Wavelet Analysis based Stability Conditions of a Prediction Model

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Abstract: Prediction models found a wide application in advanced control systems, intelligent systems of information decision support, play a significant role in any activity concerned with signal processing procedures, involving detecting failures of different technological processes. Methods based on the wavelet analysis are characterized by a unique ability of detailed frequency analysis in the time. The paper presents stability conditions of a prediction model, which are developed on the basis of the multi-scale wavelet transform, as well an example of the prediction model applied in the oil refining process.

1 INTRODUCTION

Under solving identification problems one may emphasize a broad class of process to control which constructing linear models is not enough. These processes may have some particularities in certain time instants. In engineering systems, such particularities frequently have a cyclic feature. Solving the problem of constructing prediction models for time-varying processes of such a kind looks vital (Sakrutina and Bakhtadze, 2015).

Within lattes two decades, to analyse time-varying process in different areas the wavelet transform has been broadly expanded, what numerous publications confirm (as an example, Toledo et al., 1998; Yuan and Shi. 2008; Wen and Zhou, 2009; Wen et al., 2010; Castello et al., 2015; Breidenstein et al., 2017; Muto et al., 2019). First studies on the wavelet analysis of time (space) series with manifested heterogeneity have appeared in the middle of 1980s (Grossman and Morlet, 1984). The method was positioned as an alternative to the Fourier transform localizing frequencies but not providing the process time resolution.

At present, the wavelet analysis is applied for processing and synthesis of time-varying signals, solving problems of compression and coding of information, image processes, in particular, in medicine and many other spheres. The approach is effective for studying functions and signals being time-varying or space heterogeneous, when analysis results are to contain not only frequency signal characteristics (power signal distribution over frequency components), but, as well, information about local coordinates at which certain groups of the frequency components manifest themselves, or at which fast changes of the frequency signal components are the case.

The wavelet analysis are used mainly for the identification (Ghanem and Romeo, 2000, 2001) of non-linear systems with a specific structure, where unknown time-varying coefficients can be represented as a linear combination of basis wavelet functions (Tsatsanis and Giannakis, 2002; Wei and Billings, 2002).

The present paper is devoted to applying the wavelet analysis under constructing prediction models providing the prediction without accounting future states of the prediction ground, in particular, determining the stability.

2 PREDICTION MODEL OF NON-LINEAR TIME-VARYING PLANT

A feature of the performance of advanced control systems of manufacturing processes is applying soft- and algorithmic complexes referred as virtual analysed. The virtual analysers implement constructing a prediction model of a specific

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manufacturing process, using (besides current and archived technological data) models at other manufacturing control levels.

Two aspects are features of virtual analysers. Firstly, under their performance the adaptive approach to the model tuning is implemented. Secondly, as an additional a priori information source to identify an investigated process models of other manufacturing processes can be applied, and, besides that, recommended control actions of different regulators (which, perhaps, perform in the mode of a technological process operator adviser).

To check the accuracy of defining the input and output parameters of the process model, checking the hypothesis on the model parameters significance is implemented. To evaluate the process model accuracy, checking the hypothesis on the model adequacy is implemented. The model accuracy is defined in the dependence of model prediction errors. In a number of problems the admissible measurement error is set by standards, technological regulations, and other requirements. To analyse the prediction quality, the empirical error functions are frequently used (Kassam, 1977; Kim et al., 2017): mean absolute percentage error – MAPE, mean absolute error – MAE, mean squared error – MSE.

Let a prediction associative model (Bakhtadze et al., 2013) of a non-linear time-varying plant meets the equation:

$$y(t) = a_0 + \sum_{i=1}^{m} a_i y(t-i) + \sum_{s=1}^{S}\sum_{j=1}^{r_s} b_{s,j} x(t-j)_s, \qquad (1)$$

where $y(t)$ is the plant output prediction at the time instant t, $x(t)$ is the input actions vector, m is the output memory depth, r_s is the input memory depth, s is the input vector dimension, $a_i, b_{s,j}$ are tuned coefficients, $x(t-j)_s$ are selected not in the chronological decreasing order.

Let us write a virtual prediction model (1) in the standardized scale:

$$\hat{y}(t) = \sum_{i=1}^{m} \hat{a}_i \hat{y}(t-i) + \sum_{s=1}^{S}\sum_{j=1}^{r_s} \hat{b}_{s,j} \hat{x}(t-j)_s, \qquad (2)$$

where $\hat{y} = \frac{y-M[y]}{\sigma_y}$, $\hat{x} = \frac{x-M[x]}{\sigma_x}$, $M[\hat{y}] = M[\hat{x}] = 0$, $\sigma_{\hat{y}} = \sigma_{\hat{x}} = 1$, $\hat{a}_i, \hat{b}_{s,j}$ are standardized coefficients.

For a detailing level selected L for a current input vector in the standardized scale we obtain the multi-scale expansion (Mallat, 1999):

$$\hat{x}(t) = \sum_{k=1}^{N} c_{L,k}^{\hat{x}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{\hat{x}}(t)\psi_{l,k}(t),$$
$$\hat{y}(t) = \sum_{k=1}^{N} c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t),$$

where: L is the multiscale expansion depth ($1 \le L \le L_{max}$, where $L_{max} = \lfloor \log_2 N^* \rfloor$ and N^* is the power of the state set in the base of knowledge about the system dynamic); $\varphi_{L,k}(t)$ are scaling functions; $\psi_{l,k}(t)$ are wavelet functions that are obtained from the mother wavelets by the stretching/compression and shift:

$$\psi_{l,k}(t) = 2^{l/2}\psi_{\text{mother}}(2^l t - k),$$

where, as the mother wavelets, the Haar wavelets are considered; l is the detailing analysis level; $c_{L,k}$ are scaling coefficients, $d_{l,k}$ are detailing coefficients. The coefficients are calculated by use of the Mallat algorithm (Mallat, 1999).

