Analysis of Free Time Intervals between Buyers at Cash Register using Generating Functions

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Abstract: The optimization of bursty business processes requires stochastic models with measurable parameters. Often simplifications worked out in the analysis of such models lead to inaccuracies when events occur in a bursty manner. In this work, a novel approach based on generating functions is introduced for modelling the bursty appearance of buyers via gap processes when paying at the cash register. The obtained approach is verified by analysing the payment process at the cash register by taking the free time intervals (gaps) between buyers as well as the payment processing time at the cash register into account. As both payment-related processes can be described by gap processes, the use of generating functions allows close-form solutions when analysing the payment process at the cash register. As an example the payment process is analysed in two supermarket of different sizes in Lithuania. The obtained results show that the free time intervals at the cash register are quite bursty independent of the size of the shop whereas the payment processing intervals at the cash register are quite regularly distributed.

1 INTRODUCTION

In order to model bursty business systems accurately when optimizing the performance of such systems using e. g. the well-known queuing theory, stochastic processes with measurable parameters have to be found. Such optimizations based on gap-processes have been studied successfully when analyzing bit-errors in telecommunication systems (e. g. wireless systems) (Wilhelm, 1976; Wilhelm, 2018) as well as packet arrivals in Ethernet-based data networks (Kessler et al., 2003) or when analyzing the internet traffic (Kresch and Kulkarni, 2011; Zukerman et al., 2003), where data packets arrive in bursts as well. However the concept of gap processes has never been applied to model the payment process as carried out in this work.

Bursts correspond to an enhanced activity level over a short period of time followed by long periods of inactivity and often leads to bottlenecks. In business systems, bottlenecks limit the flow of customers, services or products, etc. It happens when single business processes within the business system operate at their capacity limit or beyond. Given the diversity of systems in which burstiness emerges, the modelling of burstiness plays an important role as bottlenecks are still an indicator for customers dissatisfaction.

Burstiness in shop sales, as studied in this work, can be addressed when the components have a measurable activity pattern (such as to buy or not to buy). Fig. 1 illustrates the customer behaviour in shop sales. When analyzing the buying process chain, the customer arrival at the shop, the selection of goods, the payment process as well as the customer or buyer departure is meant. In this work a customer is a person who visits the shop but does not buy anything.

This work is aiming to achieving customer quality improvement through prevention of queuing by analyzing the payment process at the cash register (Miceviciene et al., 2018; Mittal and Kamakura, 2001; Kumar et al., 2016). The payment process itself can be divided into the process of buyers' waiting to the cash register described by free time intervals between buyers as well as the payment processing time (also referred as buyers' service time). Fig. 2 highlights the two parameters influencing the payment process at the

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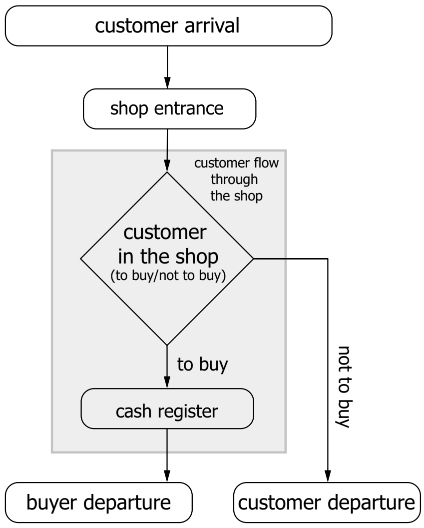

Figure 1: Customer behaviour in shop sales.

cash register. As the payment processing time is usually determined by technological limits (scanning of the purchased goods, the paying process itself) and the experience of the sales staff, only the free time intervals between buyers to the cash register as an indicator of bottlenecks throughout the whole process of buying are studied in this work.

