Information Retrieval in a Concept Lattice by using Uncertain Logical Gates

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Keywords: Data Mining, Formal Concept Analysis, Possibility Theory, Uncertain Logical Gates.

Abstract: Formal Concept Analysis (FCA) is an approach of data mining which consists in extracting formal concepts in order to provide a hierarchy of concepts also called a concept lattice. It is useful for understanding data. A formal concept is a set of objects which share the same properties. When the number of formal concepts is too high, it is difficult to explore all formal concepts in order to look for information. The use of a query to extract relevant information is a solution to this problem. A logical combination of Boolean criteria, which can be represented by a logical circuit, can serve as the condition of the query. In the uncertain formal context, we are not sure if the objects own a property. As a consequence, we must take into account uncertainties in the computation of formal concepts and in queries. We propose in this paper to use possibility theory to handle these uncertainties. As a result, we compute a necessity degree for each formal concept. We can use a query in which the condition can be computed by using possibilistic networks and uncertain logical gates. Finally, we illustrate our approach by the analysis of a satisfaction questionnaire for a course in bachelor.

1 INTRODUCTION

Formal Concept Analysis (FCA) was presented by Rudolf Wille as a mathematical theory (Wille, 1982). This method of data analysis consists in extracting formal concepts in a formal context. The latter can be obtained from human investigation such as measures, questionnaire, etc. It is often represented as a table. The formal context can be defined as a triplet composed of a set of objects, a set of properties, and a binary relation which provides the properties owned by the objects.

All formal concepts can be compared by using a partial order operator. As a result, we can build a concept lattice from which we can extract knowledge or rules. Another advantage of FCA is to avoid the loss of information as in statistics summaries. The formal concepts are very easy to interpret by a person who is not an expert, thus avoiding wrong interpretation. They highlight the common properties for a set of objects, this is useful for analysing information. Nevertheless, the number of formal concepts grows exponentially when the size of the formal context increases. As a consequence, the task of knowledge discovering is more and more complex. So we can use a query to extract formal concepts according to the user's expectations.

Another problem is how to deal with uncertainties when the formal context is uncertain. Several studies have been performed by using fuzzy set theory, accuracy degree, probability theory, or possibility theory (Dubois and Prade, 2015; Yang and Qin, 1507). We will focus our interest on possibility theory. We can define an uncertain formal context by using a pair of necessity measures as in (Dubois et al., 2007; Dubois and Prade, 2009; Dubois and Prade, 2015) and propose to extract uncertain formal concepts. The certainty of all formal concepts can be computed.

Nevertheless, if there are too many formal concepts, we can extract information by using a query, but we must take into account uncertainties. In fact, the condition of a query can be a logical combination of criteria. Moreover, the criteria may be imprecise and uncertain. As in Boolean logic, we propose to represent the condition of the query by a logical circuit composed of gates AND, OR and NOT. Then, in order to take into account imprecisions and uncertainties, we propose to use uncertain logical gates of possibility theory. The latter were proposed by the authors of (Dubois et al., 2015) as an analogy of noisy gates in probability theory. Uncertain logical gates allow us to compute automatically the conditional possibility tables of the possibilistic networks and to avoid eliciting all conditional possibilities of

Petiot, G.
Information Retrieval in a Concept Lattice by using Uncertain Logical Gates.
DOI: 10.5220/0008065902890296
In *Proceedings of the 11th International Joint Conference on Knowledge Discovery, Knowledge Engineering and Knowledge Management (IC3K 2019)*, pages 289-296
ISBN: 978-989-758-382-7

the table. For example, if a variable has 2 modalities and 7 parents with 2 modalities, we have $2^8 = 256$ parameters to elicit. The uncertain logical gates also allow us to represent uncertainty and missing knowledge in the models.

The goal of our experimentation is to use FCA for the analysis of a satisfaction questionnaire for a course of professionalization in bachelor. In this questionnaire, there is one open question and several closed questions. Whereas the students answer the open question with their own words, closed questions provide a set of possible answers. The main difficulty is the processing of answers to the open question.

To do this, we propose to present in a first part possibility theory, which will be used in the next parts dealing with formal concept analysis and uncertain logical gates. In the last part, we propose to perform a natural language processing of the answers in order to classify them. As the classification generates uncertainties, we must propagate them in the computation of formal concepts. Then, we must use a query to extract uncertain formal concepts and show the results in a graph which highlights uncertainties.

