Research and Development of High Performance Finite Element for Large Scale Acoustic Analysis Method

E. Takaoka[a], Y. Murakami and A. Takei[b]
Faculty of Engineering, University of Miyazaki, Miyazaki, Japan

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Abstract: In recent years, numerical methods such as the finite element method are developed for sound field prediction of architectural spaces. However, large-scale analysis is often necessary because of the larger domains and the higher frequencies that we deal with in engineering applications. Therefore, we often only consider larger domains at low frequencies or high frequencies in smaller domains. In this paper, the iterative domain decomposition method (IDDM) is applied to solve transient and steady-state acoustic problems. By applying the IDDM, the effectiveness of the large-scale acoustic analysis can be shown. In addition, the results of introducing high-order elements into the analysis are shown.

1 INTRODUCTION

Prediction of acoustic performance is important for the design of concert halls (Otsuru, 2002). As representative examples of the prediction, there are scale model experiments and computer simulations. Experiments have been used for a long time in many fields and are applied to visualization and audibility. However, they take a lot of time and are expensive to run. On the other hand, computer simulations can develop models in the virtual space which are a lot less expensive than experiments. However, due to the larger problems considered and the increased frequencies, the scale of models must also be increased. Hence, large scale analysis is required.

So far, acoustic analysis methods using the finite difference time-domain (FDTD) (Sendo, 2002), the boundary element method (BEM) (Sakuma, 2009), and the finite element method (FEM) (Okuzono, 2010) have been developed. In the FDTD method, mesh division is performed using a structured grid. Therefore, analysis with complicated shapes can be problematic and application to the analysis of architectural space is difficult. In the boundary element method, the boundary of the analyzed region is divided into elements and analyzed. The method leads to dense matrices which requires a large amount of memory. On the other hand, the finite element method can use unstructured mesh grids and can deal with complex shapes. For these reasons, an acoustic analysis using the finite element method is developed.

One of the problems of the finite element method is that the scale of the analysis increases with the increased frequency. The accuracy of the finite element method improves as the number of divisions with respect to the wavelength, increases. As the frequency increases, the wavelength decreases, and the number of elements required to obtain sufficient resolution increases (Urata, 2004). For this reason, there is a limitation on the wavelengths that can be analyzed with the finite element method. From such a background, it is necessary to develop a method capable of high-accuracy analysis while reducing the number of elements necessary for analysis, and a method capable of large-scale analysis. In this study, by introducing the high-performance finite element, we aim to reduce the number of elements and develop the technology corresponding to the expansion of the analysis domain and the increased frequency.

A parallel finite element steady-state acoustic analysis method applying the iterative domain decomposition method (IDDM) as a parallelization method is proposed. The method have been tested and shown to perform the steady analysis with a maximum error of about 1.4 [%] compared to a reference solution in a benchmark problem

[a] https://orcid.org/0000-0002-5932-2093
[b] https://orcid.org/0000-0001-9014-8727

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(Murakami, 2019). This has enabled large-scale calculations with high accuracy as a numerical acoustic analysis method in steady problems.

In this paper, a transient acoustic analysis method is developed using time domain parallel finite element method based on this parallelization method. We report here that this method has sufficient accuracy in a benchmark problem and that analysis is also possible in real environment applications such as a live house.

2 FORMULATION

2.1 Helmholtz Equation

Let us consider a 3D region with a boundary Γ and a region Ω inside the boundary. The wave equation for the velocity potential in the sound field is expressed by the following equation.

$$\frac{\partial^2\emptyset}{\partial x^2}+\frac{\partial^2\emptyset}{\partial y^2}+\frac{\partial^2\emptyset}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\emptyset=q \tag{1}$$

Where $\emptyset$ is the velocity potential, c [m/s] is the velocity of sound, and q is the distribution function of a sound source.