The free time intervals between buyers to the cash register, as indicated in Fig. 3, as well as the payment processing time intervals are described and modelled by gaps. The gaps are characterized by the gap distribution function $u(k)$, defined as the probability that a gap Y between two buyers is greater than or at least equal to a given number of non-buying customers k, i. e.

$$u(k) = P(Y \geq k) \quad . \tag{1}$$

Alternatively, the gap-density function $v(k) = P(Y = k)$ can be used as well, denoting the probability, that a gap Y of length k appears.

By describing the process of buyers' waiting to the cash register and the process of payment individually by gaps, the payment process can be modelled by two independent gap-processes with different parameters as highlighted in Fig. 4. As the payment processing times are quite regularly distributed, the focus in this work is put on the free-time intervals between buyers

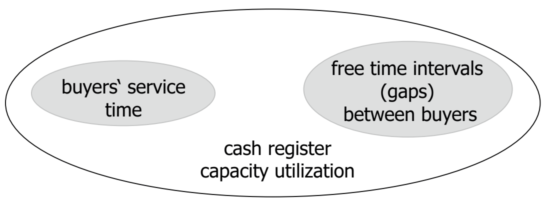

Figure 2: Parameters influencing cash register capacity utilization.

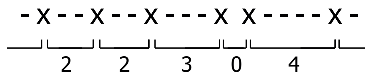

Figure 3: Modelling free-time intervals between buyers to the cash register (a buyer (represented by ”x”) within a sequence of non-buying customers (represented by ”-”)).

to the cash register.

Often the performance has been studied under the assumption that the gaps are exponentially distributed. However, under bursty conditions such exponential gap distribution functions (Weisstein, 1999) become inaccurate (Feldmann, 2000; Kessler et al., 2003; Ahrens et al., 2019b). Thus, for modelling bursty behaviour, the gap distribution function should be different from the exponential one. Against this background, in this work a novel approach for modelling bursty as well as non-bursty business processes by using generating functions, which will be used to describe the payment process, is presented.

The concept of generating functions is well-developed in the research community (Chattamvelli and Shanmugam, 2019). Generating functions have been identified as an extremely beneficial tool when analyzing discrete sequences of infinite length (Zhang et al., 2016; Li, 1992). Instead of analyzing a sequence of infinite length, a single function can be derived, representing the sequence of infinite length. For example, the gap-distribution sequence $u(k) = (1,1,1,1,\ldots)$ can be represented by the power series

$$U(t) = \sum_{k=0}^{\infty} u(k)\, t^k = 1 + t + t^2 + \ldots \quad . \tag{2}$$

This power series converges for $|t| < 1$ to

$$U(t) = \sum_{k=0}^{\infty} u(k)\, t^k = \frac{1}{1-t} \quad . \tag{3}$$

The function $U(t)$, defined in (3), provides an alternative description for the gap distribution function $u(k) = (1,1,1,1,\ldots)$ of infinite length. The elements of the sequence $u(k)$ are the coefficients of the infinite polynomial defined by (2). Such closed form solutions are known for exponential gap distribution functions. However, it is rather difficult to find closed-form solutions for gap distribution function that are different from the exponential one (Wilhelm, 1976; Wilhelm, 2018).

The novelty of this paper is given by the use of generating functions applied to the system model, namely free time intervals between buyers at the cash register, for simulating bursty buyers' behaviour. As an illustrative example the free time intervals between buyers at the cash register are studied in two supermarkets of different sizes in Lithuania.

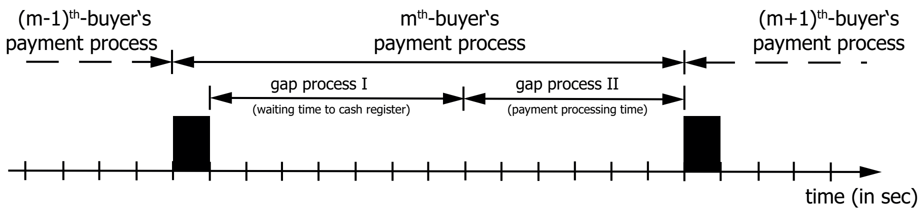

Figure 4: Modelling the payment process by two independent gap processes.

