An Efficient and Accurate Contact Model for Rough Surfaces Considering of Microscopic Interaction

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Abstract: Traditional rough surface contact models either neglect the interaction between asperities, resulting in inaccurate analysis results, or adopt exhaustive method because of considering interaction, resulting in a huge amount of calculation. In order to establish an efficient and accurate contact model for rough surfaces, the deformation distribution of asperities considering interaction is analyzed by exhaustive method, based on Green function. The results show that the deformation of the asperity still obeys the normal distribution approximately. Therefore, a fitting function is established to describe the distribution of asperities under micro-interaction through data analysis. Then, a new contact model for rough surfaces considering of microscopic interaction is established. Compared with the exhaustive method, the correctness of the model is verified, and the efficient and accurate analysis of rough surface is realized.

1 INTRODUCTION

The contact between rough surfaces has complex mechanical properties. The identification of stiffness and other parameters of the contact between rough surfaces is of great significance to the dynamic and static analysis of structures. In order to reveal the deformation mechanism in the contact process, it is necessary to study the contact characteristics of the interface on the micro scale.

The earliest rough surface contact model was proposed by GREENWOOD (Greenwood, 1966), which is called GW model. The model only considers the elastic deformation of the asperities. In order to consider the plastic deformation of asperities, Chang et al. (Chang, 1987) proposed a CEB model, which divides the deformation process of asperities into elastic deformation and plastic deformation. Zhao et al. (Zhao, 2007) believed that there should be an elastic-plastic deformation transition stage between elastic deformation and plastic deformation. So a ZMC model was proposed to supplement and improve the whole process of asperities deformation. On the basis of these three models, many scholars have further analyzed and applied the rough surface contact model (Li, 2016, Xiao, 2019). Ciavarella et al. (Ciavarella, 2006) established a contact model for rough surfaces considering interaction and compared it with GW model. The result shown that there was a large error between the two models when the loads were large, which proved that the interaction between asperities could not be neglected. Iida et al. (Iida, 2003) considered the interaction of asperities on the basis of GW model, and calculated the actual contact force of the interface by exhaustive method. Although these models have established rough surface contact models considering microscopic interaction and can reflect contact characteristics from the contact mechanism, the use of exhaustive method makes the calculation amount increase with the number of asperities on the contact surface, which is difficult to be analyzed effectively for contact characteristics of large surfaces such as bolt joint surface.

In order to comprehensively consider the accuracy and efficiency of rough surface contact model, the Iida exhaustive contact model is used to study the deformation distribution law of the asperities on the rough surface when considering the interaction. It is found that the deformation distribution of the asperities considering the interaction still obeys the normal distribution. Therefore, a fitting function is established to describe the distribution of asperities under micro-interaction through data analysis. Then, a new contact model for rough surfaces considering

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of microscopic interaction is established. The proposed model is compared with the exhaustive method and the model neglecting the micro-interaction. The results illustrate the importance of the interaction on the analysis results and verify the correctness of the proposed model. Therefore, the efficient and accurate analysis of rough surface is realized according to the proposed model.

2 OPTIMIZED ZMC CONTACT MODEL

The micro-model between two rough surfaces is very complex. It is difficult to analyze the micro-model directly. It is found that the contact between two rough surfaces can be replaced by the contact between an equivalent rough surface and a rigid smooth surface. Therefore, this equivalent model is adopted in general contact model.

The assumption of GW model is adopted in ZMC contact model, which are: (1) The micro-morphology of the surfaces is isotropic. (2) The interaction between the asperities on the surface is neglected. (3) The top of all asperities is spherical and the curvature radius is the same. (4) The height of asperities is random distribution. (5) Only the deformation of asperities in contact is considered, while the macro-matrix is not deformed.

Based on the above assumptions and statistical probability theory, if there are N asperities on nominal contact area A_n, the expected number of asperities contacting with the rigid smooth surface is n, which is

$$n = N \int_d^{\infty} g(z)\mathrm{d}z = \eta A_n \int_d^{\infty} g(z)\mathrm{d}z \tag{1}$$

Where η is the distribution density of the number of asperities, d is the distance between the smooth rigid plane and the average line of the height of asperities, z is the distance between the height of each asperity and the average line of the height of asperities, $g(z)$ is the probability density function of the height distribution of asperities. A large number of studies have shown that the height of the asperities on the engineering surface obeys the normal distribution. Therefore, the expected number n of asperities contacted is

$$n = \eta A_n \int_d^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(z-\theta)^2}{2\sigma^2}\right) \mathrm{d}z \tag{2}$$

Where θ is the average height of the asperity peaks, σ is the standard deviation of the height of the asperity peaks.

