

Output Control and Disturbances Compensation using Modified Backstepping Algorithm

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Abstract: The article deals with a problem of output control for linear systems under unknown mismatched disturbances. This algorithm is based on the modified backstepping method and the auxiliary loop method. The proposed control scheme is a robust approach intended to unknown mismatched disturbances estimation and compensation. Efficiency of the method is verified by computer modelling and practical approbation of the algorithm using a laboratory platform called "Twin Rotor MIMO System".


1 INTRODUCTION


Backstepping is one of the widely used methods in the control theory, because of the wide applicability area and high accuracy achieving without high-gain methods usage (Fradkov et al, 2013). This method is proposed in (Kokotovic, 1992). One of the main features of the backstepping approach is an opportunity to deal with nonlinear plants, which can be uncontrollable by classical linear controllers along with systems that are not feedback linearizable. For example, in (Kokotovic, 1992) the case of "peaking effect" is considered to demonstrate the weakness of high-gain control. On the other hand, the backstepping is not easy to implement in practice, because the algorithm synthesis requires an iterative calculation and consecutive analysis of Lyapunov functions for each separate state equation to provide a control law. In addition, in (Fradkov et al, 2013) it is discussed that backstepping-based approaches require to design a set filters in control system, which increases the order of the closed-loop system and enlarge a set of parameters needed to be tuned. Many modifications of the backstepping are designed to provide a more simple structure and enlarge the


application area by using adaptive and robust solutions (Krstic et al, 1995) (Nikiforov, 1997), (Queiroz et al, 2014), (Zhou et al, 2009), (Furtat and Tupichin, 2016), (Vrazhevsky, 2018). Another drawback of the algorithm is the loss in stability under unknown mismatched disturbances presence. In (Furtat et al, 2015) it is shown that the backstepping provides the stability only in the case when external mismatched disturbances supposed to be known.

Design of control techniques that took into account mismatched (unmatched) disturbances presence is a standalone and intense developing area in last years because of its high practical value. In (Yang et al, 2017) authors discuss the set of practical cases where mismatched disturbances have significant influence. These cases include, for example, power converter systems (Zhang et al, 2015), (Wang J. et al, 2017). There are three common way of solutions can be considered: disturbance observer based control (DOBC), active disturbance rejection control (ADRC) and sliding mode control (Aranovskiy et al, 2007), (Li S. et al, 2016), (Gao, 2006), (Wang X. et al, 2016), (Li G. et al, 2016), (Chang, 2016), (Yu P. et al, 2016), (Aparicio et al, 2016), (Han et al, 2009). Backstepping based

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methods to deal with mismatched disturbances are presented in (Furtat et al, 2015), (Sun, 2015). The similar solution is also designed in (Yang et al, 2017).

In current research, a relatively new robust approach to control plants under mismatched disturbances by using a combination of backstepping technique and the auxiliary loop method. The auxiliary loop algorithm is a simple and effective method to disturbances compensation that was first presented in (Tsykunov, 2007). This approach is a robust variation of the reference model methods, which makes it more convenient to work with complex unknown non-modelable dynamics compared to adaptive approaches like augmented error techniques or high order tuners (Astolfi et al, 2007). The advantage of the auxiliary loop algorithm is the ability to simply evaluate and compensate an undesired dynamics produced by a wide class of external and internal disturbances with unknown amplitude without using high-gain elements in control loop. There are a number of works consider the application of the auxiliary loop method in various fields (Belyaev et al, 2013), (Fradkov and Furtat, 2013), (Furtat and Chugina, 2016). However, in the original form, the method does not compensate mismatched disturbances, which is demonstrated in (Furtat et al, 2015).

Current research address to the problem of mismatched disturbances compensation is solved by using a control algorithm based on both the modified backstepping algorithm (Furtat, 2009) and the auxiliary loop method by (Tsykunov, 2007). In (Furtat, 2009) the modification leads to significant reducing of regulator degree is proposed. Based on this result, the algorithm called Modified Backstepping Algorithm with Disturbances Compensation (MBADC) is proposed in (Furtat et al, 2015) and ensures unknown mismatched disturbances compensation. In (Furtat et al, 2017) the method is applied for linear systems under unknown bounded delays in state vector. Currently, the applicability of MBADC in linear systems is limited by the assumption that the state vector of the plant is known along with the fact that only stabilization task is considered in previous works.