Considering a steady-state problem Eq. (2) can be reduced to the Helmholtz equation (Eq. (3)).

$$\emptyset=\Phi e^{-j\omega t} \tag{2}$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}+\frac{\omega^2}{c^2}\Phi=q \tag{3}$$

Where j is the imaginary number, and ω is the angular frequency. The sound pressure can be calculated by the following equation using an imaginary number j, and medium density ρ [kg/m^3] after calculating the velocity potential Φ.

$$p=j\omega\rho\Phi \tag{4}$$

2.2 Finite Element Formulation

To derive a weak form, the Galerkin method is applied to Eq. (3). Finite element approximation and discretization gives the following equation.

$$-k^2[M]\{\Phi\}-j\omega\rho[C]\{\Phi\}+[K]\{\Phi\}=\{q\} \tag{5}$$

Where [·] is a matrix, { · } is a vector. Now, let the element matrix of Eq. (5) be $[M]_e$, $[C]_e$, $[K]_e$. When the shape function $\{N\}$ and its transpose $\{N\}^T$ are used, they are expressed as the following equations.

$$\begin{aligned}[M]_e&=\iiint_{\Omega_e}\{N\}\{N\}^T d\Omega_e\\ [K]_e&=\iiint_{\Omega_e}\nabla\{N\}\nabla\{N\}^T d\Omega_e\\ [C]_e&=-\frac{1}{Z_n}\iint_{\Gamma_e}\{N\}\{N\}^T d\Gamma_e\end{aligned} \tag{6}$$

Where $[M]_e$ and $[K]_e$ are volume integrals, and $[C]_e$, is a surface integral to the sound absorbing boundary surface, while Z_n is a specific acoustic impedance. Matrices $[M]$ and $[K]$ are symmetric sparse matrices. Matrix $[C]$ has a complex impedance. Therefore, the entire coefficient matrix is a complex symmetric matrix.

2.3 Time Domain Problem

To convert to time domain representation, Inverse Fourier transform is applied to Eq. (5). Thus the following equation is achieved

$$[A]\{\Phi\}_t+\frac{1}{c^2}[M]\{\ddot{\Phi}\}_t+\rho[C]\{\dot{\Phi}\}_t=\{q\} \tag{7}$$

Where, $\ddot{}$ and $\dot{}$ are first-order and second-order derivatives related to time.

To solve the velocity potential $\{\Phi\}$ in the time domain problem, linear acceleration is applied in the time dimension of Eq. (8). It is assumed that velocity potentials $\{\Phi\}$, $\{\dot{\Phi}\}$ and $\{\ddot{\Phi}\}$ are known at a time t. Applying the Newmark β method (AIJ, 2011), $\beta=\frac{1}{6}$ and $\gamma=\frac{1}{2}$ are used in the time stepping form t to Δt can be approximated in the form of the following equations.

$$\begin{aligned}&\{\Phi\}_{t+\Delta t}=\\ &\{\Phi\}_t+\{\dot{\Phi}\}_t\Delta t+\frac{1}{3}\{\ddot{\Phi}\}_t\Delta t^2+\frac{1}{6}\{\ddot{\Phi}\}_{t+\Delta t}\Delta t^2\end{aligned} \tag{8}$$

$$\{\dot{\Phi}\}_{t+\Delta t}=\{\dot{\Phi}\}_t+\frac{1}{2}\left(\{\ddot{\Phi}\}_{t+\Delta t}+\{\ddot{\Phi}\}_t\right)\Delta t \tag{9}$$

From Eq. (7) to (9), the simultaneous linear equation is obtained.

$$\begin{aligned}&\{\ddot{\Phi}\}_{t+\Delta t}=\\ &\left(\{q\}_{t+\Delta t}-[A]\{Q\}-\rho[C]\{P\}\right)[L]^{-1}\end{aligned} \tag{10}$$

Where,

$$\{Q\} = \left(\{\Phi\}_t + \{\dot{\Phi}\}_t \Delta t + \frac{1}{3}\{\ddot{\Phi}\}_t \Delta t^2\right)$$
$$\{P\} = \left(\{\dot{\Phi}\}_t + \frac{1}{2}\{\ddot{\Phi}\}_t \Delta t\right) \tag{11}$$
$$[L] = \left(\frac{1}{6}[A]\Delta t^2 + \frac{1}{c^2}[M] + \frac{1}{2}\rho[C]\Delta t\right)$$

From Eq. (8) to (10), the solutions of $\{\Phi\}_{t+\Delta t}$ and $\{\dot{\Phi}\}_{t+\Delta t}$ can be obtained.

The velocity potential of the second derivative of the unknown can be obtained using the iterative method of Conjugate Orthogonal Conjugate Gradient (COCG) for the derived Eq. (11). The solution of the time domain problem is obtained by repeating these calculations step by step.