Therefore, the total contact load F of the interface is:

$$F = F_e + F_{ep} + F_p \tag{3}$$

Where F_e, F_{ep}, F_p represent the sum of the loads of the asperities in the elastic, elastic-plastic and plastic deformation stages respectively. The expressions of F_e, F_{ep}, F_p are based on Eq. (2) and have been deduced by Zhao (Zhao, 2007) and Li (Li, 2016) according to Abbott et al's theses (Abbott, 1995, Francis, 1976, Johnson, 1987, Kogut, 2002, Lin, 2005, Liou, 2010, Timoshenko 1990).

3 STUDY ON DEFORMATION DISTRIBUTION OF ASPERITIES AND EQUIVALENT MODELING

Iida considered that it was unreasonable to neglect the interaction between asperities on the interface in GW contact model. Therefore, Iida studied the influence of interaction between asperities on contact loads between two rough surfaces by exhaustive method, based on Green's function. The normal deformation of the benchmark of asperity i caused by the contact load of asperity j can be expressed by Green function (Iida, 2003):

$$u_{zij} = \frac{F_j}{\pi E* \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}} \tag{4}$$

Where u_{zij} is the deformation of the benchmark of asperity i in Z direction caused by the contact load of asperity j, F_j is the contact load of asperity j, $(x_i, y_i), (x_j, y_j)$ are the coordinates of the asperity i and j in the XY plane respectively.

According to Eq. (4), the deformation u_{zi} of the benchmark of asperity i in Z direction caused by the contact loads of all the other asperities can be obtained.

$$u_{zi} = \sum_{j=1}^{N} u_{zij} \quad (j \neq i) \tag{5}$$

Therefore, the actual deformation w_i of the asperity i is

$$w_i = z_i - d - u_{zi} \tag{6}$$

The relationship between the contact load and the deformation of a single asperity can be achieved according to the contact load functions of each asperity (Zhao, 2007, Li, 2016). Therefore, total contact load on rough surfaces considering interaction can be obtained by exhaustive method:

$$F = \sum_{j=1}^{N} f(w_i) \tag{7}$$

Where $f(w_i)$ is the contact load calculation function of the asperity i.

Iida exhaustive contact model considers the interaction of asperities on the basis of ZMC contact model, which makes the result more accurate. However, for common contact surfaces, such as bolt joints, the number of asperities on the analysis surfaces reaches tens of thousands or even more. Meanwhile, the contact load F_j in Eq. (4) is also an unknown parameter related to the deformation w_j and needs to be obtained by iteration. Therefore, there is of great calculation complexity when this method is used.

In order to solve the problem of huge amount of calculation in Iida exhaustive contact model, the deformation distribution of asperities on rough surface considering interaction is studied based on the Iida exhaustive contact model. According to the research result, an equivalent model is established to improve calculation efficiency and ensure calculation accuracy.

3.1 Study on Distribution Law of Deformation of Asperities Considering Interaction

In order to study the deformation distribution of asperities considering the interaction, the surface in Iida's paper (Iida, 2003) is analyzed. The parameters are shown in Table 1. The material properties of the two contact surfaces are shown in Table 2. The contact area analyzed in this paper is square.

When studying the deformation distribution of asperities, the deformation distribution of asperities in $(-\infty, \infty)$ range is considered, that is, negative deformation exists in asperities. Eq. (6) is substituted into Eq. (7) and the deformation distribution of asperities considering the interaction is calculated by iteration. In order to make the results more obvious, the deformation distribution of 1000 groups of asperities is calculated. The results are shown in Figure 1. It can be found that the deformation distribution of the asperities still obeys the normal distribution. Therefore, the deformation distribution function of the asperities considering interaction obeys $N(\theta_1, \sigma_1)$.

The relationship among the mean value θ_1, the standard deviation σ_1 of the deformation distribution function of the asperities and the contact parameters, such as the contact area, is further analyzed. The influence of the change of the distance d between the smooth rigid plane and the average line of the height of asperities and the contact area on the mean value θ_1 and the standard deviation σ_1 of the deformation distribution function is studied. The analysis results are shown as Figure 2.