The remaining part of this paper is organized as follows: Section 2 introduces the theoretical basis for modelling buyers' behaviour. Section 3 briefly reviews the basics of using generating functions followed by approaches to model bursty as well as non-bursty buyers' behaviour. Section 4 is dedicated to the use of generating functions for gap modelling to analyse bursty as well as non-bursty buyers' behaviour. The associated results of an empirical study of different grocery shops in Lithuania are discussed in Section 5. Finally, some concluding remarks are provided in Section 6.

2 BURSTY BUSINESS PROCESSES

Bottlenecks in supermarkets, created by bursty customers (i. e. buyers), can limit the capacity of the whole shop since e. g. more buyers appear at the cash register than can be served. Conventionally, bottlenecks can be measured by indicators or parameters such as the buyers' probability and buyers' concentration. These parameters can be obtained when analysing the gaps between the buyers, i. e. the free time intervals between the buyers to the cash register as shown in Fig. 3. Here, stochastic processes with measurable parameters are needed when analysing the free time intervals between buyers.

Practically, when analysing the free time intervals between buyers at the cash register as depicted in Fig. 5, the gap distribution function $u(k)$ can be derived by introducing a suitable time interval t_{A} and discretising the free time intervals t_ℓ. After mapping them to the discrete parameter t_ℓ/t_{A}, the subsequent rounding delivers the discrete gap parameter k_ℓ.

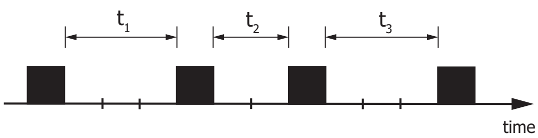

Figure 5: Free time intervals at the cash register.

Alternatively, the gap density function $v(k)$, defined as the probability that a gap Y between two buyers to the cash register is equal to a given number of non-buying visitors k, is of high interest, too. Taking the gap density function $v(k) = P(Y = k)$ into account, (1) can be re-written as

$$u(k) = v(k) + v(k+1) + v(k+2) + \dots \quad . \tag{4}$$

Fig. 6 shows different gap density functions $v(k)$ when analysing the free time intervals (gaps) between buyers to the cash register at buyer probability of $p_{\mathrm{e}} = 10^{-2}$. With an increasing level of burstiness as demonstrated in Fig. 6, the probability that after a buyer in the distance $k = 0$ another buyer appears, i. e. $v(0)$, increases. In this situation, the buyers appear more and more concentrated.

Intuitively, the buyers' behaviour can be described by a probability of purchase or buyer probability p_{e} (as a percentage of the visitors in the shop who buy something). However, the buyer probability does not give any indication of how concentrated the buyers are. In this case the model has to be extended by at least a second parameter as shown by Wilhelm (Wilhelm, 1976; Wilhelm, 2018) and Ahrens (Ahrens et al., 2019a) by introducing buyer concentration $(1-\alpha)$.

A process will appear bursty if the probability of short gaps is higher and lower for longer gaps if compared with a process with no burstiness (Fig. 6). This results in many short intervals (gaps) of high activity (probability) separated by longer intervals (gaps) of

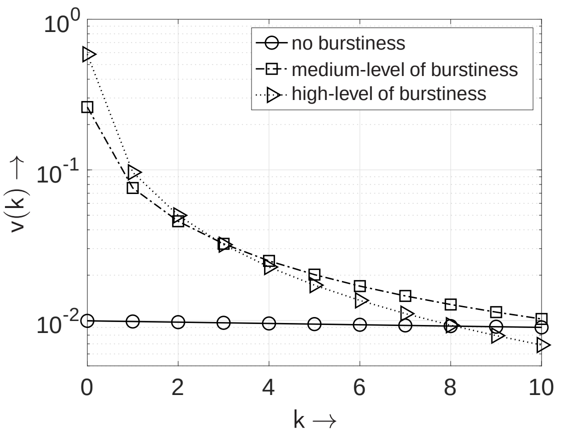

Figure 6: Gap density functions $v(k)$ for different levels of burstiness at buyer probability of $p_{\mathrm{e}} = 10^{-2}$.

inactivity.

