

Efficiency of Meme Usage in Evolutionary Algorithm

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Abstract: Emergency medical service system design attracts attention of a broad researcher and practitioner community due to increasing public demand for more safe life. A basic model of the design problem is known as the weighted p -median problem. Large instances of the problem as are hard to solve in general and very often it is necessary to obtain a series of solutions to be able to offer a spectrum of various solutions. For this purpose, an evolutionary metaheuristic seems to be a suitable tool, as it processes simultaneously a family of solutions. A standard evolutionary algorithm is based on developing a population using some nature inspired operations with solutions-members of the population, which produce candidates for population updating. This evolutionary process is characterized by fast improvement of the best-found-solution at the beginning and very slow improvement at the end, when the best-found-solution is near to the optimal one. To improve the first phase of the evolutionary process, numerous authors recommend to plug increasing procedures called memes in the process. Within this paper, we will study an impact of the meme plugin on acceleration of the evolutionary process, when big instances of the weighted p -media problem are solved. The study will be performed on instances of an emergency service system design problem solved by genetic algorithm with elite set and the studied meme will be based on exchange neighborhood searching.

1 INTRODUCTION

The emergency service system design problem represents a hard solvable combinatorial problem, where p service center locations should be selected from a finite set of possible center locations so that disutility perceived by system users be minimal (Brotcorne et al., 2003, Doerner et al., 2005, Jánošíková and Žarnay, 2014, Jánošíková et al., 2019, Marianov and Serra, 2002, Reuter-Oppermann et al., 2017). Disutility is often computed as a sum of weighted time-distances from individual users' locations to the nearest service center, where the weights correspond to frequency of emergency events at the individual users' locations. The emergency service system design problem can be presented and solved as a weighted p -median problem by an exact optimization algorithm (Avella et al., 2007, Current et al., 2002, Elloumi et al., 2004, García et al., 2011, Guerriero et al. 2016, Janáček, 2008, Sayah and Irnich, 2016). The exact approaches were mostly based on so called radial formulation of the p -median problem and this formulation also enables construction of a fast approximate approach (Janáček and Kvet, 2016).

In spite of existence of the exact and approximate fast approaches, there are many situations, which are not covered by enough efficient solving tools. It concerns very large instances of the p -median problem or necessity of producing a series of good different solutions. Such a demand can be satisfied by metaheuristic approaches (Gendreau, and Potvin, 2010).

As the problem can be studied as searching across a set of unit hypercube vertices, genetic algorithm represents a suitable tool for obtaining good solution in predetermined computational time (Daskin, 2015, Reeves, 2010, Sastry and Goldberg, 2005, Rybičková et al., 2016). Nevertheless, the progress of the best-found-solution objective function value along computational time resembles convex decreasing function, which converges slowly to the optimal value. To accelerate convergence of an evolutionary algorithm in general, many authors recommend plugging an improving heuristic called meme in the evolutionary process (Resende, 2004, Moscato and Cotta, 2010, Gupta and Ong, 2019,). We concentrate our effort on memes based on neighborhood searching, where the neighborhood of a current solution is represented by all p -median solutions,

which can be obtained from the current solution by replacing a current center location with an unoccupied one. The increasing neighborhood searching process can be restricted by several means, e.g. by limited number of objective function evaluations. Within the paper, we deal with meme efficiency in the above mentioned evolutionary process. We want to answer the question, under which conditions can meme usage improve the process convergence and when the meme complexity represents such a computational burden that the process becomes inefficient.

The paper is organized in the following way. The next section concisely describes a genetic algorithm including a special implementation of its basic operations of crossover and mutation. The third section gives an insight in an efficient increasing meme construction used for hybridization of the genetic algorithm. The fourth section introduces a way of measuring of convergence speed and comments on possible meme application in the genetic algorithm. The fifth section contains results of the numerical experiments aimed at solving the meme efficiency problem. The last section summarizes the obtained findings.