2 POSSIBILITY THEORY

Possibility theory was invented by L. A. Zadeh (Zadeh, 1978) in 1978. This theory allows us to represent the imprecision of knowledge and uncertainty. Authors in (Dubois and Prade, 1988) define a possibility distribution π as a state of knowledge. For example, if Ω is the universe and π_x a possibility distribution of a variable x defined from Ω in $[0,1]$, then if $\pi_x(u) = 0$ then $x = u$ is impossible, else if $\pi_x(u) = 1$ then $x = u$ is possible. We can define the possibility measure Π and the necessity measure N from the set of subsets of Ω (noted $P(\Omega)$) in $[0,1]$:

$$\forall A \in P(\Omega), \Pi(A) = \sup_{x \in A} \pi(x). \quad (1)$$

$$\forall A \in P(\Omega), N(A) = 1 - \Pi(\neg A) = \inf_{x \notin A} 1 - \pi(x). \quad (2)$$

Possibility theory is not additive but maxitive:

$$\forall A, B \in P(\Omega), \Pi(A \cup B) = \max(\Pi(A), \Pi(B)). \quad (3)$$

3 FORMAL CONCEPT ANALYSIS

FCA, introduced by R. Wille (Wille, 1982), is built on mathematical lattice theory. It organizes formal concepts, which are the sets of objects, and their shared properties into a concept lattice. Formal concepts are defined by the intent and the extent. The intent is the definition of the concept or the set of properties and the extent denotes the elements to which the properties apply.

The structured data which must be provided as input in formal concept analysis are called a formal context. The latter is presented as a table where the lines are the objects and the columns are the properties also called attributes.

In fact, the formal context is a triple $(O, P, \mathfrak{R})$ where $O = \{o_1, ..., o_n\}$ is the set of objects, $P = \{p_1, ..., p_m\}$ is the set of properties, and $\mathfrak{R}$ is a relation such as $\mathfrak{R} \subseteq O \times P$. If $(o, p) \in \mathfrak{R}$, then the object o has the property p. In this case, the value of the table is 1 or else 0.

A formal concept of $(O, P, \mathfrak{R})$ is a pair (X, Y) such that $X \in O$ and $Y \in P$ where Y is the set of properties shared by all objects of X. For example, in the following formal context, we obtain 6 formal concepts.

Table 1: Example of a formal context.

$\mathfrak{R}_1$	p_1	p_2	p_3
o_1	0	1	0
o_2	0	1	1
o_3	0	0	1
o_4	1	1	0
o_5	1	1	1

The formal concepts are $(\{o_2, o_5\}, \{p_2, p_3\})$, $(\{o_4, o_5\}, \{p_1, p_2\})$,$(\{o_5\}, \{p_1, p_2, p_3\})$, $(\{o_1, o_2, o_4, o_5\}, \{p_2\})$, $(\{o_2, o_3, o_5\}, \{p_3\})$ and $(\{o_1, o_2, o_3, o_4, o_5\}, \{\emptyset\})$.

The set of all formal concepts of $(O, P, \mathfrak{R})$ is noted $\beta(U, V, \mathfrak{R})$. To compare the formal concepts we can define a partial order $\leq$ such that for $(X_1, Y_1), (X_2, Y_2) \in \beta(U, V, \mathfrak{R})$, then $(X_1, Y_1) \leq (X_2, Y_2)$ if $X_1 \subseteq X_2$ or $Y_2 \subseteq Y_1$. The lattice concept can be defined by using this partial order and visualized by using a Hasse diagram. The following figure shows the concept lattice of the previous example:

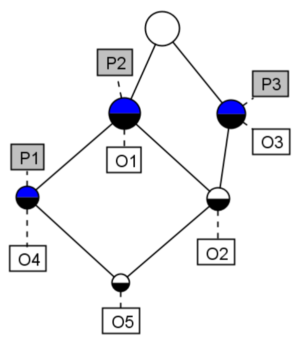

Figure 1: Concept lattice of the example.

When the properties are many-valued, we must perform a transformation of the context into a binary formal context. We can take as an example the following many-valued context:

Table 2: Example of a many-valued context.