This paper deals with the tracking output control problem for linear plants under unknown bounded mismatched disturbances. The control system synthesis consists of n steps. The first state equation is analysed separately, then the similar calculations hold for other state variables, and, on the last step, the control law is introduced. This analysis technique corresponds to the backstepping procedure (Kokotovic, 1992). As a modification, the auxiliary loop method is used in each step to estimate the value of undesired dynamics and compensate it by including obtained estimates in control law with

reverse sign. As a result, high robustness towards the disturbances in each state equation is obtained (regardless of whether there is a real control signal in particular state equation or not). In addition, the practical approbation of the obtained algorithm using the real laboratory platform called «Twin Rotor MIMO System» is considered.

The paper is organized as follows. The problem statement is presented in Section 2. The synthesis of output control MBADC algorithm is proposed in Section 3. In Section 4 the efficient of the algorithm is illustrated by computer modeling along with results of practical approbation using laboratory stand. The conclusion is given in Section 5.

2 PROBLEM STATEMENT

Consider a linear system defined by

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} I_{[n-1] \times [n-1]} x + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u + H\varphi, \\ y &= [1 \ 0 \ \dots \ 0]x, \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state vector which is not available for measurement, $u(t) \in R$ is a control signal, $y(t) \in R$ is an output signal of the system, $b > 0$ is an unknown constant, $\varphi(x, f) \in R^n$ is an unknown bounded function of external disturbances and undesired internal dynamic produced by parametric uncertainties, H is a lower triangular matrix, $f \in R^n$ is a vector of unknown parameters. Function $\varphi(x, f)$ is assumed to be bound (or bounded on t and Lipschitz in x).

The control goal is

$$\|y - y_{ref}\| < \delta, t > T, \quad (2)$$

where y_{ref} is a reference signal, $\delta > 0$ is a required accuracy, $T > 0$ is a transient time.

3 THE ALGORITHM SYNTHESIS

Introduce the auxiliary loops as systems

$$\begin{aligned} \dot{z}_i &= -c_i z_i + \hat{x}_{i+1}, i = \overline{1, n-1}, \\ \dot{z}_n &= -c_n z_n + \alpha u, \end{aligned} \quad (3)$$

that correspond to each state equation of (1)

$$\begin{aligned}\dot{x}_i &= x_{i+1} + \varphi_i, i = \overline{1, n-1}, \\ \dot{x}_n &= bu + \varphi_n,\end{aligned}\quad (4)$$

where $z_i(t)$ are state vector variables that represent the desired dynamic of (1) in case $\varphi_i(t) = 0$, $\hat{x}_i(t)$ is an estimate of the state vector, $\alpha, c_i > 0$ are positive numbers. Comparing (3) and (4), consider a set of mismatch errors and observation errors

$$\xi_i = e_i - z_i, i = \overline{1, n}, \quad (5)$$

$$\varepsilon_i = x_i - \hat{x}_i, i = \overline{2, n}, \quad (6)$$

where $\xi_i(t)$ are mismatch errors, $e_i(t)$ are tracking errors between the desired and the existing dynamic of corresponding state vector variable, $\varepsilon_i(t)$ are observation errors. Each error ξ_i , ε_i , e_i can be defined and tended to small enough zero neighborhood consistently in n steps.

