PI Piecewise Continuous Observer Design for Sampled and Delayed Linear Systems with Variable Sampling Time Period

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Abstract: In recent years, network control systems and visual servo systems have received a lot of attention, but due to network delay and the low sampling rate of visual sensors, it has caused problems for control. In order to reduce the effects of the sampling and delay, this paper deals with proportional integration piecewise continuous observer (PI-PCO), which is based on the theory of a particular class of hybrid systems, called linear piecewise continuous hybrid system (LPCHS). This proposed PI-PCO can estimate the continuous and non-delay state by using the sampled and delayed measurements with variable sampling time period. To show the proposed PI-PCO performance, some numerical simulations with compared results are demonstrated.

1 INTRODUCTION

The networked vision servo control system (NVSS) consists of a networked control system and a visual servo system (VSS). Compared with the traditional control systems, NVSSs have greater flexibility. However, output measurements can only be obtained at discrete sampling moments in NVSS, and the network may introduce time delay, which will degrade the performance of the system and cause instability. To avoid these problems, many observer design methods are proposed. The most famous one is the Kalman-Filter: a Switching-Kalman-Filter is developed in (Chroust, S. & Vincze, M., 2003); a Fussy-Kalman-Filter is proposed in (Perez, C., 2007). Besides, a continuous observer based on constructing a Lyapunov-Krasovskii function is designed in (Shen et al., 2016). A sampled-output observer is designed by compensating the time-delay and sampling with an output predictor (Kahelras, M., 2016). What's more, an equivalent system method is considered in (Natori et al., 2008). The delay generated in the network is equated with adding a network disturbance in the original system to convert a delay system into a network disturbance system, then design a communication disturbance observer (CDOB) for delay compensation. In recent years, a hybrid system approach is developed: paper (Zhang et al., 2016) designs a sampled data observer for a class of upper triangular nonlinear systems with sampling and delay measurements; the work (Wang et al., 2015, 2016), which we study on, proposed piecewise-continuous observer. It makes possible to estimate the non-delayed continuous state using sampled and delayed output.

The proportional integration observer (PIO) was first introduced by Wojciechowski in (Wojciechowski, 1978) for single-input single-output systems. Compared with the Luenberger observer, PIO adds an integral loop to the estimation error feedback. The integral part in the feedback provides freedom for the estimation in two aspects: on the one hand, it improves the robustness of the estimation (Shafai et al., 1996, 2015); on the other hand, it serves as a state and disturbance observer (Chang, 2006) to estimate the state and unknown input simultaneously, and able to improve robustness. Paper (Wu et al., 2018) proposed a proportional integral extended state observer by introducing an integral term to the linear extended state observer. Paper (Vahedforough & Shafai, 2008) extends the traditional proportional adaptive observer to the proportional integral adaptive observer. In paper (Son et al., 2015; Kim & Son, 2017), a double reduced PI observer was proposed. A robust PI observer is proposed in paper (Kim et al., 2016). In this paper, to improves the robustness of

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the estimation, the PIO design method in (Shafai, & Saif, M., 2015) is adopted.

Thus based on our previous proposed PCO in (Wang et al., 2015) and integration of the PIO theory (Shafai, & Saif, M., 2015), a proportional integration piecewise continuous observer (PI-PCO) is proposed to estimate the continuous undelayed state with unknown varing time delay and sampling period, and the other side to improve its robustness. Compared with the proportional Luenberger observer, the PI observer has a stronger ability to suppress system state reconstruction errors, resulting in more accurate estimation of performance. The performance comparison of PCO and PI-PCO is shown in the simulation results.

2 PROBLEM DESCRIPTION

In this paper, we consider the case where the NVSS sampling period T_i equals to the time delay, whose values are unknown and variables. Its corresponding architecture is illustrated in Figure 1.

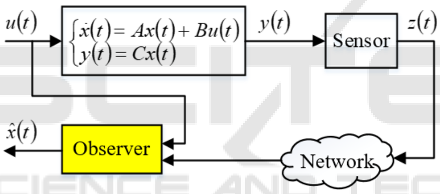

Figure 1. Networked vision servo system.

