	18 000

4.2 Analysis of Times Between Failures

The failures trend was validated with three methods: the Laplace test, the Military Handbook test (MIL-HDBK-189) and the Cramér-von Mises test. Table 2 illustrates the trend tests performed on failures data. It is possible to notice that the null hypothesis H_0 which admits a homogeneous Poisson process is rejected by the Laplace test, but accepted by the Military Handbook test. With the application of the third test, the Cramér-von Mises goodness-of-fit test, it is possible to settle on this dilemma. The execution of this last test demonstrates that the null hypothesis H_0 is rejected, which admits a rejection of the non-homogeneous Poisson process. Then, the results favor the assumption that failures data are without trend and identically distributed.

The next step is the correlation verification between failures. The dependency test was calculated with a two lag parameters (k = 1 and k = 2) and the correlation coefficients maximum between r_1 and r_2 was considered.

By analyzing the test results shown in Table 3, it is obvious that the null hypothesis H_0 which admits a lack of correlation (r = 0) is rejected since the value of t_0 is greater than the citrical value. In this case, failures data follow a BPP.

Table 2: Computed value for trend tests.

Rejection of null hypothesis H_0 at 5 % level of significance		
Laplace	Rejected (-2.68 < -1.96)	p-value 0.007
MIL-HDBK-189	Not rejected (79.08 > 68.45 > 43.19)	p-value 0.788
Cramér–von Mises	Rejected (0.642 > 0.217)	

Table 3: Computed value for dependency test.

Rejection of null hypothesis H_0 at 5 % level of significance			
r_1	r_2	p-value	Rejected
0.04	0.38	0.04	(0.219 > 0.205)

4.3 Application of the BPP

The definition of the failure intensity depends on the subsidiary distribution $\{Y_i\}$ and the distribution S. With some initial assumptions, the distribution of $\{Y_i\}$ and the parameters associated with the BPP can be estimated from the failure data. As seen in Figure 2, the chart of the logarithmic reliability function $\ln R_T(t)$ was done initially with the plotting of the empirical values represented by the function from equation (9). From these empirical values, the tail of the distribution has been defined from the point truncation, chosen arbitrarily. The truncation point was selected at interarrival time t = 250 h from where there is a decrease on the number of events representing the subsidiary failures. This new plot is represented by the tail distribution on the chart. With the tail distribution line, it is possible to estimate the parameters from the branching Poisson model. The main parameters of the BPP model, like expected values from processes, are available in Table 4.

Table 4: Parameters estimation of BPP.

λ [slope]	Intercept	$\mathbb{E}(Z)$	a	b	$\mathbb{E}(Y)$	β
0.00263	-0.9308	380	1.1	5.82	65	0.01533

Then, the logarithm of the reliability from BPP model, according to the exponential and gamma distributions (k = 2), was calculated and represented on Figure 2. Also, it has been added the curve of the reliability function represented by HPP model. As shown in Figure 2, it is obvious that the gamma distribution applied to the BPP model does not fit well with failures data. Moreover, since the curve $\ln R_T(t)$ is concave upward, it is possible to rule out the gamma function ($k > 1$) for the distribution of $\{Y_i\}$ (Lewis, 1964a).

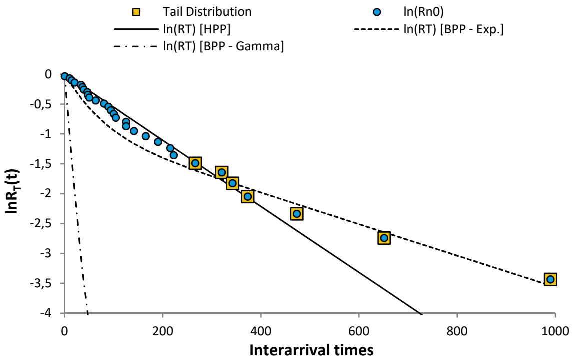

Figure 2: Log survivor of reliability function.

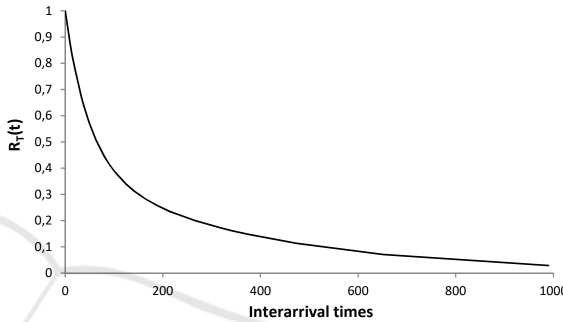

Figure 3: Reliability function of BPP model.

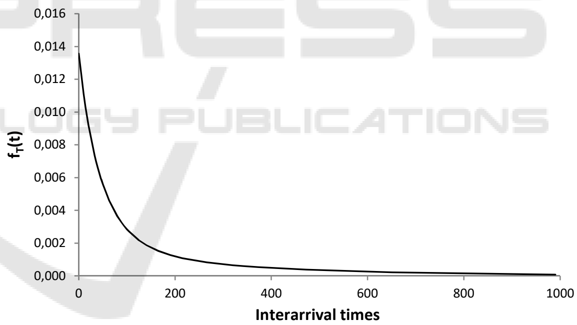

Figure 4: Probability density function of BPP model.

In this case study, the branching Poisson process with an exponential distribution is the best model according to failures data. Figures 3 and 4 present respectively the reliability function $R_T(t)$ and the probability density function $f_T(t)$ modeling the BPP process from the reparable mechanical system.

The smaller value of $\mathbb{E}(Y)$ compared to mean between failures $1/\lambda$ of the main process is consistent: before the source of the system failure is located and adequately repaired, there is a probability that unsuccessful maintenance events can reduce system operation after attempting a failure repair. Calculated by the parameter a, the higher intensity function from the mechanical system could be predominantly due to a series of imperfect repairs. For this case, it is better to wait for more maintenance data to validate if an improvement from the mechanical system is visible and validate if the stochastic process that defines the failures is then a renewal process. Otherwise, it will be necessary to consider modifying the maintenance policy by implementing more preventive action in order to follow up and repair the state of the system. However, beyond the reality of imperfect maintenance, the correlation between data can come from anomalies may be related to insufficient data collection or other situations not representative of the failure process.

5 CONCLUSION

In this study, the reliability and the probability density from repairable mechanical system were evaluated. The correlation test accepts the assumption of dependency for failures data and thereby, they follow a branching Poisson process. This process could be modeled from the graph based on a logarithmic scale and the equations defined in mathematical formulation section. For practical purposes, the estimated parameters of first order properties from the modeling are sufficient to give an interpretation of the branching process Poisson followed by the failure data. Given the verification of the Branching Poisson process model from the mechanical system, the interest remains in the value of the failure rate λ and the efficiency of the system repair $\mathbb{E}(S)$. Then, the main utility of the branching Poisson process is that it can be used to give a physical interpretation of the deviation for the time between failures.

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