An Approximate Method for Integrated Stochastic Replenishment Planning with Supplier Selection

Rabin Kumar Sahu[1,2][a], Clarisse Dhaenens[2][b], Nadarajen Veerapen[2][c] and Manuel Davy[1]

[1]*Vekia, Euratechnologies-Bâtiment Urbawood, 58 Allée Marie-Thérèse Vicot-Lhermitte, F-59000 Lille, France*

[2]*Université de Lille, CNRS, Centrale Lille, UMR 9189 - CRIStAL - Centre de Recherche en Informatique Signal et Automatique de Lille, F-59000 Lille, France*

Keywords: Stochastic Demand, Supplier Selection, Replenishment Planning, Heuristic, Integrated Planning.

Abstract: A practical methodology for integrated stochastic replenishment planning with supplier selection is proposed for the single item inventory system. A rolling horizon strategy is adopted to implement the ordering decisions. Our method works in two stages. The first stage is a general black box stage that gives the minimum expected "coverage period" cost. The second stage uses a dynamic programming approach to compute the minimum expected cost for the rolling horizon. The proposed method is applicable for both stationary and non-stationary demand distributions and even for problems with minimum order quantity constraints. We also propose to examine the benefits of a dynamic supplier selection approach in comparison to selecting a common supplier. We conduct extensive numerical analyses on synthetic data sets for validation.

1 INTRODUCTION

Integrated replenishment planning with supplier selection is one of the core problems faced by retailers. With growing competitiveness in the current market, inclusion of purchasing price and thereby supplier selection in inventory optimization becomes very important. The inherent multi stage stochastic programming (MSSP) (Homem-de Mello and Bayraksan, 2014) problem for the multi-period inventory optimization problem is very difficult to solve optimally due to the well known curse of dimensionality (Defourny et al., 2012). Supplier selection adds additional decision or action states and further increases the complexity. In this paper, we first analyze the economic benefits of dynamic supplier selection and afterwards develop an approximate method to solve this problem.

Supplier selection has received considerable attention in the inventory optimization literature post 2003. Initially, supplier selection or multiple sourcing options have been seen as a measure of supply chain risk mitigation. However, in addition to that, multiple suppliers can have significant monetary benefits in terms of costs. Most of the replenishment planning models with multiple suppliers optimize the total cost. This cost is the summation of the replenishment costs, inventory holding costs and shortage costs. Replenishment costs consist of purchasing cost of the item, and fixed order/setup costs for placing an order. Typically, the fixed order cost is independent of order quantity and charged for every order placed. The purchasing cost can be different for each supplier and influenced by any discounting schemes. Shortage costs represent the costs paid by the buyer when it is unable to fulfill its demand. It can be either for backordering, lost sales or a mix of both. Additionally, costs such as disposal costs for perishable items, miscellaneous operational costs (i.e. investments, operations, maintenance costs) are also taken into account in some literature. A detailed review of supplier selection problems can be found in (Yao and Minner, 2017).

Retailers plan their inventory with a short review period. Availability of multiple suppliers for the same item poses greater challenge for cost effective operation. Since the total cost (and thereby the profitability) is closely related to the purchase price of an item, an integrated planning method becomes essential. A single supplier for a planning horizon is more practical than dynamic supplier selection. However, any decision must be taken with due consideration to its cost implications. Contributions of this article in this regard are as follows.

[a] https://orcid.org/0000-0003-1597-6324
[b] https://orcid.org/0000-0002-6590-7215
[c] https://orcid.org/0000-0003-3699-1080

Sahu, R., Dhaenens, C., Veerapen, N. and Davy, M.
An Approximate Method for Integrated Stochastic Replenishment Planning with Supplier Selection.
DOI: 10.5220/0008970500800088
In *Proceedings of the 9th International Conference on Operations Research and Enterprise Systems (ICORES 2020)*, pages 80-88
ISBN: 978-989-758-396-4; ISSN: 2184-4372

1. We analyze the impact of dynamic a supplier selection approach on overall cost over selecting a common supplier for the whole planning horizon.
2. We propose a practical framework for dynamic supplier selection and replenishment planning with stochastic demand.