Let us expand equation (2) over the wavelets:

$$\sum_{k=1}^{N} c_{L,k}^{\hat{y}}(t)\varphi_{Lk}(t) + \sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) =$$
$$= \sum_{i=1}^{m}\left(\sum_{k=1}^{N} \hat{a}_i c_{L,k}^{\hat{y}}(t-i)\varphi_{L,k}(t-i)\right) +$$
$$+ \sum_{i=1}^{m}\left(\sum_{l=1}^{L}\sum_{k=1}^{N} \hat{a}_i d_{l,k}^{\hat{y}}(t-i)\psi_{l,k}(t-i)\right) +$$
$$+ \sum_{s=1}^{S}\sum_{j=1}^{r_s}\left(\sum_{k=1}^{N} \hat{b}_{s,j} c_{L,k}^{\hat{x}_s}(t-j)\varphi_{L,k}(t-j)\right) +$$
$$+ \sum_{s=1}^{S}\sum_{j=1}^{r_s}\left(\sum_{l=1}^{L}\sum_{k=1}^{N} \hat{b}_{s,j} d_{l,k}^{\hat{x}_s}(t-j)\psi_{l,k}(t-j)\right)$$

In the last equality, we will group members containing as co-factors identical wavelets. Meanwhile we account that due to the associative search procedure (Bakhtadze and Sakrutina, 2015) the coefficients $\hat{a}$ и $\hat{b}$ may differ of zero for inputs $\hat{x}$ selected from the archive in accordance to the associative procedure rather than the chronological sequence,

$$\sum_{k=1}^{N} c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L}\sum_{k=1}^{N} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) =$$
$$= \sum_{k=1}^{N}\left\{\sum_{i=1}^{m} \hat{a}_i c_{L,k}^{\hat{y}}(t-i)\varphi_{L,k}(t-i)\right.$$
$$\left. + \sum_{s=1}^{S}\sum_{j=1}^{r_s} \hat{b}_{sj} c_{L,k}^{\hat{x}_s}(t-j)\varphi_{L,k}(t-j)\right\} + \qquad (3)$$
$$+ \sum_{l=1}^{L}\sum_{k=1}^{N}\left\{\sum_{i=1}^{m} \hat{a}_i d_{l,k}^{\hat{y}}(t-i)\psi_{l,k}(t-i)\right.$$
$$\left. + \sum_{s=1}^{S}\sum_{j=1}^{r_s} \hat{b}_{s,j} d_{l,k}^{\hat{x}_s}(t-j)\psi_{l,k}(t-j)\right\}.$$

The dynamic plant described by relationship (6) will be stable if simultaneously the following N equations (meeting the relationships with respect each of the addendums over k ($k = 1, \ldots, N$) in the left and right parts of (3):

$$c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) =$$
$$\left\{\sum_{i=1}^{m} \hat{a}_i c_{L,k}^{\hat{y}}(t-i)\varphi_{L,k}(t-i)\right. \qquad (4)$$
$$\left. + \sum_{s=1}^{S}\sum_{j=1}^{r_s} \hat{b}_{s,j} c_{L,k}^{\hat{x}_s}(t-j)\varphi_{L,k}(t-j)\right\}$$

$$+\sum_{l=1}^{L}\left\{\sum_{i=1}^{m}\hat{a}_i d_{l,k}^{\hat{y}}(t-i)\psi_{l,k}(t-i) + \sum_{s=1}^{S}\sum_{j=1}^{r_s}\hat{b}_{s,j} d_{l,k}^{\hat{x}_s}(t-j)\psi_{l,k}(t-j)\right\}.$$

3 MODEL STABILITY CONDITIONS

Let $P = \max_{s=\overline{1,S}}(r_s)$. In subsections 3.1-3.4 there will be considered models (4) of the kind: $m > P$, $m < P$, $m = P \neq 1$, $m = P = 1$.

3.1 Stability Condition under $m > P$

If the input memory depth is less than the output memory depth, then (4) is transformed to the form:

$$\begin{aligned}
& c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) = \\
& = \hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1) + \cdots \\
& + \hat{a}_P c_{L,k}^{\hat{y}}(t-P)\varphi_{L,k}(t-P) + \cdots \\
& + \hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m) \\
& + \sum_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1) + \cdots \\
& + \sum_{s=1}^{S}\hat{b}_{s,P} c_{L,k}^{\hat{x}_s}(t-P)\varphi_{L,k}(t-P) \\
& + \sum_{l=1}^{L}\Big\{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1) + \cdots \\
& + \hat{a}_P d_{l,k}^{\hat{y}}(t-P)\psi_{l,k}(t-P) + \cdots \\
& + \hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m) \\
& + \sum_{s=1}^{S}\hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1) + \cdots \\
& + \sum_{s=1}^{S}\hat{b}_{s,P} d_{l,k}^{\hat{x}_s}(t-P)\psi_{l,k}(t-P)\Big\}.
\end{aligned} \tag{5}$$

Let us consider separately the approximating and detailing parts of equality (5) correspondingly:

$$\begin{aligned}
& c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) = \\
& = \left[\hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\right]\varphi_{L,k}(t-1) \\
& + \cdots \\
& + \left[\hat{a}_P c_{L,k}^{\hat{y}}(t-P) + \sum_{s=1}^{S}\hat{b}_{s,P} c_{L,k}^{\hat{x}_s}(t-P)\right]\varphi_{L,k}(t-P) \\
& + \hat{a}_{P+1} c_{L,k}^{\hat{y}}(t-P-1)\varphi_{L,k}(t-P-1) + \cdots \\
& + \hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m),
\end{aligned} \tag{6}$$

where $k = \overline{1,N}$;

$$\begin{aligned}
& d_{lk}^{\hat{y}}(t)\psi_{lk}(t) = \\
& = \left[\hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S}\hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\right]\psi_{l,k}(t-1) + \\
& \ldots + \left[\hat{a}_P d_{l,k}^{\hat{y}}(t-P) + \sum_{s=1}^{S}\hat{b}_{s,P} d_{l,k}^{\hat{x}_s}(t-P)\right]\psi_{l,k}(t-P) \\
& + \hat{a}_{P+1} d_{l,k}^{\hat{y}}(t-P-1)\psi_{l,k}(t-P-1) + \cdots \\
& + \hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m),
\end{aligned} \tag{7}$$

where $k = \overline{1,N}, l = \overline{1,L}$.