3.1 Step 1

The first tracking error $e_1(t)$ is defined as

$$e_1 = x_1 - y_{ref}. \quad (7)$$

The derivative of the mismatch error $\xi_1(t)$ takes the form

$$\dot{\xi}_1 = c_1 z_1 + \tilde{\varphi}_1 + \varepsilon_2. \quad (8)$$

where $\tilde{\varphi}_1 = \varphi_1 - \dot{y}_{ref}$ is a new disturbance function for the first state equation. From (8) it follows that the function $\tilde{\varphi}_1(t)$ can be estimated by

$$\tilde{\varphi}_1 = \dot{\xi}_1 - c_1 z_1 - \varepsilon_2. \quad (9)$$

Expression (9) yields to represent the first state equation by

$$\dot{e}_1 = x_2 + \xi_1 - c_1 z_1 - \varepsilon_2. \quad (10)$$

According to the backstepping procedure, assume that $x_2(t)$ is a virtual control signal in (10) and there exist virtual control law v_1 such that if $x_2 = v_1$ then the dynamics of (10) satisfies the goal (2). Introduce the first virtual control law by

$$v_1 = -c_1 \xi_1 - \tilde{\xi}_1, \quad (11)$$

where $\tilde{\xi}_1(t)$ is an estimate of the function $\dot{\xi}_1(t)$. Substituting (11) into (10), we get

$$\dot{e}_1 = -c_1 e_1 - \varepsilon_2 + \eta_1, \quad (12)$$

where $\eta_1 = \dot{\xi}_1 - \tilde{\xi}_1$ is an estimation error.

3.2 Step $i, i = \overline{2, n-1}$

The following calculations hold for each state equation of (4) with $i = \overline{2, n-1}$. Tracking errors take the form

$$e_i = x_i - v_{i-1}, i = \overline{2, n-1}, \quad (13)$$

where $v_{i-1}(t)$ is the virtual control law defined at the previous step in the form

$$v_{i-1} = -c_{i-1} \xi_{i-1} - \tilde{\xi}_{i-1}. \quad (14)$$

Taking into account (4), the i th tracking error can be represented as

$$\dot{e}_i = x_{i+1} + \tilde{\varphi}_i, \quad (15)$$

where $\tilde{\varphi}_i = \varphi_i - \dot{v}_{i-1}$ is a new disturbance function for i th state equation. From (3)-(6) and (15) it follows

$$\dot{\xi}_i = c_i z_i + \tilde{\varphi}_i + \varepsilon_{i+1}, i = \overline{1, n-1}. \quad (16)$$

From (16) it follows that the function $\tilde{\varphi}_i$ can be rewritten by

$$\tilde{\varphi}_i = \dot{\xi}_i - \varepsilon_{i+1} - c_i z_i. \quad (17)$$

Expression (17) yields to represent the i th tracking error equation by

$$\dot{e}_i = x_{i+1} + \xi_i - \varepsilon_{i+1} - c_i z_i. \quad (18)$$

Assume that $x_{i+1}(t)$ is a virtual control signal in (18) and there exist virtual control law v_i such that if $x_{i+1} = v_i$ then the dynamics of (18) satisfies the goal (2) by tending the error (13) to zero. The i th virtual control law can be introduced by

$$v_i = -c_i \hat{\xi}_i - \tilde{\xi}_i, \quad (19)$$

Where $\hat{\xi}_i(t)$ and $\tilde{\xi}_i(t)$ are estimates of the functions $\xi_i(t)$ and $\dot{\xi}_i(t)$ respectively. Substituting (19) into (18) we get

$$\dot{e}_i = -c_i e_i + \eta_i + c_i \varepsilon_i - \varepsilon_{i+1}, \quad (20)$$

where $\eta_i = \dot{\xi}_i - \tilde{\xi}_i$ is an estimation error.

3.3 Step n

The tracking error of n -th state equation is defined by

$$e_n = x_n - v_{n-1}, \quad (21)$$

where $v_{n-1}(t)$ is the virtual control law defined on the previous step. Taking into account (4), the n -th tracking error can be represented in the form

$$\dot{e}_n = bu + \tilde{\varphi}_n, \quad (22)$$

where $\tilde{\varphi}_n = \varphi_n - \dot{v}_{n-1}$ is a new disturbance function for n -th state equation. From (3)-(5) and (22) it follows

$$\dot{\xi}_n = c_n z_n + \tilde{\varphi}_n + (b - \alpha)u. \quad (23)$$

From (23) it follows that the function $\tilde{\varphi}_n(t)$ can be rewritten by