The system for disposal can be modelled as

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ z(t)=y^{*}(t-T_i)\end{cases} \tag{1}$$

Where $*$ represents variable sampling period T_i.

For simplification, $z(t)$ can be denoted as y_{i-1}.

3 PI-PIECEWISE CONTINUOUS OBSERVER DESIGN

In this section, a PI-PCO with five-step algorithm is designed, which is based on the LPCHS (Koncar & Vasseur, 2003; Wang et al., 2015, 2016), and its corresponding structure is shown in Figure 2.

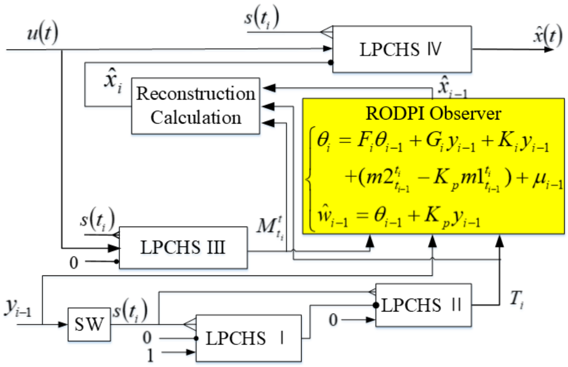

Figure 2. PI-PCO architecture.

3.1 PI-PCO Design

In this section, a PI-PCO with five-step algorithm is designed. In order to suppress the measurement noise and improve the robustness of the PCO, a PIO (Shafai, & Saif, M., 2015) is used to combine with the reduced order discrete Luenberger (RODL).

Firstly, use the LPCHS I of $\sum\left[\{t_i\},0,1,1,1\right]$ with the inputs of $u_s(t)=1$ and $v_s(t)=0$, the time interval between two successive sampling instants is integrated. Then, LPCHS II of $\sum\left[\{t_i\},0,0,I_n,I_n\right]$ with the inputs of $u_s(t)=0$ and $v_s(t)=v(t)$ is used as a zero-order-holder to generate the variable sampling interval and delay T_i.

Secondly, use the LPCHS III of $\sum\left[\{t_i\},A,B,I_n,I_n\right]$ with the inputs of $u_s(t)=u(t)$ and $v_s(t)=0$, then sampling the output by using a ZOH with variable period T_i, one obtains

$$M_{t_{i-1}}^{t_i}=\begin{bmatrix} m1_{t_{i-1}}^{t_i}\\ m2_{t_{i-1}}^{t_i}\end{bmatrix}=\int_{t_{i-1}}^{t_i}e^{A(t_i-\tau)}Bu(\tau)d\tau \tag{2}$$

With $m1\in R^m$ and $m2\in R^{n-m}$.

Thirdly, a reduced-order dimensional PI observer is designed.

In time piece $[t_{i-1},t_i]$, the state of the system can be calculated as:

$$x_i=A_d(T_i)x_{i-1}+\begin{bmatrix} m1_{t_{i-1}}^{t_i}\\ m2_{t_{i-1}}^{t_i}\end{bmatrix}. \tag{3}$$

Assume $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, and $x_1\in R^l$, $x_2\in R^{n-l}$, $x_1=y$, $x_2=w$.

$$A_d\left(T_i\right) = e^{AT_i} = \begin{bmatrix} A_{11}\left(T_i\right) & A_{12}\left(T_i\right) \\ A_{21}\left(T_i\right) & A_{22}\left(T_i\right) \end{bmatrix} \tag{4}$$

Where $A_{11}\left(T_i\right) \in R^{l \times l}$, $A_{12}\left(T_i\right) \in R^{l \times (n-l)}$, $A_{21}\left(T_i\right) \in R^{(n-l) \times l}$, $A_{22}\left(T_i\right) \in R^{(n-l)\times(n-l)}$ are variable constant matrices.