Let us introduce the following notations:

$$\tilde{\mathbf{x}}(t) = \begin{bmatrix} [\tilde{x}_1(t)] \\ \ldots \\ [\tilde{x}_m(t-m+1)] \end{bmatrix}, \tilde{\mathbf{x}}(t) \in \mathbf{R}^m,$$

$$\tilde{\mathbf{y}}(t) = \begin{bmatrix} [\tilde{y}_1(t)] \\ \ldots \\ [\tilde{y}_m(t-m+1)] \end{bmatrix}, \tilde{\mathbf{y}}(t) \in \mathbf{R}^m,$$

where:

$$\begin{gathered}
\tilde{x}_1(t) = \varphi(t);\ \tilde{x}_2(t) = \varphi(t-1);\ \ldots; \\
\tilde{x}_m(t) = \varphi(t-m+1), \\
\tilde{y}_1(t) = \psi(t);\ \tilde{y}_2(t) = y(t-1);\ \ldots; \\
\tilde{y}_m(t) = \psi(t-m+1),
\end{gathered}$$

then:

$$\begin{aligned}
\tilde{\mathbf{x}}(t) &= \begin{bmatrix} [\varphi(t)] \\ \ldots \\ [\varphi(t-m+1)] \end{bmatrix};\ \tilde{\mathbf{x}}(t-1) = \begin{bmatrix} [\varphi(t-1)] \\ \ldots \\ [\varphi(t-m)] \end{bmatrix}; \\
\tilde{\mathbf{y}}(t) &= \begin{bmatrix} [\psi(t)] \\ \ldots \\ [\psi(t-m+1)] \end{bmatrix};\ \tilde{\mathbf{y}}(t-1) = \begin{bmatrix} [\psi(t-1)] \\ \ldots \\ [\psi(t-m)] \end{bmatrix}.
\end{aligned} \tag{8}$$

Let us introduce notations for the coefficients in (6):

$$\begin{gathered}
\chi_0 = c_{L,k}^{\hat{y}}(t) \neq 0, \\
\chi_1 = \hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1), \\
\ldots \\
\chi_P = \hat{a}_P c_{L,k}^{\hat{y}}(t-P) + \sum_{s=1}^{S}\hat{b}_{s,P} c_{L,k}^{\hat{x}_s}(t-P), \\
\chi_{P+1} = \hat{a}_{P+1} c_{L,k}^{\hat{y}}(t-P-1), \\
\ldots \\
\chi_m = \hat{a}_m c_{L,k}^{\hat{y}}(t-m).
\end{gathered} \tag{9}$$

Let us introduce notations for the coefficients in (7):

$$\begin{gathered}
\mu_0 = d_{l,k}^{\hat{y}}(t) \neq 0, \\
\mu_1 = \hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S}\hat{b}_{s,1} d_{lk}^{\hat{x}_s}(t-1), \\
\ldots \\
\mu_P = \hat{a}_P d_{l,k}^{\hat{y}}(t-P) + \sum_{s=1}^{S}\hat{b}_{s,P} d_{l,k}^{\hat{x}_s}(t-P), \\
\mu_{P+1} = \hat{a}_{P+1} d_{l,k}^{\hat{y}}(t-P-1), \\
\ldots \\
\mu_m = \hat{a}_m d_{l,k}^{\hat{y}}(t-m).
\end{gathered} \tag{10}$$

By virtue of notations (9) and (10), let us rewrite (6) and (7) correspondingly in the following form:

$$\begin{gathered}
\chi_0\varphi(t) = \chi_1\varphi(t-1) + \cdots + \chi_P\varphi(t-P) + \\
+\chi_{P+1}\varphi(t-P-1) + \cdots + \chi_m\psi(t-m), \\
\mu_0\psi(t) = \mu_1\psi(t-1) + \cdots + \mu_P\psi(t-P) + \\
+\mu_{P+1}\psi(t-P-1) + \cdots + \mu_m\psi(t-m),
\end{gathered}$$

or

$$\begin{aligned}
& \chi_0\varphi(t) - \frac{\chi_1}{2}\varphi(t-1) - \cdots - \frac{\chi_P}{2}\varphi(t-P) \\
& - \frac{\chi_{P+1}}{2}\varphi(t-P-1) - \cdots - \frac{\chi_{m-1}}{2}\psi(t-m+1) \\
& = \frac{\chi_1}{2}\varphi(t-1) + \cdots + \frac{\chi_P}{2}\varphi(t-P) + \frac{\chi_{P+1}}{2}\varphi(t-P-1) \\
& + \cdots + \frac{\chi_{m-1}}{2}\psi(t-m+1) + \chi_m\psi(t-m),
\end{aligned} \tag{11}$$

$$\begin{aligned}&\mu_0\psi(t)-\frac{\mu_1}{2}\psi(t-1)-\cdots-\frac{\mu_P}{2}\psi(t-P)\\&-\frac{\mu_{P+1}}{2}\psi(t-P-1)-\cdots-\frac{\mu_{m-1}}{2}\psi(t-m+1)\\&=\frac{\mu_1}{2}\psi(t-1)+\cdots+\frac{\mu_P}{2}\psi(t-P)+\frac{\mu_{P+1}}{2}\psi(t-P-1)\\&+\cdots+\frac{\mu_{m-1}}{2}\psi(t-m+1)+\mu_m\psi(t-m).\end{aligned}\tag{12}$$