As shown in (Wilhelm, 1976) and (Ahrens, 2000; Ahrens et al., 2019a) a good gap distribution function for bursty as well as non-bursty buyers' behaviour is given by

$$u(k) = [(k+1)^{\alpha} - k^{\alpha}] \cdot e^{-\beta \cdot k} \quad . \tag{5}$$

depending on the buyer probability p_e and the buyer concentration $(1-\alpha)$. The parameter β defined in (5) has to fulfil the following equation

$$p_e \approx \beta^{\alpha} \tag{6}$$

as shown by (Wilhelm, 1976; Wilhelm, 2018). Practically, relevant buyer concentration is in the range of $0.0 < (1-\alpha) \leq 0.5$, whereas the buyer concentration of $(1-\alpha) = 0$ describes the situation with non-bursty buyers (also refers to memoryless buyer scenario), where the buyer probability is sufficient to describe the buying process.

Unfortunately, no generating function for the gap distribution function $u(k)$, defined in (5), can be derived (Wilhelm, 1976; Wilhelm, 2018), except for $(1-\alpha) = 0$.

3 GENERATING FUNCTIONS

Generating functions can be used for describing an infinite sequence of numbers by treating them as the coefficients of a power series (Zhang et al., 2016). The concept of generating functions is well-established in the research community and used in this work for modelling the free time intervals (gaps) between buyers at the cash register.

Bursty as well as non-bursty free time intervals between buyers to the cash register is described by the gap distribution function $u(k)$ defined in (1). The generating function associated to (1) is the power series

$$U(t) = \sum_{k=0}^{\infty} u(k) t^k \quad . \tag{7}$$

In the following sections, close-form solutions are derived for both non-bursty as well as bursty free time intervals between buyers at the cash register.

3.1 Non-bursty Buyers Behaviour to the Cash Register

For situations with independent events, i. e. buyers to the cash register, the gap distribution function $u(k)$ can be defined by the buyers probability p_e solely as shown in (Ahrens et al., 2019b) and results in

$$u(k) = (1-p_e)^k \quad . \tag{8}$$

Consequently, the generating function is obtained as

$$U(t) = \sum_{k=0}^{\infty} u(k) t^k = \sum_{k=0}^{\infty} (1-p_e)^k t^k \quad . \tag{9}$$

The generating function is a geometric series with the quotient $q = (1-p_e)\,t$ and leads for $|q| < 1$ to

$$U(t) = \sum_{k=0}^{\infty} q^k = \frac{1}{1-q} = \frac{1}{1-(1-p_e)\cdot t} \quad . \tag{10}$$

With the approximation $1 - p_e \approx e^{-p_e}$ for small p_e, the generating function can be re-formulated as

$$U(t) = \frac{1}{1 - e^{-p_e} \cdot t} \quad . \tag{11}$$

Given the generating function $U(t)$ defined in (11) the corresponding elements of the sequence $u(k)$ can be calculated based on Taylor's theorem by the k-th derivative of the generating function $U(t)$ at the position $t = 0$. The elements of the sequence $u(k)$ result in

$$u(k) = \frac{U^k(0)}{k!} \tag{12}$$

Differentiating the generating function $U(t)$, defined in (11), the elements of the series $u(k)$ can be obtained as

$$\begin{aligned} u(0) &= 1 \\ u(1) &= e^{-p_e} \approx (1-p_e) \\ u(2) &= e^{-p_e} \cdot e^{-p_e} \approx (1-p_e)^2 \\ \vdots &= \vdots \end{aligned}$$