The rest of this article is organized as follows. Section 2 presents the context of the problem discussed and the motivations behind it. Then in Section 3, we present the framework for comparison between dynamic supplier selection and selecting a common supplier for the whole planning horizon. We also propose a near-optimal method for dynamic supplier selection which can be used for real-world applications. In Section 4 we present the results of the experiments and discuss their relevance. At last we conclude the article and propose some future research directions.

2 CONTEXT AND MOTIVATIONS

The problem considered in this paper was motivated by a real-world retailer. It has multiple point of sales at different places along with a central warehouse. Because of economy of scale, all point-of-sales receive the items from the central warehouse. The central warehouse in turn orders from the external suppliers (Refer Figure 1). The demand information known at the point of sales is stochastic. Therefore, the demand at the central warehouse can also be interpreted as a stochastic process. For each item there are multiple suppliers. Those suppliers differ by the price they charge per unit item, available batch sizes, lead time and fixed cost charged per order. The fixed cost is charged for the transportation and administrative expenses. The retailer aims to minimize total cost incurred during a product life cycle. The usual costs incurred are purchase cost, fixed ordering cost, inventory cost and shortage cost. Any order placed by the warehouse to any supplier is delivered immediately without any lead time. Any product left over after the end of the planning horizon can still be used. Therefore, the salvage value is not taken into consideration. Only inventory cost is charged at the end of the planning horizon.

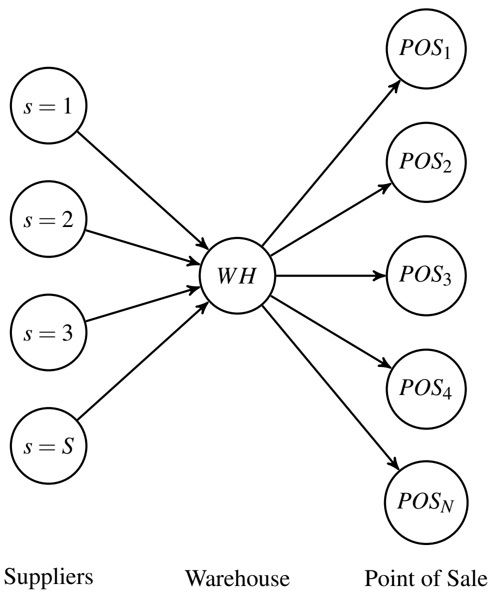

Figure 1: The supply chain network under study.

From the ease of practical applications, two approaches arise. First, when the retailer chooses only one supplier for a planning horizon (usually shorter than the product life cycle), and orders from that suppliers only. This approach is easier to implement in practice and the computation process of order quantities is comparatively less expensive than its multi-supplier counterpart. The second approach is to select suppliers dynamically during each ordering decision. This approach is computationally more expensive than the previous one due to increase in number of possible decisions in a dynamic programming setting. Beside, this approach is difficult to implement in practice. However, dynamic supplier selection has the potential to be more economical. In this article, we aim to first analyze the economic benefits of different supplier selection approaches and propose the retailer a cost benefit analysis. Practical difficulty can be offset by higher economic gain.

The methods discussed in the previous paragraph give rise to MSSPs. Those MSSPs can become intractable with increase in number of time periods and with increase in number of suppliers. Previous methods given in (Cheaitou and Van Delft, 2013), (Berling and Martínez-de Albéniz, 2015) and (Berling and Martínez-de Albéniz, 2016) consider demand distributions to be independent across time. In practice we often encounter dependent or correlated demand and distributions not following parametric distributions. Such application conditions requires new methods. The aim of this article is to develop a general framework replenishment planning problem with multiple suppliers, that can be implemented in practice.

3 PROBLEM FORMULATION

In this section, we propose the optimization models for both, the common supplier selection and the dynamic supplier selection. Afterwards, we present an approximate optimization framework based on dynamic programming for minimizing the total cost over the rolling horizon. We also present some preliminary concepts used in our methodology. The notations used throughout the paper are presented in Table 1. We consider a planning horizon of length $\hat{T}$ and S suppliers.

Table 1: Notations for the parameters and variables.