A sufficient condition to meet equations (11) and (12) is simultaneous meeting the equalities:

$$\begin{gathered}\chi_0\varphi(t)=\frac{\chi_1}{2}\varphi(t-1),\\-\frac{\chi_1}{2}\varphi(t-1)=\frac{\chi_2}{2}\varphi(t-1),\\\dots\\-\frac{\chi_P}{2}\varphi(t-P)=\frac{\chi_{P+1}}{2}\varphi(t-P-1),\\\dots\\-\frac{\chi_{m-1}}{2}\psi(t-m+1)=\chi_m\psi(t-m);\\\mu_0\psi(t)=\frac{\mu_1}{2}\psi(t-1),\\-\frac{\mu_1}{2}\psi(t-1)=\frac{\mu_2}{2}\psi(t-1),\\\dots\\-\frac{\mu_P}{2}\psi(t-P)=\frac{\mu_{P+1}}{2}\psi(t-P-1),\\\dots\\-\frac{\mu_{m-1}}{2}\psi(t-m+1)=\mu_m\psi(t-m).\end{gathered}$$

Equalities (11) and (10), by virtue of above introduced notation (8), can be represented in the form:

$$\begin{bmatrix}\chi_0 & 0 & \cdots & 0\\ 0 & -\frac{\chi_1}{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & -\frac{\chi_{m-1}}{2}\end{bmatrix}\tilde{\mathbf{x}}(t)=\begin{bmatrix}\frac{\chi_1}{2} & 0 & \cdots & 0\\ 0 & \frac{\chi_2}{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \chi_m\end{bmatrix}\tilde{\mathbf{x}}(t-1),\tag{13}$$

$$\begin{bmatrix}\mu_0 & 0 & \cdots & 0\\ 0 & -\frac{\mu_1}{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & -\frac{\mu_{m-1}}{2}\end{bmatrix}\tilde{\mathbf{y}}(t)=\begin{bmatrix}\frac{\mu_1}{2} & 0 & \cdots & 0\\ 0 & \frac{\mu_2}{2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mu_m\end{bmatrix}\tilde{\mathbf{y}}(t-1),\tag{14}$$

Let the matrices in the left hand sides of (13) and (14) be invertible, then

$$\tilde{\mathbf{x}}(t)=\begin{bmatrix}\frac{\chi_1}{2\chi_0} & 0 & \cdots & 0\\ 0 & -\frac{\chi_2}{\chi_1} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & -\frac{2\chi_m}{\chi_{m-1}}\end{bmatrix}\tilde{\mathbf{x}}(t-1),\tag{15}$$

$$\tilde{\mathbf{y}}(t)=\begin{bmatrix}\frac{\mu_1}{2\mu_0} & 0 & \cdots & 0\\ 0 & -\frac{\mu_2}{\mu_1} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & -\frac{2\mu_m}{\mu_{m-1}}\end{bmatrix}\tilde{\mathbf{y}}(t-1).\tag{16}$$

One can interpret relationships (15) and (16) as a representation of a system in the state space. The system stability is defined by the characteristic polynomial of diagonal matrix in the right hand sides of (15) and (16) (Kwakernakk and Sivan, 1972).

Thus, we obtain that the stability criterion of plant (5) (and, hence, (6) and (7) for $\forall\, k=\overline{1,N}, l=\overline{1,L}$) is assured by meeting the equalities:

$$\left|\frac{\chi_1}{2\chi_0}\right|<1,\left|-\frac{\chi_2}{\chi_1}\right|<1,\dots,\left|-\frac{2\chi_m}{\chi_{m-1}}\right|<1;\tag{17}$$

$$\left|\frac{\mu_1}{2\mu_0}\right|<1,\left|-\frac{\mu_2}{\mu_1}\right|<1,\dots,\left|-\frac{2\mu_m}{\mu_{m-1}}\right|<1.\tag{18}$$

The system of inequalities (17) and (18) by virtue of earlier introduced notations can be rewritten for the approximating part in the form of (19) $\forall\, k=\overline{1,N}$, and for the detailing part in the form of (20) for $\forall\, k=\overline{1,N}, l=\overline{1,L}$.

$$\begin{gathered}\left|\frac{\hat{a}_1 c^{\hat{y}}_{L,k}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}c^{\hat{x}_s}_{L,k}(t-1)}{2c^{\hat{y}}_{L,k}(t)}\right|<1,\\\left|-\frac{\hat{a}_2 c^{\hat{y}}_{L,k}(t-2)+\sum_{s=1}^{S}\hat{b}_{s,2}c^{\hat{x}_s}_{L,k}(t-2)}{\hat{a}_1 c^{\hat{y}}_{L,k}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}c^{\hat{x}_s}_{L,k}(t-1)}\right|<1,\\\dots,\\\left|-\frac{\hat{a}_{P+1}c^{\hat{y}}_{L,k}(t-P-1)}{\hat{a}_P c^{\hat{y}}_{L,k}(t-P)+\sum_{s=1}^{S}\hat{b}_{s,P}c^{\hat{x}_s}_{L,k}(t-P)}\right|<1,\\\left|-\frac{\hat{a}_{P+2}c^{\hat{y}}_{L,k}(t-P-2)}{\hat{a}_{P+1}c^{\hat{y}}_{L,k}(t-P-1)}\right|<1,\\\dots,\\\left|-\frac{2\hat{a}_m c^{\hat{y}}_{L,k}(t-m)}{\hat{a}_{m-1}c^{\hat{y}}_{L,k}(t-m+1)}\right|<1.\end{gathered}\tag{19}$$