and confirm (8). The validation of the generating function can be carried out when analysing the average gap length (free time intervals) between two buyers to the cash register. Having completely independent buyers, the average gap length between two buyers can be expressed by the buyer probability p_e as

$$\frac{1}{p_e} - 1 = \sum_{k=0}^{\infty} k \cdot v(k) \quad . \tag{13}$$

With

$$\sum_{k=0}^{\infty} k \cdot v(k) = \sum_{k=0}^{\infty} u(k) - 1 \tag{14}$$

we get

$$\frac{1}{p_e} = \sum_{k=0}^{\infty} u(k) = \sum_{k=0}^{\infty} u(k) \cdot 1^k \quad . \tag{15}$$

This equation can be re-written as

$$\frac{1}{p_e} = \sum_{k=0}^{\infty} u(k) \cdot 1^k = U(1) \tag{16}$$

and is confirmed by $U(1)$ as $U(1)$ results with (11) in

$$U(1) = \frac{1}{1-\mathrm{e}^{-p_\mathrm{e}}} \approx \frac{1}{p_\mathrm{e}} \quad . \tag{17}$$

Therefore, the generating function should be suitable for describing non-bursty buyers behaviour to the cash register.

3.2 Bursty Buyers Behaviour

For bursty free time intervals between buyers to the cash register, the following gap distribution function is identified to be useful

$$u(k) = [(k+1)^\alpha - k^\alpha] \cdot \mathrm{e}^{-\beta \cdot k} \quad , \tag{18}$$

with the parameter $(1-\alpha)$ describing the buyer concentration and $p_\mathrm{e} = \beta^\alpha$ defined as buyer probability (Wilhelm, 2018; Ahrens et al., 2019a). Unfortunately, there is no known closed-form solutions for the corresponding generating function (Wilhelm, 1976). Therefore an approximation for $u(k)$ defined in (18) has to be applied (Wilhelm, 1976; Wilhelm, 2018). Taking the series expansion of the expression

$$(k+\Delta k)^\alpha = k^\alpha \,(1+\frac{\alpha}{k}\,\Delta k + \ldots) \tag{19}$$

into account, the expression (18) simplifies with $\Delta k = 1$ to

$$u(k) \approx \sum_{k=0}^{\infty} \alpha k^{\alpha-1} \mathrm{e}^{-\beta \cdot k} \quad . \tag{20}$$

Using the integral instead of the sum, the generating function $U(t)$ results in

$$U(t) = c\alpha \int_{k=0}^{\infty} k^{\alpha-1}\, \mathrm{e}^{-\beta \cdot k} t^k \, \mathrm{d}k \tag{21}$$

and can be re-defined as

$$U(t) = c\alpha \int_{0}^{\infty} k^{\alpha-1}\, \mathrm{e}^{(\ln(t)-\beta)\,k} \, \mathrm{d}k \quad . \tag{22}$$

The parameter c has to be determined in order to fulfil the condition $u(0) = 1$. Using the integral

$$\int_{0}^{\infty} k^{\delta-1}\, \mathrm{e}^{-\omega k} \, \mathrm{d}k = \frac{\Gamma(\delta)}{\omega^\delta} \quad . \tag{23}$$

the following solution with $\omega = \beta - \ln(t)$ and $\delta = \alpha$ can be obtained

$$U(t) = c\,\alpha\,\Gamma(\alpha) \cdot \frac{1}{(\beta - \ln(t))^\alpha} \quad , \tag{24}$$

with the parameter $\Gamma(\cdot)$ describing the Gamma function. With

$$\beta - \ln(t) = (1-(1-[\beta - \ln(t)])) \tag{25}$$

the expression $\beta - \ln(t)$ can be represented for $|\beta - \ln(t)| \ll 1$ as

$$\beta - \ln(t) \approx 1 - \mathrm{e}^{-(\beta-\ln(t))} = 1 - \mathrm{e}^{-\beta} t \tag{26}$$

and the generating function $U(t)$ results in

$$U(t) = c\,\alpha\,\Gamma(\alpha) \cdot \frac{1}{(1-\mathrm{e}^{-\beta} t)^\alpha} \tag{27}$$