Substituting (5) into (3), one has

$$\begin{bmatrix} y_i \\ w_i \end{bmatrix} = \begin{bmatrix} A_{11}\left(T_i\right) & A_{12}\left(T_i\right) \\ A_{21}\left(T_i\right) & A_{22}\left(T_i\right) \end{bmatrix} \begin{bmatrix} y_{i-1} \\ w_{i-1} \end{bmatrix} + \begin{bmatrix} m1_{t_{i-1}}^{t_i} \\ m2_{t_{i-1}}^{t_i} \end{bmatrix} \tag{5}$$

Then, one obtains

$$y_i = A_{11}\left(T_i\right) y_{i-1} + A_{12}\left(T_i\right) w_{i-1} + m1_{t_{i-1}}^{t_i} \tag{6}$$

$$w_i = A_{21}\left(T_i\right) y_{i-1} + A_{22}\left(T_i\right) w_{i-1} + m2_{t_{i-1}}^{t_i} \tag{7}$$

Let

$$Z = y_i - A_{11}\left(T_i\right) y_{i-1} - m1_{t_{i-1}}^{t_i} \tag{8}$$

$$V = A_{21}\left(T_i\right) y_{i-1} + m2_{t_{i-1}}^{t_i} \tag{9}$$

Substituting (6) and (7) into (8) and (9), we can get the state space expression of the $(n-l)$ dimension subsystem whose state vector is w:

$$Z = A_{12}\left(T_i\right) w_{i-1} \tag{10}$$

$$w_i = A_{22}\left(T_i\right) w_{i-1} + V \tag{11}$$

In the formula, V is the subsystem's input vector, Z is the output vector, $A_{22}\left(T_i\right)$ is the coefficient matrix, and $A_{12}\left(T_i\right)$ is the output matrix. Since the original system is fully observable, the subsystem must also be observable.

The reduced-order dimensional PI observer can be defined as follows:

$$\hat{w}_i = A_{22}\left(T_i\right) \hat{w}_{i-1} + V + K_p(Z - \hat{Z}) + L_{i-1} \tag{12}$$

$$\hat{Z} = A_{12}\left(T_i\right) \hat{w}_{i-1} \tag{13}$$

$$L_i = L_{i-1} + K_i(Z - \hat{Z}) \tag{14}$$

Where L_i is the vector representing the integral of the estimation error of Z, and the matrices K_p and K_i are selected to ensure the stability of the observer.

In the interval $\left[t_{i-1}, t_i\right]$, the state space expression of the reduced-dimensional observer can be written as follows:

$$\begin{cases} \theta_i = F_i \theta_{i-1} + G_i y_{i-1} + (m2_{t_{i-1}}^{t_i} - K_p m1_{t_{i-1}}^{t_i}) + L_{i-1} \\ \hat{w}_{i-1} = \theta_{i-1} + K_p y_{i-1} \end{cases} \tag{15}$$

With

$$F_i = A_{22}\left(T_i\right) - K_p A_{12}\left(T_i\right) \tag{16}$$

$$G_i = F_i K_P + A_{21}\left(T_i\right) - K_P A_{11}\left(T_i\right) \tag{17}$$

Fourthly, calculate the non-delayed but sampled state $\hat{x}_i$ with the following equation:

$$\hat{x}_i = A_d\left(T_i\right) \hat{x}_{i-1} + M_{t_{i-1}}^{t_i} \tag{18}$$

Lastly, the continuous and non-delayed state $\hat{x}$ can be obtained by using the LPCHS IV of $\sum\left[\{t_i\}, A, B, I_n, I_n\right]$ with the inputs of $u_s(t) = u(t)$ and $v_s(t) = \hat{x}_i$:

$$\hat{x}(t) = e^{A(t-t_i)} \hat{x}_i + \int_{t_i}^{t} e^{A(t-\tau)} B u(\tau) d\tau \tag{19}$$

3.2 PI-Piecewise Continuous Observer Stability Analysis

The stability analysis of the PI-PCO can be achieved by the analysis of the state estimation error.