$$\begin{gathered}\left|\frac{\hat{a}_1 d^{\hat{y}}_{l,k}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}d^{\hat{x}_s}_{lk}(t-1)}{2d^{\hat{y}}_{l,k}(t)}\right|<1,\\\left|-\frac{\hat{a}_2 d^{\hat{y}}_{l,k}(t-2)+\sum_{s=1}^{S}\hat{b}_{s,2}d^{\hat{x}_s}_{lk}(t-2)}{\hat{a}_1 d^{\hat{y}}_{l,k}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}d^{\hat{x}_s}_{lk}(t-1)}\right|<1,\\\dots,\\\left|-\frac{\hat{a}_{P+1}d^{\hat{y}}_{l,k}(t-P-1)}{\hat{a}_P d^{\hat{y}}_{l,k}(t-P)+\sum_{s=1}^{S}\hat{b}_{s,P}d^{\hat{x}_s}_{lk}(t-P)}\right|<1,\\\left|-\frac{\hat{a}_{P+2}d^{\hat{y}}_{l,k}(t-P-2)}{\hat{a}_{P+1}d^{\hat{y}}_{l,k}(t-P-1)}\right|<1,\\\dots,\\\left|-\frac{2\hat{a}_m d^{\hat{y}}_{l,k}(t-m)}{\hat{a}_{m-1}d^{\hat{y}}_{l,k}(t-m+1)}\right|<1.\end{gathered}\tag{20}$$

3.2 Stability Condition under $m < P$

If the input memory depth is more than the output memory depth, then (4) is transformed to the form:

$$
\begin{aligned}
&c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t)+\sum\nolimits_{l=1}^{L} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t)= \\
&=\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} c_{L,k}^{\hat{x}_s}(t-m)\varphi_{L,k}(t-m)+\ldots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,P} c_{L,k}^{\hat{x}_s}(t-P)\varphi_{L,k}(t-P) \\
&+\sum\nolimits_{l=1}^{L}\Big\{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} d_{l,k}^{\hat{x}_s}(t-m)\psi_{l,k}(t-m)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,P} d_{l,k}^{\hat{x}_s}(t-P)\psi_{l,k}(t-P)\Big\}.
\end{aligned} \tag{21}
$$

Let us consider separately the approximating and detailing parts of equality (21) correspondingly:

$$
\begin{aligned}
&c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t)= \\
&=\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} c_{L,k}^{\hat{x}_s}(t-m)\varphi_{L,k}(t-m)+\ldots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,P} c_{L,k}^{\hat{x}_s}(t-P)\varphi_{L,k}(t-P)
\end{aligned} \tag{22}
$$

where $k=\overline{1,N}$;

$$
\begin{aligned}
&d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t)= \\
&=\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\sum_{s=1}^{S}\hat{b}_{s,m} d_{l,k}^{\hat{x}_s}(t-m)\psi_{l,k}(t-m)+\cdots \\
&+\sum_{s=1}^{S}\hat{b}_{s,P} d_{l,k}^{\hat{x}_s}(t-P)\psi_{l,k}(t-P)
\end{aligned} \tag{23}
$$

As a result of transformations being equivalent to those of Subsection 3.1, we obtain sufficient conditions for the approximating part in the form of (24) $\forall\ k=\overline{1,N}$, and for the detailing part, in the form of (25) for $\forall\ k=\overline{1,N}, l=\overline{1,L}$.

$$
\begin{aligned}
&\left|\frac{\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}c_{L,k}^{\hat{x}_s}(t-1)}{2c_{L,k}^{\hat{y}}(t)}\right|<1, \\
&\left|-\frac{\hat{a}_2 c_{L,k}^{\hat{y}}(t-2)+\sum_{s=1}^{S}\hat{b}_{s,1}c_{L,k}^{\hat{x}_s}(t-2)}{\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}c_{L,k}^{\hat{x}_s}(t-1)}\right|<1, \\
&\ldots, \\
&\left|-\frac{\sum_{s=1}^{S}\hat{b}_{s,m+1}c_{L,k}^{\hat{x}_s}(t-m-1)}{\hat{a}_m c_{L,k}^{\hat{y}}(t-m)+\sum_{s=1}^{S}\hat{b}_{s,m}c_{L,k}^{\hat{x}_s}(t-m)}\right|<1, \\
&\left|-\frac{\sum_{s=1}^{S}\hat{b}_{s,m+2}c_{L,k}^{\hat{x}_s}(t-m-2)}{\sum_{s=1}^{S}\hat{b}_{s,m+1}c_{L,k}^{\hat{x}_s}(t-m-1)}\right|<1, \\
&\ldots, \\
&\left|-\frac{2\sum_{s=1}^{S}\hat{b}_{s,P}c_{L,k}^{\hat{x}_s}(t-P)}{\sum_{s=1}^{S}\hat{b}_{s,P-1}c_{L,k}^{\hat{x}_s}(t-P+1)}\right|<1;
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
&\left|\frac{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}d_{l,k}^{\hat{x}_s}(t-1)}{2d_{l,k}^{\hat{y}}(t)}\right|<1, \\
&\left|-\frac{\hat{a}_2 d_{l,k}^{\hat{y}}(t-2)+\sum_{s=1}^{S}\hat{b}_{s,2}d_{l,k}^{\hat{x}_s}(t-2)}{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)+\sum_{s=1}^{S}\hat{b}_{s,1}d_{l,k}^{\hat{x}_s}(t-1)}\right|<1, \\
&\ldots, \\
&\left|-\frac{\sum_{s=1}^{S}\hat{b}_{s,m+1}d_{l,k}^{\hat{x}_s}(t-m-1)}{\hat{a}_m d_{l,k}^{\hat{y}}(t-m)+\sum_{s=1}^{S}\hat{b}_{s,m}d_{l,k}^{\hat{x}_s}(t-m)}\right|<1, \\
&\left|-\frac{\sum_{s=1}^{S}\hat{b}_{s,m+2}d_{l,k}^{\hat{x}_s}(t-m-2)}{\sum_{s=1}^{S}\hat{b}_{s,m+1}d_{l,k}^{\hat{x}_s}(t-m-1)}\right|<1, \\
&\ldots, \\
&\left|-\frac{2\left(\hat{a}_m d_{l,k}^{\hat{y}}(t-m)+\sum_{s=1}^{S}\hat{b}_{s,m}d_{l,k}^{\hat{x}_s}(t-m)\right)}{\hat{a}_{m-1} d_{l,k}^{\hat{y}}(t-m+1)+\sum_{s=1}^{S}\hat{b}_{s,m-1}d_{l,k}^{\hat{x}_s}(t-m+1)}\right| \\
&<1.
\end{aligned} \tag{25}
$$