$$\tilde{\varphi}_n = \dot{\xi}_n - c_n z_n - (b - \alpha)u. \quad (24)$$

Expression (24) yields to represent the n -th tracking error equation by

$$\dot{e}_n = \dot{\xi}_n - c_n z_n + \alpha u. \quad (25)$$

From (25) it follows that the control law can be introduced by

$$u = -\frac{1}{\alpha} (c_n \hat{\xi}_n + \tilde{\xi}_n). \quad (26)$$

where $\hat{\xi}_n(t)$ and $\tilde{\xi}_n(t)$ are estimates of the functions $\xi_i(t)$ and $\dot{\xi}_n(t)$ respectively. Substituting (26) into (25) we get

$$\dot{e}_n = -c_n e_n + c_n \varepsilon_n + \eta_n, \quad (27)$$

where $\eta_n = \dot{\xi}_n - \tilde{\xi}_n$ is an estimation error.

To estimate unknown signals $\xi_i(t)$, $\dot{\xi}_i(t)$ and $x_i(t)$ the following dirty differential filter (DDF)

$$W(p) = \frac{p}{\mu p + 1} \quad (28)$$

where $\mu > 0$ is a small enough number. The choice of the parameter μ determines the rate of transients in the observers by increasing the overshoot parameter.

3.4 Theorem

There exist constants $c_i > 0$, $i = \overline{1, n}$, and $\mu_0 > 0$ such that for any $0 < \mu \leq \mu_0$ the control system consisting of auxiliary loops (3), virtual control laws (11), (19), control law (26) and observers (28) provides goal (2) for plant (1).

The proof of the theorem is similar to one from (Furtat et al, 2017) with additional consideration dynamics of observation errors $\varepsilon_i(t)$. Efficiency of proposed algorithm is demonstrated by the following example.

4 MODELLING

The following model is used to test the algorithm via computer simulation

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} f, \\ x(0) &= \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, f = \begin{bmatrix} (1 + 0.5 \sin(t))x_1^2 \\ (x_1^2 + x_2^2) \sin(t) \\ 10 \sin(x_3^2) \end{bmatrix} \end{aligned} \quad (29)$$

Auxiliary loops are defined by

$$\dot{z} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} z + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (30)$$

In the simulation the following parameters of the observer and the coefficients of the auxiliary circuit is used: $\mu = 0.01$, $\alpha = 2$, $c_1 = 5$, $c_2 = 3.5$, $c_3 = 3$. The transients of the closed-loop system in stabilization mode are demonstrated on Fig. 1. Fig. 2 shows the transient in the tracking mode with $y_{ref} = \sin(t + 0.5\pi) + 2 \sin(2t)$.

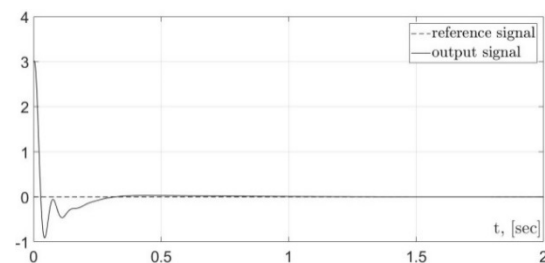


Figure 1: The result of simulation of control algorithms for the stabilization mode.

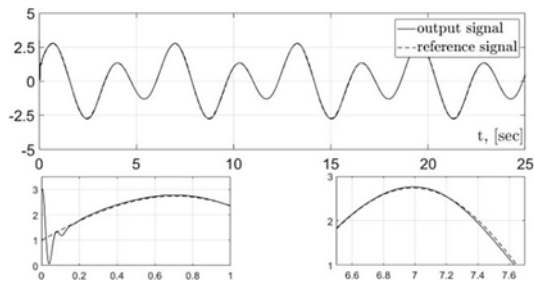


Figure 2: The result of simulation of control algorithms in tracking mode.

To demonstrate the efficiency of the algorithm in practical task, the laboratory plant called “Twin Rotor MIMO System” (TRMS) is used. TRMS is a helicopter-like system that can be represented by linear model with an unknown nonlinear function of undesired internal and external influences and two-channel independent control for each degree of freedom. General view of TRMS is presented in Figure 3.