$\Re_2$	Measure	Quality
o_1	0	low
o_2	4	medium
o_3	7	medium
o_4	8	high
o_5	9	low

We can see that the measure is numerical with a range in $[0,10]$, so we must propose a categorization of the values by defining for example three classes. The first one is low for the values in $[0,3]$, the second is medium for the values in $[4,6]$, and the last class is high for the values in $[7,10]$. It can be transformed into the following binary formal context:

Table 3: The transformation of the many-valued context into a binary formal context.

$\Re_3$	M_{low}	M_{medium}	M_{high}	Q_{low}	Q_{medium}	Q_{high}
o_1	1	0	0	1	0	0
o_2	0	1	0	0	1	0
o_3	0	0	1	0	1	0
o_4	0	0	1	0	0	1
o_5	0	0	1	1	0	0

So far, the properties were certain but if the properties are uncertain, the computation of the formal concepts must take into account these uncertainties. The authors in (Dubois et al., 2007) propose to use possibility theory (Zadeh, 1978) and to define a possibility distribution $\pi_{o_p}(u)$ with $u \in \Omega$, which is the possibility that the property p of the object o is u. This possibility distribution must be normalized. Certainty is the necessity measure in possibility theory. The authors in (Dubois and Prade, 2015) propose to use a pair of necessity measures $(\alpha(o,p),\beta(o,p))$ with $\alpha(o,p) = N((o,p) \in \Re)$ and $\beta(o,p) = N((o,p) \notin \Re)$ which represents the certainty that the object has or does not have the property. We can define the uncertain formal context in the following formula:

$$\Re' = \{(\alpha(o,p),\beta(o,p)) | o \in O, p \in P\} \quad (4)$$

Moreover, we must satisfy the property of possibility theory $min(N((o,p) \in \Re), N((o,p) \notin \Re)) = 0$. The advantage of this solution is to provide a theoretical frame to represent ignorance, which can be partial or full. Indeed, if the pair is $(1,0)$ or $(0,1)$ in the uncertain formal context, we are sure that the object has the property or not. Otherwise, we have two cases to describe. In the first case if $1 > max(\alpha(o,p),\beta(o,p)) > 0$, ignorance is partial. In the second case, if we have $(0,0)$, ignorance is total.

For our first experimentation, we will transform the uncertain context by replacing the values $(\alpha(o,p),0)$ by 1 and $(0,\beta(o,p))$ by 0. Thus, we obtain a new formal context for which we can easily compute the formal concepts. Once the formal concepts are extracted, we can compute the necessity measure (the certainty) of a formal concept $C = (X,Y)$ by using the following formula:

$$N(C) = \min_{o \in X, p \in Y} N((o,p) \in \Re) \quad (5)$$

To illustrate this computation, we provide the following example:

Table 4: Example of an uncertain formal context.

$\Re'$	p_1	p_2	p_3
o_1	(0,1)	(1,0)	(0.2,0)
o_2	(0,0.5)	(1,0)	(1,0)
o_3	(0.5,0)	(1,0)	(0,0.9)
o_4	(1,0)	(1,0)	(0.8,0)
o_5	(1,0)	(1,0)	(1,0)

If we perform for this uncertain formal context the transformation of the uncertain values into sure ones, we obtain:

Table 5: Transformation of the uncertain formal context into a binary formal context.

$\Re_4$	p_1	p_2	p_3
o_1	0	1	1
o_2	0	1	1
o_3	1	1	0
o_4	1	1	1
o_5	1	1	1

In this example, we can see that $(\{o_1,o_2,o_4,o_5\},\{p_2,p_3\})$, $(\{o_3,o_4,o_5\},\{p_1,p_2\})$, $(\{o_4,o_5\},\{p_1,p_2,p_3\})$ and $(\{o_1,o_2,o_3,o_4,o_5\},\{p_2\})$ are formal concepts of this formal context. We can now compute the certainty of these formal concepts:

Table 6: Computation of the formal concept certainties.

Formal concepts	**Certainty**
$(\{o_1,o_2,o_4,o_5\},\{p_2,p_3\})$	0.2
$(\{o_3,o_4,o_5\},\{p_1,p_2\})$	0.5
$(\{o_4,o_5\},\{p_1,p_2,p_3\})$	0.8
$(\{o_1,o_2,o_3,o_4,o_5\},\{p_2\})$	1

In our experimentation, among the existing algorithms described in (Kuznetsov and Obiedkov, 2003), we have chosen Ganter Algorithm *Next Closure* (Ganter, 1987) to find all intents or extents of the formal concepts.