3.3 Stability Condition under $m = P$

If the input memory depth is equal to the output memory depth, then (4) is transformed to the form:

$$
\begin{aligned}
&c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t)+\sum\nolimits_{l=1}^{L} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t)= \\
&=\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} c_{L,k}^{\hat{x}_s}(t-m)\varphi_{L,k}(t-m)+ \\
&+\sum\nolimits_{l=1}^{L}\Big\{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} d_{l,k}^{\hat{x}_s}(t-m)\psi_{l,k}(t-m)\Big\}.
\end{aligned} \tag{26}
$$

Let us consider separately the approximating and detailing parts of equality (26) correspondingly:

$$
\begin{aligned}
&c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t)= \\
&=\hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\hat{a}_m c_{L,k}^{\hat{y}}(t-m)\varphi_{L,k}(t-m) \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1)+\cdots \\
&+\sum\nolimits_{s=1}^{S}\hat{b}_{s,m} c_{L,k}^{\hat{x}_s}(t-m)\varphi_{L,k}(t-m)
\end{aligned} \tag{27}
$$

where $k=\overline{1,N}$,

$$\begin{aligned} & d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) = \\ & = \hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1) + \cdots \\ & + \hat{a}_m d_{l,k}^{\hat{y}}(t-m)\psi_{l,k}(t-m) \\ & + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1) + \cdots \\ & + \sum_{s=1}^{S} \hat{b}_{s,m} d_{l,k}^{\hat{x}_s}(t-m)\psi_{l,k}(t-m) \end{aligned} \quad (28)$$

where $k = \overline{1,N}, l = \overline{1,L}$.

As a result of transformations being equivalent to those of Subsection 3.1, we obtain sufficient conditions for the approximating part in the form of (29) $\forall\ k = \overline{1,N}$, and for the detailing part, in the form of (30) for $\forall\ k = \overline{1,N}, l = \overline{1,L}$.

$$\begin{aligned} & \left| \frac{\hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)}{2c_{L,k}^{\hat{y}}(t)} \right| < 1, \\ & \left| -\frac{\hat{a}_2 c_{L,k}^{\hat{y}}(t-2) + \sum_{s=1}^{S} \hat{b}_{s,2} c_{L,k}^{\hat{x}_s}(t-2)}{\hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)} \right| < 1, \\ & \dots, \\ & \left| -\frac{2\left(\hat{a}_m c_{L,k}^{\hat{y}}(t-m) + \sum_{s=1}^{S} \hat{b}_{s,m} c_{L,k}^{\hat{x}_s}(t-m)\right)}{\hat{a}_{m-1} c_{L,k}^{\hat{y}}(t-m+1) + \sum_{s=1}^{S} \hat{b}_{s,m-1} c_{L,k}^{\hat{x}_s}(t-m+1)} \right| \\ & < 1; \end{aligned} \quad (29)$$

$$\begin{aligned} & \left| \frac{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)}{2d_{l,k}^{\hat{y}}(t)} \right| < 1, \\ & \left| \frac{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)}{2d_{l,k}^{\hat{y}}(t)} \right| < 1, \\ & \left| -\frac{\hat{a}_2 d_{l,k}^{\hat{y}}(t-2) + \sum_{s=1}^{S} \hat{b}_{s,2} d_{l,k}^{\hat{x}_s}(t-2)}{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)} \right| < 1, \\ & \dots, \\ & \left| -\frac{2\left(\hat{a}_m d_{l,k}^{\hat{y}}(t-m) + \sum_{s=1}^{S} \hat{b}_{s,m} d_{l,k}^{\hat{x}_s}(t-m)\right)}{\hat{a}_{m-1} d_{l,k}^{\hat{y}}(t-m+1) + \sum_{s=1}^{S} \hat{b}_{s,m-1} d_{l,k}^{\hat{x}_s}(t-m+1)} \right| \\ & < 1. \end{aligned} \quad (30)$$

3.4 Stability Condition under m=P=1

Let us consider a case, when the input and output memories depths are equal to 1, then (4) is transformed to the form:

$$\begin{aligned} & c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) + \sum_{l=1}^{L} d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) = \\ & = \hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1) \\ & + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1) \\ & + \sum_{l=1}^{L} \Big\{ \hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1) \\ & + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1) \Big\} \end{aligned} \quad (31)$$

Let us consider separately the approximating and detailing parts of equality (31) correspondingly:

$$c_{L,k}^{\hat{y}}(t)\varphi_{L,k}(t) = \hat{a}_1 c_{L,k}^{\hat{y}}(t-1)\varphi_{L,k}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)\varphi_{L,k}(t-1) \quad (32)$$

where $k = \overline{1,N}$,

$$d_{l,k}^{\hat{y}}(t)\psi_{l,k}(t) = \hat{a}_1 d_{l,k}^{\hat{y}}(t-1)\psi_{l,k}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)\psi_{l,k}(t-1) \quad (33)$$

where $k = \overline{1,N}, l = \overline{1,L}$. Let us introduce notations for the coefficients in (32):

$$\begin{aligned} & \chi_0 = c_{L,k}^{\hat{y}}(t) \neq 0, \\ & \chi_1 = \hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1). \end{aligned} \quad (34)$$