3 PARALLELIZATION

3.1 Interface Problem

In this research, the IDDM is applied as a parallelization method. This is known as a method that can solve large-scale simultaneous linear system of equations efficiently. The procedure of the IDDM is shown below:

(1). The analysis area is divided into an arbitrary number of areas (partial areas).
(2). Perform finite element analysis for using iterative calculation of the interface problem in between these areas.
(3). With the end of the iterative calculations, obtain the solution of the whole analysis area.

The simultaneous linear system of equations to be solved is

$$Ku = f \tag{12}$$

Where, K is the coefficient matrix, u is the unknown vector and f is the known vector. With the smallest unit as an element, the analysis area is divided such that there is no overlap.

$$V = \bigcup_{i=1}^{N} V^{(i)} \tag{13}$$

When the domain decomposition method is applied with u_B degrees of freedom newly generated on the boundaries between the regions, and $u_I{}^{(i)}$ degrees of freedom generated on the partial areas, the following equations are obtained.

$$\begin{bmatrix} K_{II}{}^{(1)} & \cdots & 0 & K_{IB}{}^{(1)}R_B{}^{(1)T} \\ 0 & \ddots & \vdots & \vdots \\ 0 & \cdots & K_{II}{}^{(N)} & K_{IB}{}^{(N)}R_B{}^{(N)T} \\ R_B{}^{(1)}K_{IB}{}^{(1)T} & \cdots & R_B{}^{(N)}K_{IB}{}^{(N)T} & \sum_{i=1}^{N} R_B{}^{(i)}K_{BB}{}^{(i)}R_B{}^{(i)T} \end{bmatrix} \begin{Bmatrix} u_I{}^{(1)} \\ \vdots \\ u_I{}^{(N)} \\ u_B \end{Bmatrix} = \begin{Bmatrix} f_I{}^{(1)} \\ \vdots \\ f_I{}^{(N)} \\ f_B \end{Bmatrix} \tag{14}$$

Where, $R_B{}^{(i)}$ is a 0-1 matrix that limits $u_B{}^{(i)}$ to the internal degrees of freedom of the partial area. Eq. (14) leads to the following equations:

$$K_{II}{}^{(i)}u_I{}^{(i)} = f_I{}^{(i)} - K_{IB}{}^{(i)}u_B{}^{(i)}, \quad i = 1, \dots, N \tag{15}$$

$$\left[\sum_{i=1}^{N} R_B{}^{(i)}\left\{K_{BB}{}^{(i)} - K_{IB}{}^{(i)T}\left(K_{II}{}^{(i)}\right)^{-1}K_{IB}{}^{(i)}\right\}R_B{}^{(i)T}\right]u_B = \sum_{i=1}^{N} R_B{}^{(i)}\left\{f_B{}^{(i)} - K_{IB}{}^{(i)T}\left(K_{II}{}^{(i)}\right)^{-1}f_I{}^{(i)}\right\} \tag{16}$$

Where, $f_B{}^{(i)}$ is the right hand vector for u_B, and $\left(K_{II}{}^{(i)}\right)^{-1}$ is the inverse of $K_{II}{}^{(i)}$. Eq. (16) is the interface problem (Ogino, 2016) which represents a connection between areas in the domain decomposition method.

3.2 IDDM

IDDM is a method to obtain the degrees of freedom for nodes inside partial areas contained in DDM iteratively.

In this research, COCG method is applied as an iterative solution method of IDDM. Where δ is a convergence determination value and $\|\cdot\|$ is 2-norm. Figure 1 shows COCG method algorithm for interface problem.

In (I) and (II) shown in Figure 1, there is a need for a vector product operation of the Schur complement matrix. The construction of this matrix involves a lot of calculations. Therefore, as shown in Figure 1, calculations are performed by substituting the finite element method calculation of the partial area.

The finite element method calculations are performed for each partial area in each step of the iterative solution of the interface problem. Therefore, high parallelism can be expected because it can be calculated independently.

Choose u_B

$r^0 = S{u_B}^0 - g$ .... (I)

$z^0 = M^{-1}r^0$

$p^0 = z^0$

For n=0, 1, 2, ...