4 UNCERTAIN LOGICAL GATES

Possibilistic networks (Benferhat et al., 1999; Borgelt et al., 2000; Dubois et al., 2015) are based on d-separation, conditional independence (Amor and Benferhat, 2005), and factoring property. The factoring property can be defined from the joint possibility distribution $\Pi(V)$ for a directed acyclic graph $G=(V,E)$ where V is the set of variables and E the set of edges between the variables. $\Pi(V)$ can be factorized as following:

$$\Pi(X_1,...,X_n) = \bigotimes_{i=1}^{n} \Pi(X_i/Pa(X_i)). \quad (6)$$

With Pa the parents of the node X_i. The function used for $\bigotimes$ is the minimum.

If we have a set of causal variables $X_1,...,X_n$ which influence another variable Y called effect variable, we can introduce intermediate variables $Z_i s$ between each $X_i s$ and Y. These variables represent uncertainty in the causal influence of $X_i s$ on Y. For example, even if a cause is met, it is possible that an inhibitor will not produce Y. The Independence of Causal Influence (Dìez and Drudzel, 2007) can be defined as the independence of the variables $Z_i s$ given $X_1,...,X_n$. A causal mechanism is independent of all other causal mechanisms of the model.

In probability theory, the ICI model gives birth to the noisy model. In this model, there is a deterministic function f which combines the individual influences of the variables $Z_i s$. The equation of the combination is the following: $Y = f(Z_1,...,Z_n)$. The leaky ICI model is derived from the noisy model by adding a leakage variable Z_l which represents the unknown knowledge in the model. By analogy, we propose the same reasoning for possibility theory and we can define a possibilistic model with the ICI. This possibilistic model is presented in the following graph:

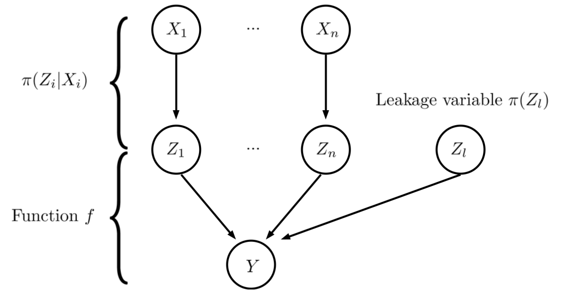

Figure 2: Possibilistic model with ICI.

We propose to calculate $\pi(Y|X_1,...,X_n)$ by marginalizing the variables $Z_i s$ as following:

$$\pi(y|x_1,...,x_n) = \bigoplus_{z_1,...,z_n} \pi(y|z_1,...,z_n) \otimes \pi(z_1,...,z_n|x_1,...,x_n) \quad (7)$$

The $\otimes$ is the minimum and the $\oplus$ is the maximum in possibility theory.

$$\pi(y|x_1,...,x_n) = \bigoplus_{z_1,...,z_n} \pi(y|z_1,...,z_n) \otimes \bigotimes_{i=1}^{n} \pi(z_i|x_i) \quad (8)$$

$$\text{where } \pi(y|z_1,...,z_n) = \begin{cases} 1 & \text{if } y = f(z_1,...,z_n) \\ 0 & \text{else} \end{cases} \quad (9)$$

As a result, we obtain:

$$\pi(y|x_1,...,x_n) = \bigoplus_{z_1,...,z_n : y=f(z_1,...,z_n)} \bigotimes_{i=1}^{n} \pi(z_i|x_i) \quad (10)$$

If we add a leakage variable Z_l in the previous model, we obtain the following equation:

$$\pi(y|x_1,...,x_n) = \bigoplus_{z_1,...,z_n,z_l : y=f(z_1,...,z_n,z_l)} \bigotimes_{i=1}^{n} \pi(z_i|x_i) \otimes \pi(z_l) \quad (11)$$

Authors in (Dìez and Drudzel, 2007) provide several examples for probability theory. They are also applicable to possibility theory. The functions f can be AND, OR, NOT, INV, XOR, MAX, MIN, MEAN, and linear combination. The Conditional Possibility Table (CPT) is obtained by the calculation of the above formula. For Boolean variables, the possibility table between the variables X_i and Z_i is as follows:

Table 7: Possibility table for Boolean variables.