Let us introduce notations for coefficients in (34):

$$\begin{aligned} & \mu_0 = d_{l,k}^{\hat{y}}(t) \neq 0, \\ & \mu_1 = \hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1). \end{aligned} \quad (35)$$

By virtue of notations (34) and (35) introduced, let us rewrite (32) and (33) correspondingly in the following form:

$$\chi_0 \varphi(t) = \chi_1 \varphi(t-1). \quad (36)$$

$$\mu_0 \psi(t) = \mu_1 \psi(t-1). \quad (37)$$

One can interpret relationships (36) and (37) as a representation of a system in the state space.

$$\left| \frac{\chi_1}{\chi_0} \right| < 1; \quad (38)$$

$$\left| \frac{\mu_1}{\mu_0} \right| < 1. \quad (39)$$

The systems of inequalities (38) and (39), by virtue of the notations earlier introduced, can be rewritten for the approximating part in the form (40) $\forall\ k = \overline{1,N}$, and for the detailing part in the form of (41) for $\forall\ k = \overline{1,N}, l = \overline{1,L}$.

$$\left| \frac{\hat{a}_1 c_{L,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} c_{L,k}^{\hat{x}_s}(t-1)}{c_{L,k}^{\hat{y}}(t)} \right| < 1, \quad (40)$$

$$\left| \frac{\hat{a}_1 d_{l,k}^{\hat{y}}(t-1) + \sum_{s=1}^{S} \hat{b}_{s,1} d_{l,k}^{\hat{x}_s}(t-1)}{d_{l,k}^{\hat{y}}(t)} \right| < 1. \quad (41)$$

4 MODELLING OIL REFINING PROCESS

On the basis of preliminary data analysis, a prediction linear model of the following type has been built:

$$\begin{aligned} & T(t) = \sum_{i=1}^{4} b_i F_i(t-1) + b_5 F_5(t-3) + \\ & b_6 F_6(t-5) + \sum_{j=7}^{12} b_j F_j(t-7), \end{aligned} \quad (42)$$

where $T(t)$ is the prediction of the temperature of boiling away of 10% fraction “150-250ºC” (a detailed description of the variables is presented in the paper of Kalashnikov and Sakrutina (2018).

The associative model will have the structure of linear model (42), but a principal distinction of the associative model is forming at each step a new model

Table 1: The comparison of the associative models quantity in accordance to the number of vectors selected from the plant knowledge base.

Number of vectors in the associative model	MAPE	MAE	MSE	Maximal absolute error	Minimal absolute error
195	0,30886%	0,50004	0,42292	3,32514	0,00011
170	0,30058%	0,48662	0,40238	3,17054	0,00093
152	0,29356%	0,47525	0,38331	2,84228	0,00020
133	0,28576%	0,46262	0,36426	2,65459	0,00064
113	0,27167%	0,43978	0,33527	2,21305	0,00010
101	0,26629%	0,43105	0,32104	2,33122	0,00019
86	0,25112%	0,40656	0,29165	2,53347	0,00002
65	0,22790%	0,36897	0,24949	2,46776	0,00027
61	0,22249%	0,36024	0,23835	2,52234	0,00002
60	0,22230%	0,35992	0,23673	2,49180	0,00042
58	0,22063%	0,35721	0,23372	2,44692	0,00003
55	0,21527%	0,34854	0,22429	2,46557	0,00007
54	0,21637%	0,35035	0,22685	2,41414	0,00013
50	0,21267%	0,34437	0,21904	2,24581	0,00021
46	0,20609%	0,33370	0,20835	2,21200	0,00002
42	**0,19653%**	**0,31823**	**0,19297**	**2,35652**	**0,00001**
41	0,19486%	0,31556	0,18879	2,17517	0,00041

on the basis knowledge about the plant, which is updated and specified in the time progress. To determine a necessary quantity of input vectors to build an accurate associative model, by use of a test sample (2400 steps) we will use a number of accuracy and prediction adequacy evaluations. Table 1 contains 17 variants of the number of input vectors, on the basis of which the associative models were being built, for which indicators of the model accuracy have been calculated: MAPE, MAE, MSE, maximal and minimal absolute errors. From the considered models the best associative model has been selected, i.e. most accurate and with smallest quantity of large errors, namely, the one built on the basis of 42 vectors selected from the plant knowledge base.

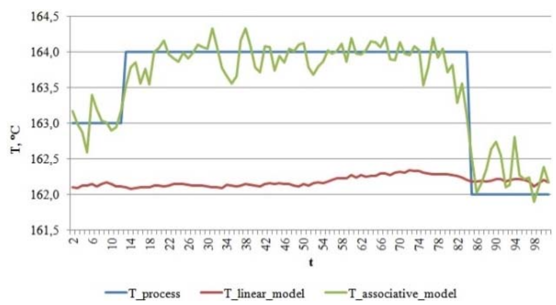

Figure 1: Boiling away point prediction of the 10% fraction "150-250ºC" at steps 2-101.

The considered process prediction was being built on the basis of the linear and associative models form 10525 steps (1 step = 10 min.). Figure 1 displays results of modelling for steps $t = \overline{2,101}$, where the dependencies of data of laboratory analysis of the boiling away temperature of the 10% fraction "150-250ºC" (**T_process**) of the time t, the dependence of predictions of the boiling away temperature of the 10% fraction “150-250ºC” on the basis of the linear model (**T_linear_model**) an associative model (**T_associative_model**) of the time t.

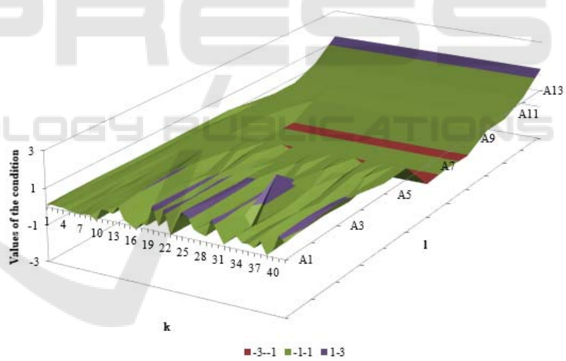

Figure 2: Stability condition of the approximating part for the prediction model in the point t=55 in the dependence of the expansion depth.