$q^n = Sp^n$.. (II)

$\alpha^n = \frac{(r^n)^T z^n}{(p^n)^T q^n}$

$u_B^{n+1} = u_B^n - \alpha^n p^n$

$r^{n+1} = r^n - \alpha^n q^n$

IF $\|r^{n+1}\| < \delta\|r^0\|$, break

$Z^n = M^{-1}r^{n+1}$

$\beta^n = \frac{(r^{n+1})^T z^{n+1}}{(r^n)^T z^n}$

$p^{n+1} = z^{n+1} - \beta^n p^n$

End For

(I) In each subdomain

Compute $u_I^{(i)0}$ by

$K_{II}^{(i)} u_I^{(i)0} = f_I^{(i)0} - K_{IB}^{(i)} {R_{IB}^{(i)}}^T u_B^0$

$r^{(i)0} = {K_{IB}^{(i)T}} u_I^{(i)0} - K_{BB}^{(i)} {R_B^{(i)}}^T u_B^0 - f_I^{(i)0}$

$$p^0 = r^0 = \sum_{i=1}^{N} R_B^{(i)} r^{(i)0}$$

(II) In each subdomain

Compute $p_I^{(i)n}$ by

$K_{II}^{(i)} p_I^{(i)n} = -K_{IB}^{(i)} {R_B^{(i)}}^T p^{(i)n}$

$q^{(i)n} = {K_{IB}^{(i)T}} u_I^{(i)n} - K_{BB}^{(i)} {R_B^{(i)}}^T p^{(i)n}$

$$q^n = \sum_{i=1}^{N} R_B^{(i)} q^{in}$$

Figure 1: COCG method algorithm for IDDM.

3.3 HDDM

The hierarchical domain decomposition method (HDDM) is a DDM having a 2-step hierarchy in which the entire analysis area is first divided into an arbitrary number of parts, and then divided into multiple subdomains (Takei, 2010). HDDM is a method of allocating divided parts to 1 process or 1 thread, performing internal finite element analysis at each step of the iterative solution method of the interface problem, and obtain a solution of the whole analysis area.

The assignment to each computation node of this method is shown in Figure 2.

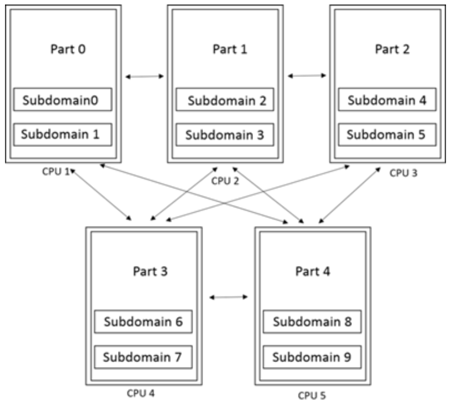


Figure 2: Assignment of HDDM to each compute node.

3.4 BDD Pre-processing

The balancing domain decomposition (BDD) pre-processing is based on the multigrid method, and can greatly improve the convergence of the interface problem (Mandel, 1993). The algorithm of BDD pre-processing is shown in Figure 3. This figure shows finding z $= M^{-1}r$ for the residual r obtained in each step of the COCG method.

Step 1 : $\lambda^{(i)}$ is calculated

$$Z^{(i)^T} D^{(i)^T} R^{(i)^T} \left(r - S \sum_{j=1}^{N} R_B^{(j)} D^{(j)} Z^{(j)} \lambda^{(j)} \right) = 0,\ i = 1, \dots, N$$

Step 2: $S^{(i)}$ is calculated

$$s = r - S \sum_{j=1}^{N} R_B^{(j)} D^{(j)} Z^{(j)} \lambda^{(j)} = 0,$$

$S^{(i)} = D^{(i)^T} R^{(i)^T} s,\quad i = 1, \dots, N$

Step 3: Solve the local problem in each region.

$S^{(i)} u^{(i)} = s^{(i)},\ \ i = 1, \dots, N$

Step 4: $\mu^{(i)}$is calculated

$$Z^{(i)^T} D^{(i)^T} R^{(i)^T} \left(r - S \sum_{j=1}^{N} R_B^{(j)} D^{(j)} \left(u^{(j)} + Z^{(j)} \mu^{(j)} \right) \right) = 0, \qquad i = 1, \dots, N$$

Step 5: z is calculated

$$z = \sum_{j=1}^{N} R_B^{(j)} D^{(j)} \left(u^{(j)} + Z^{(j)} \mu^{(j)} \right)$$

Figure 3: Algorithm of BDD pre-processing.