Sets	
$\mathcal{T}$	Set of time periods $t \in \{1, ..., \hat{T}\}$
$\mathcal{S}$	Set of suppliers $s \in \{1, ..., S\}$
Parameters	
d_t	Random demand at time t, $\in \mathbb{R}^+$
R_s	Unit purchase price from supplier s, $\in \mathbb{R}^+$
K_s	Fixed cost of ordering per order from supplier s, $\in \mathbb{R}^+$
H	Inventory holding cost per unit inventory per unit time period, $\in \mathbb{R}^+$
W	Backorder cost per unit, $\in \mathbb{R}^+$
Decision Variables	
q_{st}	Order quantity from supplier s at time t, $\in \mathbb{Z}^+$
α_{st}	Binary indicator for positive order from supplier s at time t, $\in \{0, 1\}$
I_t	Inventory at the end of time t, $\in \mathbb{Z}^+$

3.1 Preliminaries

Two preliminary concepts are presented which are used throughout the article. First, we present various control strategies for multi-period stochastic inventory optimization problems, and then we present a rolling horizon framework. The notations used in this article are presented in Table 1.

The control strategy or the uncertainty strategy for the multi-period stochastic inventory optimization problem is defined based on two conditions: when the decisions regarding the order timings (schedule) are taken, and when the corresponding order quantities are decided. Those control strategies are broadly divided into three categories: static, static-dynamic, and dynamic uncertainty (Rossi et al., 2015). When the decision maker determines both the ordering schedule and the order quantities at the very beginning of the planning horizon, it falls under the static uncertainty strategy. In case of the static-dynamic uncertainty strategy, timing of inventory reviews are fixed at the beginning of the planning horizon and the associated order quantities are decided upon only when orders are issued. The dynamic uncertainty strategy allows the decision maker to decide dynamically at each time period whether or not to place an order and how much to order. This strategy is known to be cost-optimal (Scarf, 1959). Our proposed methodology follows a static-dynamic uncertainty strategy. This is due to difficulty in practical implementation of a dynamic uncertainty strategy.

Next, we discuss rolling horizon approach. A rolling horizon is usually a planning period shorter than the planning horizon. An approach utilizing a rolling horizon scheme might compute optimal ordering decisions for the whole horizon but, implements only the first one. There are two justifications for adopting a rolling horizon approach. First is the curse of dimensionality. For any MSSP with increase in the number of stages the problem becomes computationally intractable. At the same time the optimal solution of a MSSP with few stages is usually a myopic solution of the global problem. Therefore, a compromise can be made between the computation time and solution quality by implementing the decisions in a rolling horizon manner (Rahdar et al., 2018). Second justification is that, with demand information farther in the future the forecast quality is usually less accurate. Therefore, inclusion of forecast information very far in future only adds to computation time without any substantial gain to the solution quality.

The mechanism of a rolling horizon approach is as follows. Under this, the demand information up to $\hat{T}$ (length of the planning horizon) periods is available. The total length of the planning horizon can extend up to infinity. A suitable length of rolling horizon $T \leq \hat{T}$ is then chosen. During the beginning of each period t, ordering decisions are determined considering the demand during the rolling horizon and initial inventory level at that time. Depending upon the adopted control strategy, multiple ordering decisions may be evaluated but, only the first one is implemented. The process is then repeated for each of the next periods with updated inventory level and demand information. A rolling horizon approach enables tackling dependent demand scenario with updated forecast in each order cycle.

3.2 Common Supplier Selection

The multi-period stochastic inventory optimization problem with a common supplier for every period is a finite-stage MSSP. It can be solved optimally with dynamic programming (Özen et al., 2012). This requires the end state to be known and demands across time to be independent. The optimal cost can be found for each supplier independently. Then the supplier having the minimum cost for the planning horizon can be selected. The functional equation of the resulting dynamic program is as follows. With $\mathcal{C}_t^s(I_{t-1})$ being the optimal cost for supplier s with state I_{t-1} at time

Table 2: MCPC Calculations.