Table 1. Typical Rough Surface Parameters.

Sample	Mean asperity height	Standard deviation of asperity height	Asperity radius of curvature	Asperity density
1	2nm	0.7nm	2μm	$1/\mu m^2$

Table 2. Properties of contact materials.

Material properties	Upper contact body	Lower contact body
Modulus of elasticity E[Pa]	3.85e11	7.1e10
Hardness H[Pa]	2.34e10	2.37e10
Poisson ratio	0.3	0.244

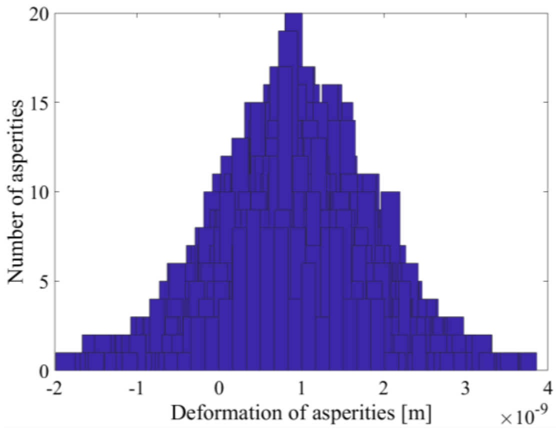

Figure 1. Deformation distribution of asperities considering interaction.

3.2 Equivalent Contact Model with High Efficiency and Accuracy

According to the curve shown in Figure 2 and data analysis, Eq. (8) and (9) are selected to fit the mean value and standard deviation of the deformation distribution of the asperities respectively.

$$\theta_1 = a_1 * exp(b_1 * l) \tag{8}$$

$$\sigma_1 = a_2 * exp(b_2 * l) + c_2 * exp(d_2 * l) \tag{9}$$

Where a_1, b_1, a_2, b_2, c_2 and d_2 are undetermined constants, l is the side length of contact area.

The parameters a_1, b_1, a_2, b_2, c_2 and d_2 are fitted with the fitting toolbox cftool in MATLAB, as shown in Table 3. The R-square of each fitting function is shown in Table 4.

According to Table 3 and 4, it can be found that each fitting function (i.e. Eq. (8) and (9)) can fit each point well.

Furthermore, the relationship between the distance d between two surfaces and the fitting functions is studied. Some results are obtained:

(1) The relationship between the parameter a_1 in Eq. (8) and the distance d satisfies: $a_1 = -0.9864d + 2.015e - 9$. The R-square of the fitting function is 1.

(2) The parameter b_1 in Eq. (8) varies slightly with distance d. Its maximum variation is less than 2.7%. Therefore, its average value is taken as the parameter in the final fitting function.

(3) According to Figure 2(b), the growth trend of standard deviation of each curve is slow when l > 4e-6m. The maximum change of adjacent points (spacing 1e-6m) is 0.6%, and the change gradually tends to zero. Meanwhile, there is little difference in standard deviation between curves, the maximum error is 1.1%. Therefore, the average value is taken as the standard deviation in the final fitting function.

Table 3. Fitting results of the parameters a_1, b_1, a_2, b_2, c_2 and d_2.

Distance d($*e-10$)		0	2	4	6	8	10
Mean value θ_1	$a_1(*e-9)$	2.014	1.817	1.621	1.424	1.226	1.027
	$b_1(*e4)$	-4.928	-4.863	-4.823	-4.799	-4.807	-4.891
Standard deviation σ_1	$a_2(*e-10)$	7.053	7.055	7.033	7.034	7.049	7.035
	b_2	1007	796.1	880.9	747.1	159.5	147.9
	$c_2(*e-10)$	-4.996	-3.541	-3.678	-3.113	-2.957	-3.559
	$d_2(*e6)$	-1.284	-1.086	-1.137	-1.03	-0.9821	-1.053