In order to calculate the parameter c, the function

$$U(t) = u(0)t^0 + u(1)t^1 + u(2)t^2 + \cdots \quad , \tag{28}$$

has to fulfil the condition

$$u(0) = 1 \quad \rightarrow \quad U(0) = 1 \quad , \tag{29}$$

and therefore the parameter c has to be set

$$c\,\alpha\,\Gamma(\alpha) = 1 \quad . \tag{30}$$

Finally, the generating function $U(t)$ results in

$$U(t) = \frac{1}{(1-\mathrm{e}^{-\beta} t)^\alpha} \quad . \tag{31}$$

Comparing (31) and (11), the equations match for non-bursty (memoryless) free time intervals at the cash register with $(1-\alpha) = 0$ (i. e. $\alpha = 1$). In this case the parameter β equals the buyer probability p_e (Ahrens et al., 2019a).

The validation of the generating function can be carried out when analysing the average gap length between two buyers using (15) with

$$U(1) = \frac{1}{p_\mathrm{e}} \quad . \tag{32}$$

Taking (31) into account, the generating function $U(1)$ simplifies for small values of β with

$$1 - \mathrm{e}^{-\beta} \approx 1 - (1-\beta) = \beta \tag{33}$$

to

$$U(1) = \frac{1}{(1-\mathrm{e}^{-\beta})^\alpha} = \frac{1}{\beta^\alpha} = \frac{1}{p_\mathrm{e}} \quad . \tag{34}$$

For bursty free time intervals between buyers to the cash register, the parameter β must satisfy the following condition

$$p_\mathrm{e} = \beta^\alpha \tag{35}$$

as shown in (Wilhelm, 1976; Ahrens et al., 2019a). The generating function defined in (31) includes the description of non-bursty free time intervals between buyers to the cash register for a buyer concentration $(1-\alpha) = 0$ or $\alpha = 1$.

4 USE OF GENERATING FUNCTION FOR GAP MODELLING

The generating function, defined in (31), can now be used to calculate the elements of the corresponding gap distribution function $u(k)$ in analogy to the gap distribution function defined in (5). Given the generating function

$$U(t) = \sum_{k=0}^{\infty} u(k)\, t^k \tag{36}$$

the corresponding elements of the sequence $u(k)$ can be calculated based on Taylor's theorem by the k-th derivative of the generating function $U(t)$ at $t = 0$. The elements of the sequence $u(k)$ result in

$$u(k) = \frac{U^k(0)}{k!} \tag{37}$$

Differentiating the generating function

$$U(t) = \frac{1}{(1-\mathrm{e}^{-\beta} t)^{\alpha}} \quad , \tag{38}$$

the elements of the series $u(k)$ can be obtained as

$$\begin{aligned} u(0) &= 1 \\ u(1) &= \frac{\alpha}{1!}\,\mathrm{e}^{-\beta} \\ u(2) &= \frac{\alpha\cdot(1+\alpha)}{2!}\,\mathrm{e}^{-2\beta} \\ \vdots &= \vdots \end{aligned}$$

For $k = 1, 2, \ldots$, the elements of the gap-distribution function $u(k)$ result in

$$u(k) = \frac{\alpha\cdot(1+\alpha)\cdot\ldots\cdot(k-1+\alpha)}{k!}\,\mathrm{e}^{-k\beta} \quad . \tag{39}$$

With

$$\frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} = \alpha\cdot(1+\alpha)\cdot\ldots\cdot(k-1+\alpha) \tag{40}$$

and $\Gamma(1+k) = k!$, the gap distribution function can be formulated as

$$u(k) = \frac{\Gamma(\alpha+k)}{\Gamma(1+k)\cdot\Gamma(\alpha)}\,\mathrm{e}^{-\beta k} \quad . \tag{41}$$

With (41) an alternative definition of the gap distribution function $u(k)$ for bursty as well as non-bursty buyers' behaviour at the cash register is provided. Fig. 7 shows the gap distribution functions defined in (5) and (41). Both approaches show a close similarity for different parameters of the buyer concentration $(1-\alpha)$ at the buyer probability of $p_\mathrm{e} = 10^{-2}$ as (41) was derived from (5).