According to equation (19), the estimate error $e(t) = x(t) - \hat{x}(t)$ can be formulated as:

$$e(t) = e^{A(t-t_i)}(x_i - \hat{x}_i) \tag{20}$$

Denoting

$$e_i = x_i - \hat{x}_i = \begin{bmatrix} 0 \\ w_i - \hat{w}_i \end{bmatrix} \overset{\Delta}{=} \begin{bmatrix} e_{y,i} \\ e_{w,i} \end{bmatrix} \tag{21}$$

The $e_{w,i}$ is expressed as:

$$\begin{aligned} e_{w,i} &= w_i - \hat{w}_i = w_i - (K_p y_i + \theta_i) \\ &= A_{21}(T_i) y_{i-1} + A_{22}(T_i) w_{i-1} + m2_{t_{i-1}}^{t_i} \\ &\quad -K_p (A_{11}(T_i) y_{i-1} + A_{12}(T_i) w_{i-1} + m1_{t_{i-1}}^{t_i}) \\ &\quad -\left(F_i\theta_{i-1} + G_i y_{i-1} + (m2_{t_{i-1}}^{t_i} - K_p m1_{t_{i-1}}^{t_i}) + L_{i-1}\right) \end{aligned} \tag{22}$$

From (22) and the second equation of (15), one can get:

$$\theta_i = w_i - e_{w,i} - K_p y_i \tag{23}$$

Thus:

$$\theta_{i-1} = w_{i-1} - e_{w,i-1} - K_p y_{i-1} \tag{24}$$

From equation (22), one gets:

$$\begin{aligned} e_{w,i} &= F_i e_{w,i-1} + (A_{22}(T_i) - K_p A_{12}(T_i) - F_i) w_{i-1} \\ &\quad + (A_{21}(T_i) - K_p A_{11}(T_i) + F_i K_p - G_i) y_{i-1} - L_{i-1} \end{aligned} \tag{25}$$

According to (16) and (17), one can get:

$$e_{w,i} = F_i e_{w,i-1} - L_{i-1} \tag{26}$$

According to the algorithm of the PI-PCO, the K_p is selected to make sure F_i is a stable matrix to ensure that $\hat{w}_i$ converges rapidly to w_i, and L_{i-1} is the integral term of the error of w_{i-1}. This guarantees a rapid decay of $e_{w,i}$, which leads to a rapid decay of e_i. Therefore, the attenuation of the estimation error $e(t)$ and the state stability of the PI-PCO are guaranteed.

4 SIMULATION RESULTS

In this section, the performance of PI-PCO is compared with PCO on a networked visual servo mobile cart system introduced in (Wang et al., 2015). The input signal is selected as $u(t) = \sin 4t$, and the measurement is disturbed with an additional measuring noise, which is a gaussian noise with a covariance value 0.001 and zero mean.

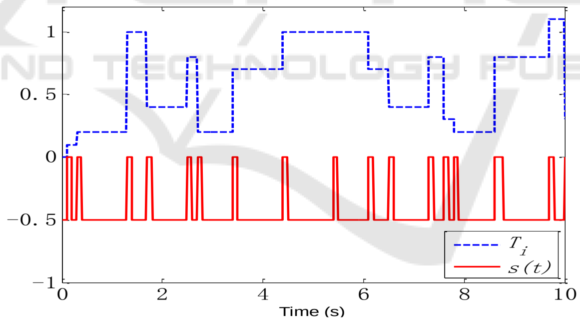

Figure 3. Variable delayed period T_i and the signal $s(t_i)$.