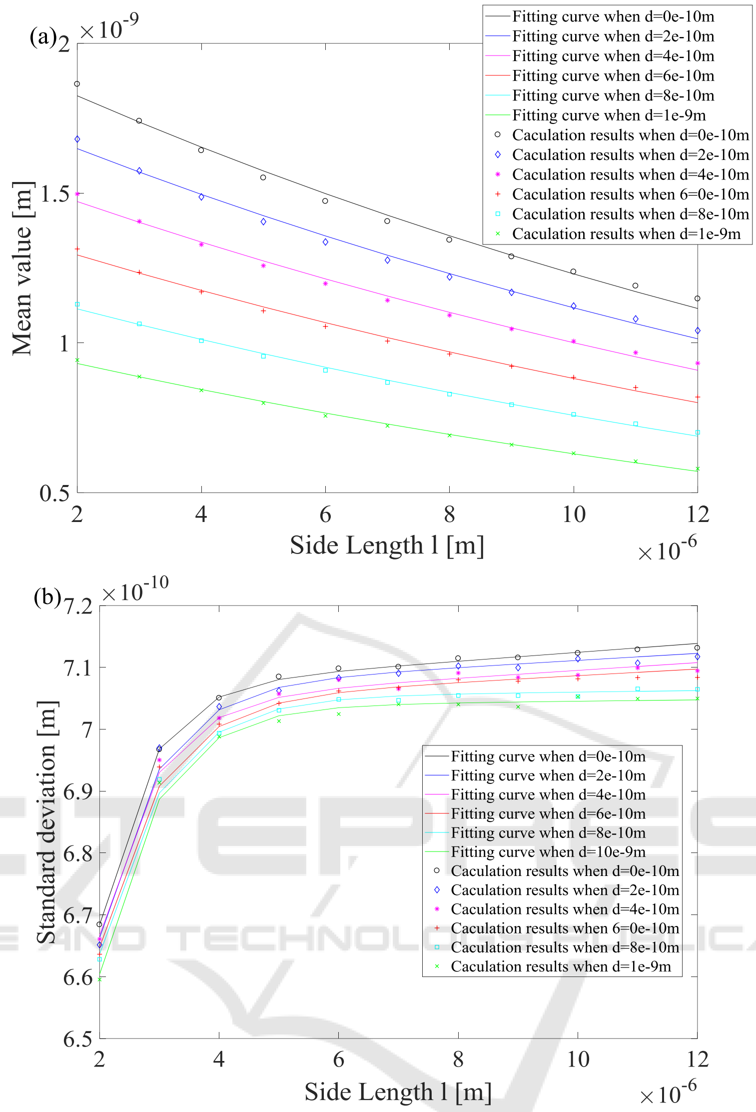

Figure 2: The relationship between the deformation distribution of asperities and the distance *d* between two surfaces and the side length *l* (contact area): (a) mean value, (b) standard deviation.

Table 4. The R-square of each fitting function.

Distance d($* e - 10$)		0	2	4	6	8	10
Fitting function R-square	Mean value θ_1	0.9909	0.9923	0.9933	0.9946	0.9961	0.9971
	Standard deviation σ_1	0.9992	0.9917	0.9934	0.9919	0.9958	0.9939

In summary, the fitting function is finally obtained:

$$\theta = \left(-0.9864d + 2.015e - 9\right) * exp\left(-4.8518\,l\right) \tag{10}$$

$$\sigma = 7.0804e - 10 \tag{11}$$

3.3 Model Validation

In order to verify the accuracy of the proposed model, the contact forces when the contact area is 10e-6m*10e-6m and the distance d between two surfaces is $d = 0$ are calculated with different models. The results are compared as shown in Table 5.

Table 5. The contact forces calculated with different models.

Calculated models	F /mN	Error with exhaustive method
Model proposed in this paper	0.5975	0.27%
Optimized ZMC Contact Model	1.1316	90%
Iida exhaustive contact model	0.5959	/

According to the calculation result, the fitting model proposed in this paper is basically consistent with the exhaustive model. Meanwhile, it can be found that the model without considering the interaction will produce great errors.

4 CONCLUSION

In this paper, the traditional micro-contact model of rough surfaces is optimized in order to consider the accuracy and efficiency of calculation comprehensively. The deformation distribution law of asperities considering interaction is studied by using Iida exhaustive model. Fitting function is established to describe the distribution of asperities under micro-interaction through data analysis. Therefore, a new contact model of rough surfaces is proposed. Compared with the optimized ZMC model and Iida exhaustive model, the correctness of the proposed model and the non-negligibility of the interaction are verified.

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