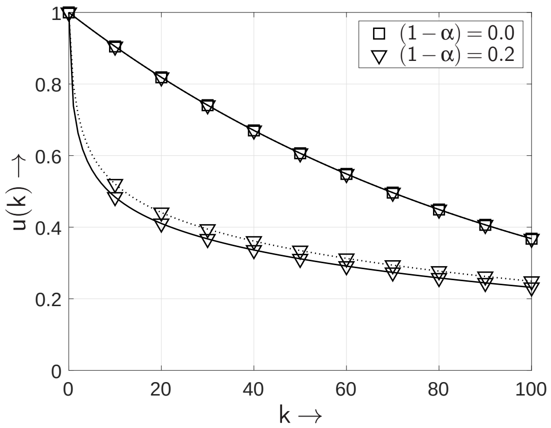

Figure 7: Comparison of the gap-distribution functions using (5) (dotted line) and (41) (solid line) for different parameters of $(1-\alpha)$ at the buyer probability of $p_\mathrm{e} = 10^{-2}$.

The buyer concentration can be estimated when analysing the probability that immediately after a buyer in the distance $k = 0$ another buyer appears. In this case the free time interval between buyers to the cash register is zero. Taking (41) into account, the buyer gap-density function $v(k) = u(k) - u(k+1)$ results in

$$v(k) = u(k)\left(1 - \frac{k+\alpha}{k+1}\,\mathrm{e}^{-\beta}\right) \quad . \tag{42}$$

Analysing the free-time intervals with $k = 0$ the expression simplifies to

$$v(k) = u(0)\cdot(1-\alpha\cdot\mathrm{e}^{-\beta}) \quad . \tag{43}$$

With $u(0) = 1$ and

$$\mathrm{e}^{-\beta} = 1-\beta \approx 1 \quad . \tag{44}$$

for small values of β the probability $v(0) = P(Y = 0)$ equals the buyer concentration

$$v(0) = 1-\alpha \quad . \tag{45}$$

The probability $v(0)$ is zero for non-bursty free time intervals between buyers to the cash register and increased to 50 % for bursty ones.

5 GROCERY SHOPS IN LITHUANIA

In order to get practically relevant data regarding the distribution of gaps, the free time intervals between buyers to the cash register in two different shops (grocery shop and supermarket) in Lithuania are studied. The buyer probability as well as buyer concentration derived from the obtained data allows identifying bottlenecks during the payment process at the cash register in shop sales.

The collected cash register data, obtained from a single cash register of each shop, contain the operation time, the amount of goods purchased, their codes and the prices paid by each buyer. The data collection was carried out in June 2018 and September 2018.

Unfortunately, the cash registers do not record the start time of the operation. Therefore, the service duration time was not available from the database. To cope with this problem the observed buyers' service durations with different quantities of goods were analysed as shown in (Ahrens et al., 2019a). By analysing the quantity of bought goods n_g and the service duration t_s the regression equation

$$t_s = 1{,}9\,n_g + 22{,}8 \quad (46)$$

was obtained. The equation yields that for one good about 1,9 seconds and additionally about 22,8 seconds for each buyer are required. The data were collected in the grocery shop, and it is assumed that all grocery shops as well as supermarkets have similar performance as they are working with similar equipment of cash registers and salespeople who are working at a similar intensity. Knowing the quantity of goods and (46), the start and end times of each buyer can be calculated. This allowed us to analyse the free time intervals between two buyers' service.