For model (42), Figure 2 display an example of meeting the stability criterion for the approximating part:

$$\left|\frac{\sum_{i=1}^{4} b_i c_{L,k}^{F_i}(t-1)}{2c_{L,k}^{T}(t)}\right| < 1.$$

in the dependence of the expansion depth.

5 CONCLUSIONS

In the paper, the results, obtained on the basis of the multi-scale wavelet transform, of the stability conditions of prediction models based on the associative search technique and proving the prediction without accounting possible future states of the prediction ground.

The stability conditions obtained can be applied to the risk potential evaluation (Kalashnikov and Sakrutina, 2018) of implementing the prediction by use, for instance, the Harrington verbal-numerical scale (Harrington, 1965).

REFERENCES

Bakhtadze, N.N., Pavlov, B.V., Sakrutina, E.A., 2013. Development of Intelligent Identification Models and Their Applications to Predict the Submarine Dynamics by Use of Computer Simulation Complexes. In *IFAC-PapersOnLine*, vol. 7, no. 1. pp. 1244-1249.

Bakhtadze, N.N., Sakrutina, E.A., 2015. The Intelligent Identification Technique with Associative Search. In *International Journal of Mathematical Models and Methods in Applied Sciences*, vol. 9, pp. 418-431.

Breidenstein, B., Mörke, T., Hockauf, R., Jörn Ostermann, J., Spitschan, B., 2017. Sensors, data storage and communication technologies. In book *"Cyber-Physical and Gentelligent Systems in Manufacturing and Life Cycle. Genetics and Intelligence - Keys to Industry 4.0"*. Academic Press, pp. 7-278.

Castello, G., Moretti, P., Vezzani, S., 2015. Retention models for programmed gas chromatography. In *Journal of Chromatography A*, vol. 1216, no. 10, pp. 1607-1623.

Ghanem, R., Romeo, F., 2000. A wavelet-based approach for the identification of linear time-varying dynamical systems. In *Journal of Sound and Vibration*, vol. 234, no. 4, pp. 555-576.

Ghanem, R., Romeo F., 2001. A wavelet-based approach for model and parameter identification of non-linear systems. In *International Journal of Non-Linear Mechanics*, vol. 36, no, 5, pp. 835-859.

Grossman, A., Morlet, J. 1984. Decomposition of Hardy functions into square integrable wavelets of constant shape. In *SIAM Journal on Mathematical Analysis*, vol. 14, no. 4, pp. 723-736.

Harrington, E.C., 1965. The desirable function. In *Industrial Quality Control*, vol. 21, no. 10, pp. 494-498.

Kalashnikov, A., Sakrutina, E., 2018. Towards Risk Potential of Significant Plants of Critical Information Infrastructure In *Proceedings of 2018 International Russian Automation Conference (RusAutoCon)*, IEEE Catalog Number CFP18RUS-ART, pp. 1-6.

Kassam, S., 1977. The mean-absolute-error criterion for quantization. In *Acoustics, Speech, and Signal Processing, 1977 IEEE International Conference on Acoustics (ICASSP '77)*, vol. 2, pp. 632-635.

Kim, K.-Y., Park, J., Sohmshetty, R. 2017. Prediction measurement with mean acceptable error for proper inconsistency in noisy weldability prediction data. In *Robotics and Computer-Integrated Manufacturing*, vol. 43, pp. 18-29.

Kwakernakk, H., Sivan, R., 1972. *Linear optimal control systems*. Wiley-interscience, NewYork.

Mallat, S., 1999. *A wavelet tour of signal processing*, Academic press, Amsterdam.

Muto, A., Anandakrishnan, S., Alley, R.B., Horgan, H.J., Parizek, B.R., Koellner, S., Christianson, K., Holschuh, N., 2019. Relating bed character and subglacial morphology using seismic data from Thwaites Glacier, West Antarctica. In *Earth and Planetary Science Letters*, vol. 507, pp. 199-206.

Sakrutina, E., Bakhtadze, N., 2015. Towards the Possibility of Applying the Wavelet Analysis to Derive Predicting Models. In *IFAC-PapersOnLine*, vol. 48, no. 1, pp. 409-414.

Toledo, E., Gurevitz, O., Hod, H., Eldar, M., Akselrod, S. 1998. The use of a wavelet transform for the analysis of nonstationary heart rate variability signal during thrombolytic therapy as a marker of reperfusion. In *Computers in Cardiology*, vol. 25, pp. 609-612.

Tsatsanis, M., Giannakis, G., 2002. Time-varying system identification and model validation using wavelets. In *IEEE Transactions on Signal Processing*, vol. 41, no. 12, pp. 3512-3523.

Wei, H.L., Billings, S.A., 2002. Identification of time-varying systems using multiresolution wavelet models. In *International Journal of Systems Science*, vol. 33, no. 15, pp. 1217-1228.

Wen, X., Zhou, X., 2009. Research and Design of Intelligent Wireless Harmonic Detection of Electric Power System. In *Proceedings of 2009 First International Workshop on Education Technology and Computer Science*, vol. 1, pp. 644-648.

Wen, F., Zhou, Z., Qiao, J., 2010. Notice of Retraction Use Matlab to Realize Acceleration Signal Processing of Armor-Piercing Bullet Penetrating Steel Target. In *Proceedings of 2010 2nd International Conference on Information Engineering and Computer Science*, pp. 1-4.

Xiao-qing Yuan, Yi-kai Shi, 2008. Characteristic spectrum research in ae signals based on wavelet analysis. In *Proceedings of 2008 Symposium on Piezoelectricity, Acoustic Waves, and Device Applications*, pp. 439- 442.