$t=1$	$t=2$	...	...	$t=T-1$	$t=T$
$\tilde{C}_s^*(I_0,q,1,1)$	$\tilde{C}_s^*(0,q,2,2)$	...	...	$\tilde{C}_s^*(0,q,T-1,T-1)$	$\tilde{C}_s^*(0,q,T,T)$
$\tilde{C}_s^*(I_0,q,1,2)$	$\tilde{C}_s^*(0,q,2,3)$	...	...	$\tilde{C}_s^*(0,q,T-1,T)$	
...	...	...	...		
...	...	...	...		
$\tilde{C}_s^*(I_0,q,1,T-1)$	$\tilde{C}_s^*(0,q,2,T)$				
$\tilde{C}_s^*(I_0,q,1,T)$					

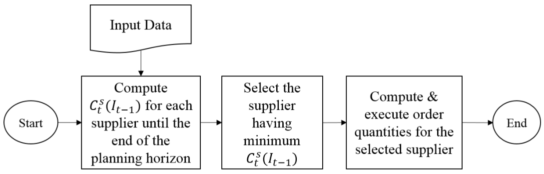

Figure 2: Working of common supplier selection.

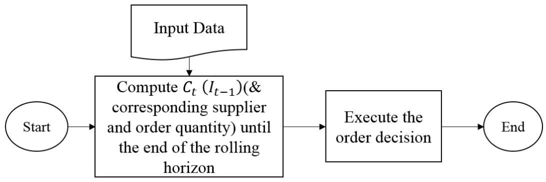

Figure 3: Working of dynamic supplier selection.

t, and q_{st} being the actions

$$C_t^s(I_{t-1}) = \min_{q_{st}} \Big\{ \mathbb{E}\big[H(I_{t-1}+q_{st}-d_t)^+ + W[-I_{t-1}-q_{st}+d_t]^+ + K_s\alpha_{st} + \sum_{s=1}^{S} R_s q_{st} + C_{t+1}(I_{t-1}+q_{st}-d_t)\big] \Big\} \quad (1)$$

$$\alpha_{st} = \begin{cases} 1 & \text{if } q_{st} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The first, second, third and fourth terms represent the expected inventory holding costs, expected shortage costs, fixed order cost and purchase costs respectively for period t. The last term represents the minimum expected cost for the next period. The above equation can be solved optimally by value iteration (Puterman, 2014). The methodology is summarized in Figure 2.

3.3 Dynamic Supplier Selection

The dynamic supplier selection problem is quite similar to the common supplier problem except, it has several sets of possible actions. In the case of common supplier selection, we considered the set of possible actions q_{st} for each supplier s separately. However, in case of a dynamic supplier selection, we consider all possible actions from all possible suppliers in a single dynamic program. The functional equation is given below. With $C_t(I_{t-1})$ being the optimal cost with state I_{t-1} at time t, and q_{st} being the actions

$$C_t(I_{t-1}) = \min_{q_{st}, s\in S} \Big\{ \mathbb{E}\big[H(I_{t-1}+\sum_{s=1}^{S} q_{st}-d_t)^+ + W[-I_{t-1}-\sum_{s=1}^{S} q_{st}+d_t]^+ + \sum_{s=1}^{S} K_s\alpha_{st} + \sum_{s=1}^{S} R_s q_{st} + C_{t+1}(I_{t-1}+\sum_{s=1}^{S} q_{st}-d_t)\big] \Big\} \quad (3)$$

In the above program, we do not consider the capacity constraint for the suppliers. Inclusion of capacity constraint can affect the ordering decisions and, we plan to study this in future research. Dynamic supplier selection can be achieved using the above formulation. Choice of supplier at an ordering epoch is affected by the inventory position, unit purchase price, fixed order cost and minimum order quantities, etc. The methodology is summarized in Figure 3.

3.3.1 Approximation Framework

The dynamic programs presented previously can become intractable when the number of periods is high. In this section we present an approximation framework to alleviate the curse of dimensionality without compromising the solution quality substantially.

We introduce a term called minimum coverage period cost (MCPC) which is formally defined as follows. Under the assumption of discrete time periods the MCPC is the minimum expected cost during a coverage period of length not less than one. A coverage period is such that any order received in the beginning would suffice till the end. Our proposed method has two major stages. The first stage is a general black box which gives the optimal order quantity and cost for any discrete coverage period. Such methods are given in (Sahu et al., 2019) for any general distributions and (Özen et al., 2012) for parametric distributions. Once we get those costs, we can proceed to minimize the cost for the whole rolling horizon in the second stage using a dynamic programming approach XDP. The process is depicted in Figure 4.