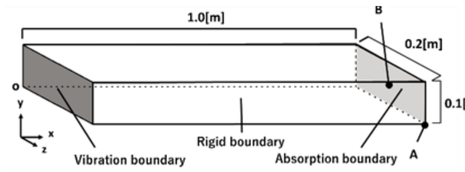


Figure 4: AHLV100.

4 PERFORMANCE EVALUATION

4.1 Benchmark Model

An acoustic problem benchmark model is used to verify our method. The analysis uses the test model AHLV100 of Code_Aster known as a representative benchmark problem of acoustic problems. The model is shown in Figure 4.

This model is an acoustic tube having a length of 1 [m], a height of 0.1 [m] and a width of 0.2 [m], and has a vibration boundary (sound source) at the left end and a sound absorption boundary at the right end. Specific acoustic impedance $Z_n = 445.9$ [kg/m$^3 \cdot$ s] is given as a sound absorbing boundary condition.

The analysis environment uses 1 PC equipped with Intel Core i7-8700 multi-core CPU and 32GB of memory.

4.2 ADVENTURE System

ADVENTURE system is a generic term for a group of system modules in a large-scale computational dynamics development project for design (ADVENTURE Project, Accessed on: April 15, 2019). The system is developed by several universities research groups. ADVENTURE project is centered at the University of Tokyo. This system has the following features.

(1). Analysis by meshes of hundreds to 100 million degrees of freedom is possible.

(2). Even in a parallel computing environment of 2,000 processors, high parallel efficiency can be achieved.

(3). It is free and open source.

(4). Scalability and maintainability are secured by standardization of modular structure and I/O.

In this research, the ADVENTURE system is used for analysis model creation, mesh generation, and setting of boundary conditions.

4.3 Analysis Result (AHLV100)

The results of a steady-state analysis of AHLV100 are shown in Figure 5, and the results of transient state analysis are shown in Figure 6.

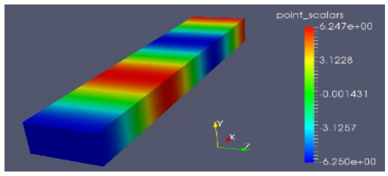


Figure 5: Result of steady-state.

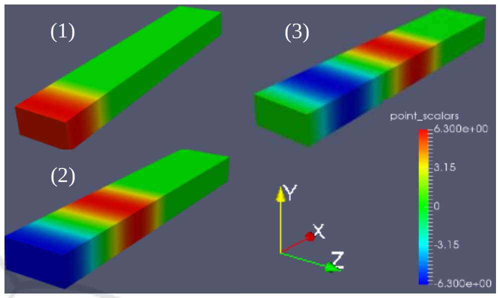


Figure 6: Result of transient state.

From Figure 6, the acoustic phenomenon of the closed tube can be confirmed. Also, it can be confirmed that the plane wave from the sound source propagates vertically in the *x*-direction.

Table 1 shows the results of accuracy verification. Here, the verification is performed at the time when the sound pressure distribution becomes the same as the solution shown in Figure 5. Comparison with the reference solution is made at node A belonging to the sound absorbing surface shown in Figure 4.

Table 1: Analysis result.

Point	A
Theoretical [Pa]	6.0237
Numerical [Pa]	5.9721
Error [%]	-0.8566

4.4 Analysis of Real Environment Model

Perform analysis based on the real environment to confirm whether large-scale analysis is possible. The analysis target is a small acoustic environment (live house). This is a model based on an existing live house. The appearance and dimensions of the analysis target are shown in Figure 7 and Figure 8.

The live house model used has a shape that simulates a sound source, a stage, human bodies, and structures inside. From the sound source, a sound pressure of 0.2 [Pa] is released where assuming the performance of the instrument is given as a tone burst. Sound absorption boundary conditions are set to the specific acoustic impedance, the floor is set to wood, and other wall surfaces and structures are set to glass wool. The cylinder that simulates the human body gives the boundary condition as total stiffness. The medium is air ρ =1.2 [kg/m^3], the speed of sound is c =340[m/s] and the frequency is 560 [Hz].

The analysis environment is a PC cluster composed of 25 PCs (100 cores) equipped with a multicore CPU of Intel Core i7 2600K (3.4GHz / L2 256KB x4 / L3 8MB) and 32GB of memory.