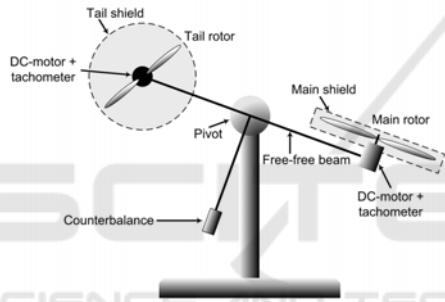


Figure 3: Laboratory plant "Twin Rotor MIMO System".

The plant dynamics is described by the following system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f_d(t), x(0) = 0, \\ y(t) = Lx(t), \end{cases} \quad (31)$$

where $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -12.354 & -2.774 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1.56 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0.847 & 0 \\ 0 & 0 \\ 0 & 0.046 \end{bmatrix}$,

$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $f_d(t)$ is an unknown function of external disturbances supposed to be bounded, $y = [\alpha \ \beta]^T$ (α is a pitch angle of the beam, β is a yaw angle of the beam). A voltage supplied to the DC motors is used as a control action.

The following parameters of the observer and the auxiliary loop are used for the synthesis of the control system: $\mu = 0.01$, $\alpha = 2.5$, $c_1 = 0.4$, $c_2 = 0.2$. Fig. 4-7 show the transients of the pitch angle, yaw angle and control signals supplied to DC motors.

5 CONCLUSIONS

This paper demonstrates a robust approach to output tracking control for plants under unknown mismatched disturbances. The algorithm is based on the backstepping method and the auxiliary loop method. The simulations show the high quality of transients and a high accuracy of regulation in the steady state. Experimental results illustrate that accuracy in steady state is comparable to TRMS resolution of encoders – up to 0.02 rad.

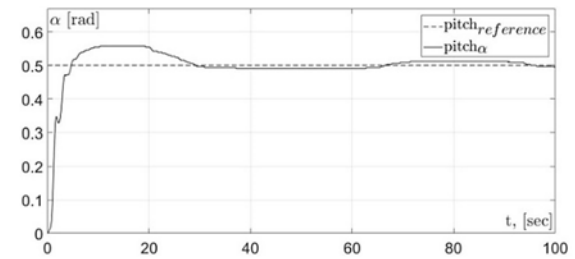


Figure 4: Transients and steady state of pitch angle stabilization.

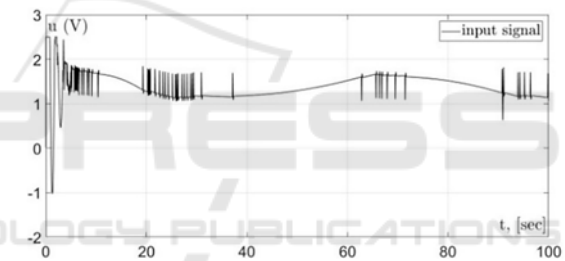


Figure 5: Control signal for pitch subsystem.

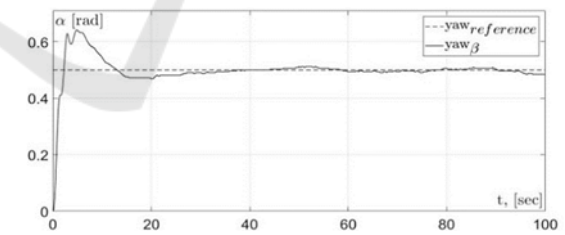


Figure 6: Transients and steady state of yaw angle stabilization.

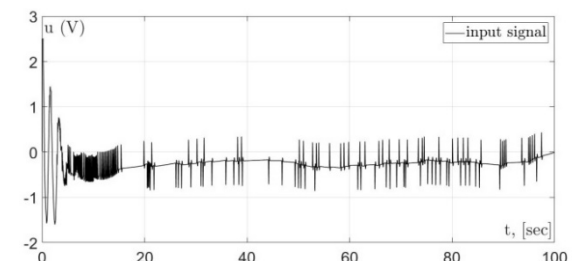


Figure 7: Control signal for yaw subsystem.

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