2 GENETIC ALGORITHM FOR P-MEDIAN PROBLEM

A genetic algorithm (GA) imitates a naturel process of species development in general. The algorithm processes a current population of individuals, where each individual corresponds to a solution of the solved problem. So called fitness of an individual usually reflects objective function value of the associated solution. The algorithm starts with creation of an initial population of different individuals-solutions and computing their fitness values. Then, the evolutionary process is simulated by creating candidates for a new population, by forming the new population and by a way of population exchange. This process is repeated until a termination rule is met, e.g. until used computational time reaches a given limit. Pool of the candidates is created using two operations performed on individuals selected from the current population, where the selection is performed randomly and probability of an individual choice depends on its fitness. The first of the operations is called crossover and it combines a pair of individuals and creates two new individuals. The new individuals are subjected to the second operation – mutation and then, they are inserted into the pool of candidates.

After the pool has been completed, the new population is formed by selection of some individuals from the pool and by including some elite individuals of the current population. Creating of the new population is completed by fitness evaluation of each individual and by updating the best-found-solution. As can be seen, no special improving algorithm is included into the basic evolutionary process. Quality of the resulting solution is achieved only by the two selections (parent’s selection for crossover and the selection of individuals for new population from the pool) and by keeping the best-found-solution or so called elite sub-set of the current population.

Efficiency of GA algorithm implementation is determined by a design of the two mentioned operations, which usually exploit characteristics of the solved problem to speed up the algorithm performance. In the studied case, the emergency system design problem is solved.

The emergency system design consists of choice of p service center locations from the set I consisting of m possible service center locations so that the min-sum objective function reflecting system users’ disutility is minimal. It is assumed that the system users are concentrated at a finite set J of users’ locations, where b_j denotes a volume of weight of user $j \in J$, e.g. b_j may correspond to an average number of emergency calls from the user’s location j . If symbol d_{ij} denotes the integer time-distance between locations $i \in I$ and $j \in J$, and if the service center deployment is described by zero-one m -dimensional vector \mathbf{y} of variables y_i denoting the center location by unit value, we can describe the problem by the formula (1).

$$\begin{aligned} \min \{ & \sum_{j \in J} b_j \min \{ d_{ij} : i \in I, y_i = 1 \} \\ & : \mathbf{y} \in \{0, 1\}^m, \sum_{i \in I} y_i = p \} \end{aligned} \quad (1)$$

As concerns the emergency system design, it is assumed that a user demand is satisfied from the nearest service center. From the mathematical point of view, the set of some vertices of m -dimensional hypercube is searched through, to obtain optimal solution.

The studied genetic algorithm is not able to find the optimal solution in general, but it tries to produce as good as possible solution using the specific crossover and mutation operations. Both the operations are designed so that the resulting offspring are feasible solutions, i.e. they contains exactly p located centers.

The suggested operation of crossover is performed with two individuals-parents x and y , where each parent is represented by a vector consisting of m zero-one components. Each component corresponds to one possible service center location and unit value of component i indicates that the associated solution-individual locates a service center at the possible service location. Thus, each vector contains exactly p units and $m-p$ zeros at the m positions. Comparing the associated components of vectors x and y , it can be found that the components can be categorized into three classes. The first class consists of components, at which positions the both vectors have the zero values. The second class consists of u components with units in the both vectors and the third class contains b components, at which one vector has unit value and the other has zero value. As totally $2p$ units are contained in the both vectors and $2u$ units are placed at components from the second class, then $b=2p-2u$ units occupy positions of the third class, i.e. b is even number.

The suggested crossover operation lets offspring's components of the first and second class unchanged and distributes the b units randomly among the offspring's components of the third class so that each offspring gets exactly $b/2$ units.

This way, each offspring represents a feasible solution of the p -median problem.

The mutation used in this genetic algorithm is based on the exchange operation, which randomly chooses a position occupied by a center and moves it at some unoccupied position, which is also chosen randomly.

As concerns the parent selection for crossover, a tournament approach is used in the studied case of GA. Elite set for the new generation completion is defined as the set of $BSize$ different individuals with the best fitness values withdrawn from the current population.

Diversity of the new population is assured by exclusion of individuals, fitness of which is equal to a fitness value of already accepted individual of the new population.