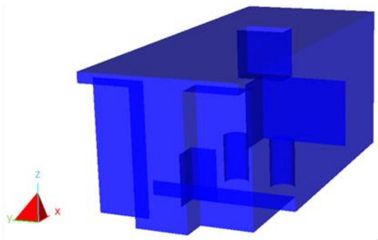
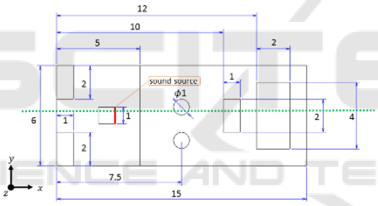
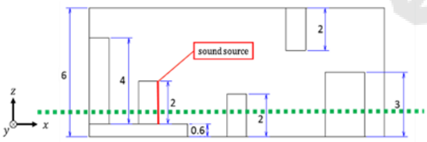


Figure 7: Appearance of model shape.



(a) Z axis viewpoint [m].



(b) Y axis viewpoint [m].

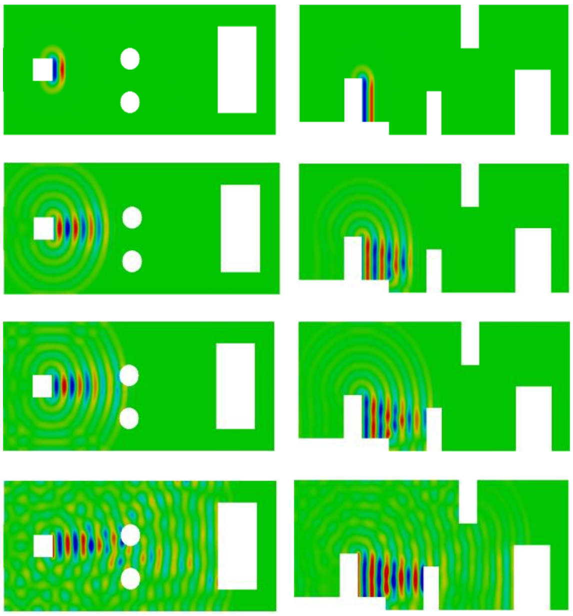
Figure 8: Dimensions of model shape.

4.5 Analysis Result (Live House)

The visualization results of the sound pressure viewed from the Y, Z axes directions are shown in Figure 9(a) and (b). Visualization is the result of the green dotted line in Figure 8.

4.6 Higher Order Elements

For large-scale analysis, in addition to the development of solvers as described above, the reduction of the number of necessary elements is also an issue. Therefore, in addition to the commonly used



(a) Z axis (b) Y axis

Figure 9: Appearance of model shape.

2nd order elements, authors introduce 3rd order elements. This is introduced to the non-parallel code which is the test code described above. The analysis target is AHLV100 shown in 4.1. Analysis frequency is 1 [kHz], 2 [kHz].

The analysis results of the error rate of 0.5 [%] are shown in Table 2. Where, NOE is number of elements, NOI is number of iterations and NOD is number of divisions.

From these results, it can be seen that the number of elements is greatly reduced by the introduction of the high-order elements, and highly accurate analysis can be performed with a small number of divisions. The impact is particularly large when the frequency is high. However, further reduction of the number of elements is necessary for practical use.

Table 2: Analysis result (high order element).

Freq.	1 [kHz]		2 [kHz]	
Nth elm.	2	3	2	3
NOE	822	142	24 808	1 487
NOI	274	122	2 867	2 180
NOD	6.92	3.86	10.8	4.22

5 CONCLUSIONS

This paper describes a large-scale acoustic analysis method using a parallel finite element method based on the IDDM, and discusses the introduction of high order elements.

In the transient analysis using the test model AHLV100, the analysis was performed correctly, and the usefulness of this method could be confirmed. Also shown that transient analysis can be performed correctly even in real environmental problems such as the live house.

Moreover, it was shown that high precision analysis is possible to reduce the number of elements by introducing high order element. In particular, it was found that the higher the frequency, the greater the benefit of high order elements. By introducing this into the parallelized code, further efficiency of analysis can be expected.

However, application to large-scale and complex sound environments requires further reduction of the number of elements. For this reason, to introduce high-performance elements such as the partition of unity FEM (PUFEM) with high tracking ability to waveforms, and promote research and development.

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