$\pi(Z_i\|X_i)$	$\neg x_i$	x_i
$\neg z_i$	1	κ_i
z_i	0	1

In the above table, the κ_i parameter can be interpreted as the possibility that an inhibitor exists if the cause is met. The value 0 in the table means that it is impossible to have an effect if the cause is not met. The possibility of the variable Z_L is $\pi(z_L) = \kappa_L$. It corresponds to an external event which causes Y without any influence of the variables X_i. Several uncertain logical gate connectors AND, OR, MIN and MAX were described in (Dubois et al., 2015). A mathematical simplification has been performed leading to optimized connectors. The connectors AND, OR and NOT allow us to build and evaluate uncertain logical circuit. In other words, it can be used for the condition of a query on formal concepts. To do this, we must provide the function f for the connectors AND and OR by taking into account the leakage variables Z_l. The first one is the function f for the uncertain leaky AND which is $f = \bigwedge_{i=1}^{n} Z_i \bigvee Z_L$. The second one is the function f for the uncertain leaky OR which is $f = \bigvee_{i=1}^{n} Z_i \bigvee Z_L$. For example, from the equation 11 we compute the following conditional tables of the uncertain leaky AND for two causal variables:

Table 8: Conditional tables for uncertain leaky AND.

$\pi(\neg y \vert X_1, X_2)$	$\neg x_1$	x_1
$\neg x_2$	1	1
x_2	1	$\kappa_1 \oplus \kappa_2$

$\pi(y \vert X_1, X_2)$	$\neg x_1$	x_1
$\neg x_2$	κ_L	κ_L
x_2	κ_L	1

We provide below the same example for the connector uncertain leaky OR:

Table 9: Conditional tables for uncertain leaky OR.

$\pi(\neg y \vert X_1, X_2)$	$\neg x_1$	x_1
$\neg x_2$	1	κ_1
x_2	κ_2	$\kappa_1 \otimes \kappa_2$

$\pi(y \vert X_1, X_2)$	$\neg x_1$	x_1
$\neg x_2$	κ_L	1
x_2	1	1

This example can be generalized to the case of n causal variables as in (Dubois et al., 2015). We can also propose the table of the NOT connector which has only one variable:

Table 10: Conditional table for NOT connector.

$\pi(Y \vert X)$	$\neg x$	x
$\neg y$	0	1
y	1	0

5 EXPERIMENTATION

The experimentation consists in the analysis of course satisfaction questionnaire in bachelor. In this questionnaire realized in the Learning Management System Moodle, there is one open question and 32 many-valued closed questions. 144 students answered the questionnaire. We proposed to perform a supervised classification of the answers to the open question by a neural network in 8 classes. For the learning phase, we constructed sets of samples by gathering a text description of the classes and samples for all classes. We chose 16% of all answers for the samples. Then, we performed a preprocessing of all sets of samples and answers in order to obtain a Document Term Matrix (DTM) for samples and answers. We present the processing in the following graph:

In order to take into account the spelling mistake in the answers, we proposed to use a measure of resemblance between the words during the computation of the DTM and during the classification. We decided

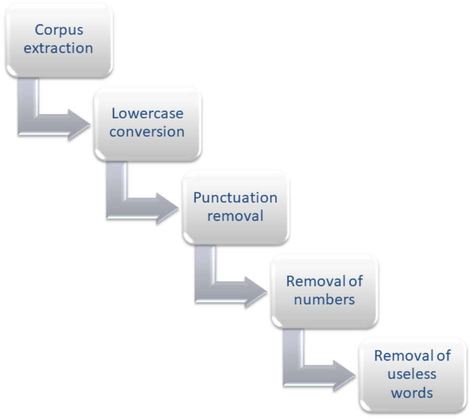

Figure 3: Processing of the corpus.

to use a measure in $[0, 1]$. Several string metrics exist to measure the resemblance of two strings (Christen, 2006; Jaro, 1989). The most famous are the distance of Levenshtein, Jaccard, Damerau-Levenshtein, Hamming, the longest common subsequence, Smith-Waterman and Jaro-Winkler (Winkler, 1999). We chose the distance of Jaro-Winkler.