3 MEME FOR P-MEDIAN PROBLEM

Generally, noun "meme" denotes an arbitrary heuristic, which can improve some input solution of the solved problem (Gupta and Ong, 2019). Within this paper, we will concentrate on a special case of memes designed as an intensification tool for

metaheuristics, which solve p -median problem. The studied meme is increasing heuristic based on neighborhood searching applied to a current p -median solution, where the neighborhood of the current solution consists of all p -median solutions, which differ from the current one in exactly one located service center. If the meme is run, the neighborhood of the current solution is searched through by inspecting results of the individual exchanges occupied center locations for unoccupied ones. The searching process proceeds until either a limited number t of inspections is reached or an admissible exchange is found. In the second case, a current solution is updated by the better one and neighborhood searching is repeated until either t inspections have been performed or the current neighborhood has been inspected unless an admissible solution has been found.

The meme can be described by following five steps, where I denotes the set of all possible center locations, P denotes the set of p chosen center locations, which determine the current solution. $F(P)$ represents value of the objective function value connected with the solution P . Definition of the function value $F(P)$ is described in the formula (1).

Meme(P, t)

0. Initialize $F^*=F(P)$, $P^*=P$, $C=I-P$, $done=false$, $s=0$, mark all elements of P and C as uninspected, and go to step 1.
1. If there is any uninspected element of P and $done=false$ and $s < t$ hold, choose an uninspected element i from P and perform step 2, otherwise go to step 4.
2. If there is any uninspected element of C and $done=false$ and $s < t$ hold, choose an uninspected element j from C and perform step 3, otherwise mark element i as inspected and all elements of C as uninspected and go to step 1.
3. Set $s=s+1$, $\underline{P} = (P - \{i\}) \cup \{j\}$ and compute $F(\underline{P})$, mark j as inspected. If $F(\underline{P}) < F^*$, then set $F^*=F(\underline{P})$ and $P^*=P$ and $done=true$. Go to step 2.
4. If $s \geq t$ or $done=false$, then terminate, the resulting solution is P^* and its objective function is F^* , else if $done=false$, then set $P=P^*$, $C=I-P$, $done=false$, mark all elements of P and C as uninspected and go to step 1.

Efficiency of the above presented standard exchange algorithm obviously depends on a way of objective function value computing. If the objective function value of a solution P is computed in a

standard way according to formula presented in model (1), then complexity of the computational process is $O(|J| \cdot |P|)$, where $|J|$ denotes cardinality of the set J of users and the cardinality $|P|$ equals to the number p of located centers.

In our implementation of the process, we make use of the fact that most of the inspected solutions \underline{P} differ from the current solution P only in one service center. It enabled us to compute the objective function value $F(\underline{P})$ with complexity $O(2|J|)$. This effect was achieved by determining and saving the first and second minimal values of the set $\{d_{ij}; i \in P\}$ for each $j \in J$. These data are used for fast computation of objective function value of any solution \underline{P} , which differs from P in only one element.

This meme can be used in the genetic algorithm for p -median problem solving, to improve some portion of individuals from the pool of candidates or elite set before creating new population.

4 MEME APPLICATION AND CONVERGENCE OF EVOLUTIONARY PROCESS

To be able to study impact of meme usage on an evolutionary process, some way of convergence evaluation must be defined. We want to study characteristics of the evolutionary process limited by given computational time. As the tested heuristic need not reach the exact optimum, but it generally produces near-to-optimal solutions in the given time, there is no use to define quality of process convergence only by the objective function value of the resulting solution. That is why, we suggest the following measure based on progress of the best-found-solution objective function value in the running time. Intuitively, we consider that the process depicted in Figure 1 by bold lines is better than the process depicted by the dash lines.

Applied measure of the convergence quality is defined here as the area below the graph determined by the progress.

In the remainder of the paper, we will study influence of meme usage on convergence of the evolutionary process. Due to big computational time demand of the suggested meme, we restricted our study only on the case, when a meme is applied at most once in population exchange and, in addition, it will be applied to the best solution of the current population. Under these assumptions, we will study following schemes of meme applications.

- a) A meme is applied to the best solution of the current population before the new population is created.
- b) A meme is applied with given constant probability to the best solution of the current population.
- c) A meme is applied with probability, which decreases with the number of performed population exchanges.