The histograms of the free time intervals at the cash register for the grocery shop as well as the supermarket are given in Fig. 8 and Fig. 9. Comparing both figures it turns out that the free times of the cash register are more bursty in the supermarket. Here, either short free time intervals or significantly longer free time intervals are observed.

According to (Goh and Barabási, 2008), the level of burstiness of free time intervals can be calculated analytically and is defined as

$$B = \frac{\sigma - m_1}{\sigma + m_1} \ , \quad (47)$$

by taking the mean value m_1 (average gap length or average length of free time intervals between two buyers at the cash register) as well as the standard deviation σ of the length of the free time intervals into account. The burstiness parameter B ranges between $-1 \leq B \leq 1$. Here, larger values of B indicate a higher level of burstiness. As shown in (Ahrens et al., 2019a), the parameter B equals the buyers' concentration $(1-\alpha)$ in the range of $0 < B \leq 1$.

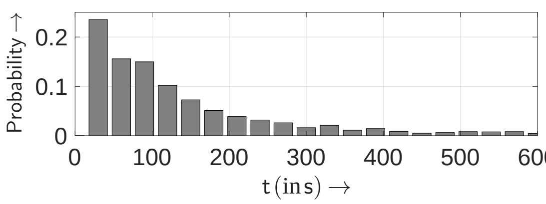

Figure 8: Distribution of free times of cash register (grouped) at grocery shop.

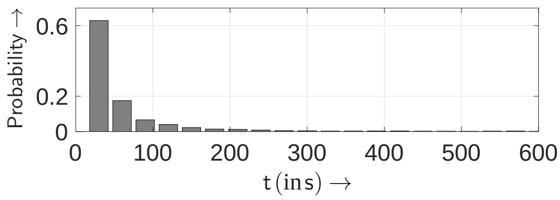

Figure 9: Distribution of free times of cash register (grouped) at the supermarket.

Tab. 1 shows the calculated level of burstiness, defined by the parameter B, of free time intervals between buyers at the cash register for the two investigated shops. The obtained data confirm a higher level of burstiness in the supermarket compared with the grocery shop when analysing the free time intervals between buyers as shown in Fig. 8 and Fig. 9.

Table 1: Burstiness of free time intervals between buyers at the cash register.

Shop	m_1	σ	B
Grocery Shop	234,1 s	620,1 s	0,45
Supermarket	96,9 s	408,7 s	0,62

As the parameter B equals the buyers' concentration $(1-\alpha)$, appropriate parameters of the underlying gap process for modelling the free time intervals to the cash register could be found. The results show a slightly higher intensity, expressed by lower value m_1 and a higher buyer concentration, at the supermarket compared with the grocery shop.

6 CONCLUSION

In this work the concept of generating functions was applied to the field of business processes. By the theoretical investigations of the inter-connections between buyers and corresponding gap-processes, a new approach based on generating functions was introduced.

Generating functions were used for the analysis of the free time intervals between buyers to the cash register as a part of the payment process. The proposed parameters, namely buyer probability and buyer concentration, allow identifying a burstiness level. In turn, a burstiness level serves as an indicator of bottlenecks. A high level of burstiness, expressed by the buyer concentration, increases the possibilities of bottleneck emergence.

For practical verification the payment process was analysed in two shops of different sizes in Lithuania.

The obtained results show that in both shops free time intervals at the cash register are quite bursty.

Practical implementation allows concluding that the proposed solutions are applicable to the field of business processes.

However the research has some limitations. In this work only two grocery shops in Lithuania were investigated. Another limitation is that only one cash register per shop was analysed.

Further work will concentrate on the joint modelling of free time intervals between buyers as well as the payment processing time at the cash register. For this, it is important for service quality improvement to analyse if the found burstiness of free time intervals between buyers affects the buyers' service time.

Future research will also focus on extending the dataset for a practical study. It includes the comparison of the implemented experimental analysis with other existing approaches. Examination of the use of the proposed approach for large shops with numerous counters will be planned and executed in order to demonstrate the use of the approach in real-life projects.

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