This distance is computed by using the Jaro distance between the words w_1 and w_2:

$$d_J(w_1, w_2) = \frac{1}{3}\left(\frac{\chi}{|w_1|} + \frac{\chi}{|w_2|} + \frac{\chi - \tau}{\chi}\right) \tag{12}$$

With $|w_i|$ the size of the word i, χ the number of matching character (the number of characters which are in the two words with a distance smaller or equal to $\left\lfloor \frac{\max(|w_1|, |w_2|)}{2} \right\rfloor - 1$). τ is the number of transposition (the number of characters inverted). The Jaro-Winkler distance is:

$$d_{JW}(w_1, w_2) = d_J(w_1, w_2) + \alpha\beta(1 - d_J(w_1, w_2)) \tag{13}$$

With α the size of the common prefix of the two words with a maximum of 4 characters and β a coefficient often equal to 0.1. So if $d_{JW}(w_1, w_2) < \eta$, then the word w_1 is different from the word w_2. The next step is the construction of the DTM. For example, for the students' answers the result is the following:

Table 11: Example of a DTM for the students' answers.

Words / Students	intelligences	gardner	questionnaire	proust	cv	...
student 1	0.0	0.0	0.0	0.0	0.0	...
student 2	1.0	0.96	0.0	0.0	0.0	...
student 3	0.0	0.0	0.0	0.0	0.0	...
...	...	...	...	...	...	...
student N	0.0	0.0	0.0	0.0	0.0	...

When the DTM is computed for the students' answers and the samples of the classes, we can perform the classification of the students' answers by the neural network. The learning of the coefficient of the neural network is performed by using a backpropagation of the gradient. The confusion matrix of the classification is the following:

Table 12: Confusion matrix.

Actual \ Predicted	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	**35**	0	0	0	0	0	0	0
C_2	0	**29**	0	0	0	0	0	0
C_3	0	0	**25**	0	0	0	0	0
C_4	0	0	0	**20**	2	0	0	0
C_5	0	0	0	0	**11**	0	0	0
C_6	0	0	0	0	1	**7**	0	0
C_7	0	0	0	0	0	0	**4**	0
C_8	0	0	0	0	2	0	0	**8**

The membership degree of all classes is transformed in order to compute a possibility measure and a pair of necessity measures. The next phase is the computation of the formal concepts. We have transformed the many-valued questions in order to obtain a binary formal context. Then, we have integrated in this formal context the pair of necessity measures of the classification by inserting one column for all classes. As a result, we have an uncertain formal context where the columns are possible answers (the properties of FCA) noted P_i and the lines the answers of the students (the objects of FCA). The first 8 properties concern the classes C_i of the open question. We present below a part of the uncertain formal context:

Table 13: A part of the uncertain formal context.

$\Re'$	P_1 (C_1)	P_2 (C_2)	P_3 (C_3)	P_4 (C_4)	P_5 (C_5)	...
Student 1	(0,1)	(0,1)	(0,1)	(0,1)	(0.49,0)	...
Student 2	(0.99,0)	(0,1)	(0,1)	(0,1)	(0,1)	...
Student 3	(0,1)	(0,1)	(0,1)	(0,1)	(0.99,0)	...
Student 4	(0.1,0)	(0,1)	(0,1)	(0,1)	(0,1)	...
Student 5	(0,1)	(0,1)	(0,1)	(0,1)	(0.91,0)	...
...	...	...	...	...	...	...
Student 144	(0,1)	(0,1)	(0,1)	(0,1)	(0.99, 0)	...