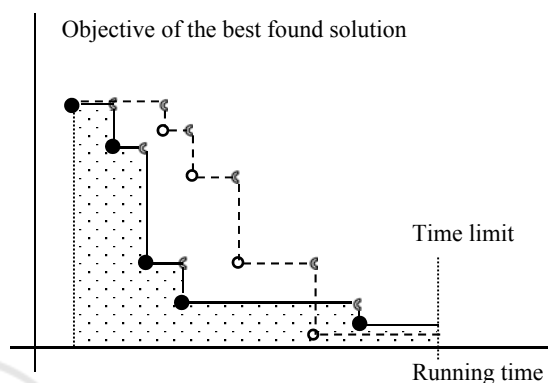


Figure 1: Possible progresses of the objective function values of the best found solutions depending on the running time of the GA. Dotted area below the full line curve represents possible evaluation of the associated process convergence.

5 NUMERICAL EXPERIMENTS

To perform the planned study, the genetic algorithm including the above described meme was programmed in programming language JAVA in NetBeansIDE 7.3 and the associated experiments were run on a PC equipped with the Intel® Core™ i7 5500U processor with the parameters: 2.4 GHz and 16 GB RAM. The used benchmarks were obtained from the road network of Slovak self-governing regions. The mentioned instances are further denoted by the names of capitals of the individual regions followed by triples (XX, m, p) , where XX is commonly used abbreviation of the region denotation, m stands for the number of possible centre locations (cardinality of the set I) and p is the number of service centres, which are to be located in the mentioned region. The list of instances follows: Bratislava (BA, 87, 14), Banská Bystrica (BB, 515, 36), Košice (KE, 460, 32), Nitra (NR, 350, 27), Prešov (PO, 664, 32), Trenčín (TN, 276, 21), Trnava (TT, 249, 18) and Žilina (ZA, 315, 29).

All cities and villages with corresponding number b_j of inhabitants were taken into account. The coefficients b_j were rounded to hundreds. The set of

communities represents both the set J of users' locations and the set I of possible center locations as well.

To verify the results obtained for the regular self-governing regions, we constructed bigger benchmarks by union of the original regions. Thus, we obtained the additional instances (ESR, 1124, 112), (WSR, 1792, 180) and (HSR, 2916, 273).

Parameters of the studied genetic algorithm were set up at the following most fitting values according to preliminary experiments. The size of the population ($PopSize$) was determined to correspond with cardinality of so called near-to-maximal uniformly deployed set of p -median problem for given m and p (Janáček and Kvet, 2019, Kvet and Janáček, 2019). The size of elite set ($BSize$) was equal to $(1/3)PopSize$ and the size of pool of candidates was $(3/2)PopSize$. The probability of mutation was set up at the value of 0.3. The computational time of one original benchmark solution was 5 seconds. The maximal solving time of the additional benchmarks was set up to 20 seconds.

Each run of the genetic algorithm was repeated 50 times with the same benchmark and average results are reported in the following tables.

Table 1 gives an overview of the benchmarks, their exact optimal solutions ($optSol$) and results of the standard version of the above described genetic algorithm without any meme application.

The table refers about size of population ($PopSize$), average resulting objective function value ($bestFit$), number of population exchanges during run of the algorithm ($noPop$). The column labeled by "RedArea" contains evaluation of the algorithm convergence described in the previous section. In the tables is given so called reduced area, which differs from the full area by subtracting the product of $bestFit$ and associated time of the run.

Table 1: Characteristics of benchmarks and results of standard version of the genetic algorithm without meme application.

Region	$optSol$	$PopSize$	$bestFit$	$RedArea$	$noPop$
BA	19325	23	19325	9	98368
BB	29873	172	30083	5233	625
KE	31200	60	31290	3172	2428
NR	34041	83	34051	1393	2891
PO	39073	232	39352	6809	464
TN	25099	137	25099	403	2939
TT	28206	212	28206	470	2384
ZA	28967	112	28971	1329	1951
ESR	40713	200	42696	82574	355
WSR	108993	200	124605	906515	111
HSR	161448	200	296142	296142	19

An individual experiment using the scheme a) and b) were organized so that the meme described in Section 3 for $t = p * 64$ was applied to the best solution of the current population with the probability $1/(2^T)$ for parameter values $T = 0, 1, \dots, 5$. It must be noted that experiments for $T = 0$ correspond to the case a), where the meme is applied once to the best solution in each population.

Table 2: Reduced area of the experiments a) and b).