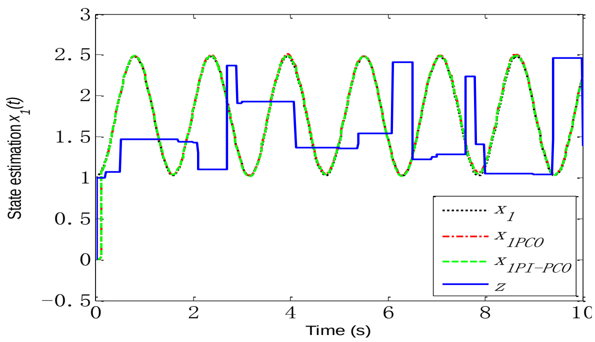

Figure 4. Position estimation with measurement noise.

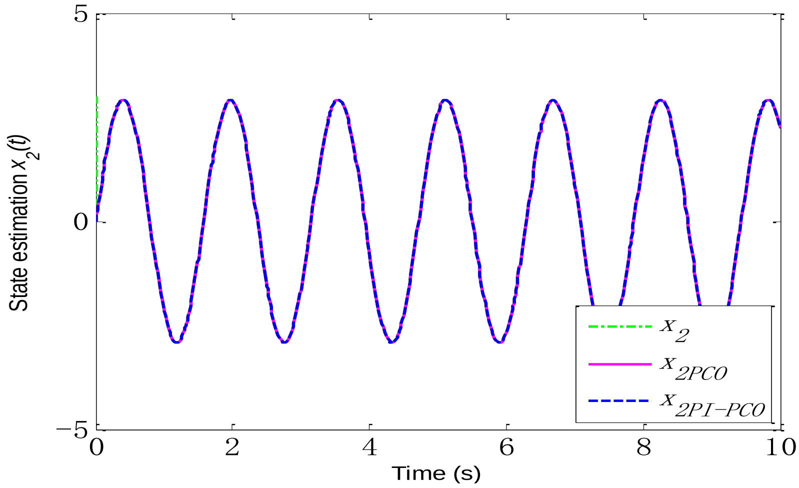

Figure 5. Speed estimation with measurement noise.

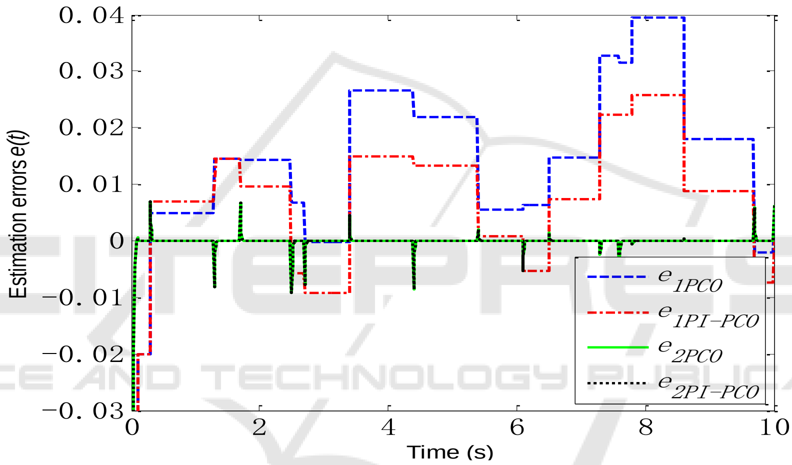

Figure 6. State estimation error of visual servo system.

The Figure 3 illustrates the variable delayed period and the square signal. Figs. 4–6 show the state estimations and the estimation errors both in PI-PCO and PCO methods, the blue curve in the Figure 4 represents the sampled and delayed output, which is used in the observer for states estimation. From the estimation errors depicted in the Figure 6, it is clear to note that under the variable sampling and delayed period, the proposed PI-PCO shows better performances and ensures minor estimation errors.

5 CONCLUSIONS

This paper dealt with the design of a new class of state observers, called PI-PCOs, which is based on the piecewise continuous systems and the concept of PIO. It makes possible to estimate the continuous and non-delayed state using sampled and delayed measurements with variable sampling time period. The proposed observer has a simple structure and can be easily implemented. However, in the actual system, the delay time and the sampling time period are variable and not connected, which should be studied in more depth.

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