If we look for formal concepts which fit best with the student's answers, we can define a score. For example, if (X,Y) is a formal concept with X the extent and Y the intent, then we can propose the following score:

$$S = \frac{|X| + |Y|}{\max\limits_{(u,v)\in\beta(U,V,\Re)} |u| + |v|} \tag{14}$$

As there is often a very large number of formal concepts, it is necessary to filter these formal concepts in order to visualize only those which are relevant. To do this, we have performed two processing operations on the formal concepts. The first one is the use of a query to extract formal concepts. The query is a logical combination of criteria which can be evaluated by using a possibilistic network which uses uncertain logical gates. The second one is the visualization of the result by using a directed graph. The orientation of the edge of the graph is defined by using the partial order. The size of the nodes is proportional to the score of relevance and the color of the node is proportional to the certainty of the formal concept. We chose to use the Gephi tool for visualization. The result of the query is generated in csv files before being imported in Gephi. For example, we present below a query :

Q^***=SELECT** c **FROM** $\beta(U,V,\Re)$
WHERE
(($c.P_1$ is true **OR** $c.P_2$ is true **OR** $c.P_3$ is true **OR** $c.P_4$ is true **OR** $c.P_5$ is true **OR** $c.P_6$ is true **OR** $c.P_7$ is true **OR** $c.P_8$ is true)
AND
$Score(c)$ is high
AND
$Card(c.X)$ is high)

With $c = (X,Y)$ a formal concept of $\beta(U,V,\Re)$, *Card* the number of properties or objects in the formal concept. Then, $c.P_i$ is true if the formal concept has the property else $c.P_i$ is false. Finally, $c.P_i$ *is true*, $Score(c)$ *is high*, and $Card(c.X)$ *is high* are possibility distributions which take into account the imprecision of knowledge. The condition of the query Q^* can be represented as a logical circuit as follows:

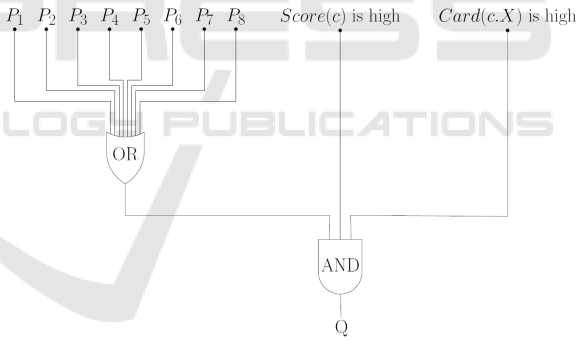

Figure 4: Logical circuit of the condition of query Q^*.

The logical circuit can be transformed in a possibilistic network with uncertain logical gates. The domain of all the variables is $(false, true)$. To evaluate the condition, we must perform several processing operations. At first, we compute the CPTs of uncertain logical gates. Then, we compute the evidences by using the possibility distributions associated to all states of the variables. Finally, we propagate the evidence in the possibilistic network by using the algorithm of message passing in a junction tree (Lauritzen and Spiegelhalter, 1988) of Bayesian networks adapted to possibilistic networks. The junction tree is composed of cliques and separators. The cliques are computed by transforming the initial graph into a moral graph and triangulated graph (Kjaerulff, 1994). Then we apply the Kruskal algorithm (Kruskal, 1956). To resume, the propagation algorithm has three steps. The initialization with the injection of evidence, then the collect with the propagation of evidence from leaf to root and the distribution with the propagation of evidence from root to leaf. We propose for the example of our query the following possibility distributions in order to compute evidence:

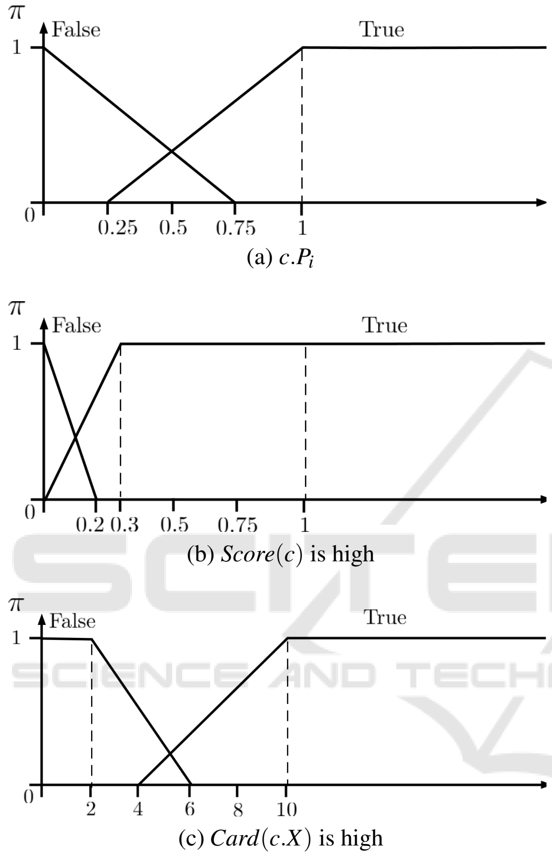

Figure 5: Possibility distributions.