Reg\T	0	1	2
BA	14	9	8
BB	5487	5092	4610
KE	3288	2911	2672
NR	1735	1466	1334
PO	7733	7116	6781
TN	509	455	415
TT	511	482	479
ZA	1562	1432	1328
ESR	104056	95287	89018
WSR	1022991	959206	908363
HSR	1496960	1475589	1457807
Reg\T	3	4	5
BA	7	6	6
BB	4604	4838	4790
KE	2657	2519	2668
NR	1113	1165	1238
PO	6647	6384	6365
TN	405	406	401
TT	454	436	449
ZA	1214	1240	1295
ESR	86416	83311	86312
WSR	905763	895335	877440
HSR	1452485	1444251	1455430

Comparing the column for $T=0$ of the Table 2 to the column $RedArea$ of the Table 1, it can be found that meme application has worsen convergence of the algorithm. This effect can be explained by big computational demand of the used meme, which has lowered the number of population exchanges (compare the column for $T=0$ of the Table 3 to the column $noPop$ of the Table 1) and thus it decreases efficiency of the evolutionary operations. Our experiments showed that lower probability of the meme application (for $T=3, 4, 5$) can considerably improve the convergence of hybridized GA algorithm.

Table 3: Number of population exchanges of the experiments a) and b).

Reg\T	0	1	2	3	4	5
BA	18636	31502	47893	64861	78899	88472
BB	402	487	538	566	586	597
KE	945	1376	1750	2051	2223	2319
NR	1341	1868	2331	2636	2832	2957
PO	321	380	418	442	451	452
TN	1667	2137	2488	2699	2832	2900
TT	1593	1932	2140	2265	2322	2377
ZA	1052	1350	1558	1694	1779	1829
ESR	245	290	318	335	345	348
WSR	79	93	102	107	109	111
HSR	17	18	19	20	20	20

Table 4: Reduced area of the experiments c).

Reg\T	0	1	2
BA	10	9	9
BB	4854	4986	4867
KE	3185	3144	3351
NR	1185	1261	1444
PO	6995	6801	6991
TN	418	438	421
TT	491	512	499
ZA	1427	1346	1389
ESR	78515	79120	79679
WSR	849673	843178	845608
HSR	1441520	1488013	1433971
Reg\T	3	4	5
BA	10	10	10
BB	4873	5162	5219
KE	3004	2713	2746
NR	1152	1241	1241
PO	6651	6798	7098
TN	448	448	461
TT	485	477	479
ZA	1370	1331	1328
ESR	82681	84750	89350
WSR	863891	884460	903266
HSR	1503572	1464120	1511966

The experiments for scheme c) were performed for the situation, when the probability Pr of meme application was dynamically lowered with the increasing number noP of performed population exchanges according to (2), where T is so called shaping parameter of the probability progress.

$$Pr = e^{(1-noP)/2^T} \tag{2}$$

In this case, the convergence of GA has been also improved in comparison to the standard GA algorithm, but the improvement was not as big as in case of the scheme b). Contrary to scheme b), it must be noted that bigger value of the parameter T in scheme c) according to (2) means slower decrease of probability depending on noP .

Table 5: Number of population exchanges of the experiments c).

Reg\T	0	1	2	3	4	5
BA	99985	100213	100248	100342	100380	100255
BB	603	615	617	609	604	599
KE	2399	2436	2420	2450	2429	2388
NR	3081	3119	3128	3113	3083	3080
PO	452	458	461	460	458	446
TN	2932	2952	2968	2958	2938	2938
TT	2303	2318	2335	2323	2319	2300
ZA	1823	1826	1815	1810	1802	1802
ESR	363	363	363	362	356	350
WSR	118	119	118	117	113	106
HSR	20	20	19	19	18	18

6 CONCLUSIONS

The paper reports on research conducted to increase the efficiency of the genetic algorithm in cases where real-time emergency service system instances need to be resolved and, in addition, a whole set of good alternative solutions is required. To estimate contribution of genetic algorithm hybridization, we studied an impact of the meme plugin on acceleration of the evolutionary process applied to the emergency system design.

The performed experiments showed that usage of meme in the evolutionary process need not inevitably contribute to acceleration of the process. Two schemes of random meme application were suggested and their influence on evolutionary process convergence was studied. It was found that suitable setting of application probability may lead to considerable improvement of the evolutionary process.

Future research may be aimed at a deeper research of the meme application and parameter tuning including learning process and other tools of artificial intelligence.

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