We apply this computation to all formal concepts. As a result, we obtain for the variable Q of the logical circuit a possibility measure and a necessity measure for all states. We can deduce the formal concepts which answer the query where $N(Q = true) > 0$. As in a web query in a search engine, where the result is a ranking of the web pages, the certainty $N(Q = true)$ can be considered as a score of relevance which allows us to perform a ranking of the formal concept from the more certain to the less certain. For our application, we propose the following result where the labels are only displayed every 10 formal concepts for more visibility:

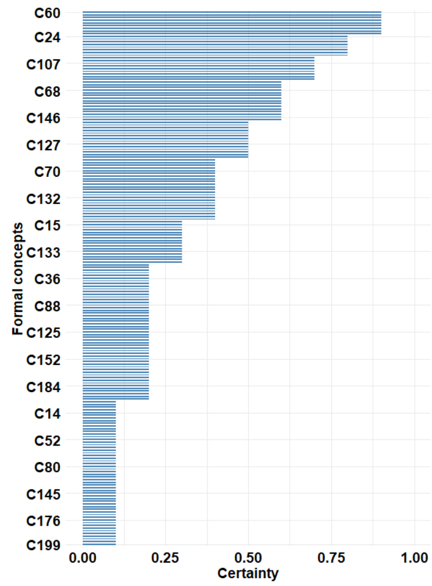

Figure 6: Score of relevance ($N(Q = true)$).

We can see that the formal concepts can be sorted as expected. Then we can propose to visualize the result of the query Q^* with the following diagram:

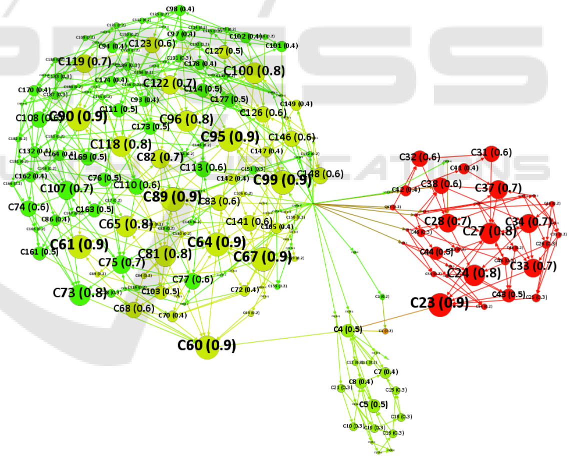

Figure 7: Example of a query result.

In this figure, we can see the certainty of the formal concepts but also their score of relevance for the query. The scores of relevance of the formal concepts are given in brackets in the labels of the nodes in the diagram. If we consider the example of the formal concept $C60$, we can see that the size of the node is one of the most important. The score of relevance is equal to 0.9 for this formal concept. The knowledge that we can extract from this formal concept is that the students have appreciated the part of the course concerning the theory of multiple intelligences of H. Gardner.

6 CONCLUSIONS

In this paper, we present an experimentation of formal concept analysis which allows us to take into account uncertainties. We have proposed to compute a certainty degree for all formal concepts by using possibility theory. We have used queries in order to extract formal concepts in the concept lattice. The condition of the queries can be a logical combination of criteria leading to a logical circuit. All criteria are transformed into a possibility distribution in order to take into account the imprecision and uncertainty of knowledge. This logical circuit can be transformed into a possibilistic network with uncertain logical gates. As a result, we computed a score of relevance for all formal concepts which allow us to present a ranking of the formal concepts. Then, we presented a visualization of the results in a diagram with a colour shading proportional to the certainty of the formal concept and a node size proportional to the score of relevance. For our future works, we would like to generalize this approach to variables with more than two states in order to extend the possible criteria. We would like to improve the performance of the computation of the formal concepts and optimize the inference of the possibilistic networks. We would like to propose further evaluation in order to better evaluate how uncertainties can be useful in applications. Finally, we have to develop an HMI with a query assistant which would allow a graphical expression of queries and a code generation to improve the usability of our tool.

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