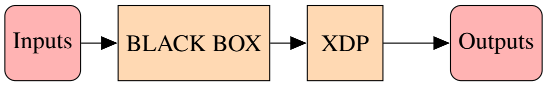

Figure 4: Illustration of the approximation framework. The inputs required are current inventory level, demand information and cost parameters. The BLACK BOX gives the MCPC for any given coverage period. The XDP used those costs to minimize the rolling horizon cost using a dynamic program.

3.3.2 Rolling Horizon Cost Optimization

We propose a dynamic programming approach to minimize the total cost over the rolling horizon. The process is presented in Figure 5.

Let us consider a rolling horizon of length T. An order can be placed at any time $t \in \{1,2,...,T\}$. When the decision is taken at time $t=1$, order can be placed at any one of the suppliers $s \in \{1,2,...,S\}$. The order quantity for any supplier can have coverage period upto $\{1,2,...,T\}$. If we go further in time at $t=2$, we can have different ordering options based on the state of inventory. However, our initial definition of coverage period states that the delivered quantity suffices till the end of that coverage period. Therefore, for each time period $t>1$, we compute the optimal order quantity and cost assuming the inventory state equal to zero. Hence, if an ordering decision is made at time $t=2$ of the same rolling horizon, the order quantity for any supplier can have coverage period up to $\{2,3,...,T\}$. Similarly, if a ordering decision is made at time t of the same rolling horizon, the order quantity for any supplier can have coverage period $\{t,t+1,...,T\}$. From the above we have, at $t=1$, there are ST ordering options, at $t=2$, there are $S(T-1)$ ordering options and so on. At $t=T$, there are S ordering options. Additionally, no order option can also be adopted. Let $\tilde{C}_s^*(I_{T^1},q,T^1,T^2)$ represent the MCPC for supplier s, for the coverage period T^1 to T^2 ($T^2 \geq T^1$). From the analysis given in the paragraph above, all possible ordering options during a rolling horizon of length T are given in Table 2. Those values are provided by the BLACK BOX along with corresponding order quantities. Mathematically, $\tilde{C}_s^*(I_{T^1},q,T^1,T^2)$ is defined as follows.

$$\tilde{C}_s^*(I_{T^1},q,T^1,T^2) = \min \mathbb{E} \sum_{t=T^1}^{T^2} \Big\{ H[I_{T^1} + q_{sT^1} - \sum_{\tau=T^1}^{t} d_\tau]^+ + W[-I_{T^1} - q_{sT^1} + \sum_{\tau=T^1}^{t} d_\tau]^+ + R_s q_{sT^1} + \alpha_{sT^1} K_s \Big\} \tag{4}$$

The possible different $\tilde{C}_s^*(I_{T^1},q,T^1,T^2)$ are presented in Table 2. We end up with $\frac{T(T+1)}{2}$ different costs for a rolling horizon of length T. After the computation of all the minimum expected costs, we can solve the multi-stage problem for the whole rolling horizon using a dynamic programming formulation as presented below. All the MCPCs are computed at zero initial inventory.

$$Z_T = \min_s \tilde{C}_s^*(0,q,T,T) \tag{5}$$

$$Z_{T-1} = \min \Big\{ \min_s \big(\tilde{C}_s^*(0,q,T-1,T-1) + Z_T\big), \min_s \big(\tilde{C}_s^*(0,q,T-1,T)\big) \Big\} \tag{6}$$

$$Z_{T-2} = \min \Big\{ \min_s \big(\tilde{C}_s^*(0,q,T-2,T-2) + Z_{T-1}\big), \min_s \big(\tilde{C}_s^*(0,q,T-2,T-1) + Z_T\big), \min_s \big(\tilde{C}_s^*(0,q,T-2,T)\big) \Big\} \tag{7}$$

$$Z_1 = \min \Big\{ \min_s \big(\tilde{C}_s^*(0,q,1,1) + Z_2\big), \min_s \big(\tilde{C}_s^*(0,q,1,2) + Z_3\big), ..., \min_s \big(\tilde{C}_s^*(0,q,1,T-1) + Z_T\big) \min_s \big(\tilde{C}_s^*(0,q,1,T)\big) \Big\} \tag{8}$$

The above formulation is a backward recursion. At $t=T$, we have the option of ordering only to cover the period T. Therefore, its expected minimum cost is the minimum of expected costs across the suppliers with coverage period T. Similar computations for the whole rolling horizon is of the order $\frac{ST(T+1)}{2}$.

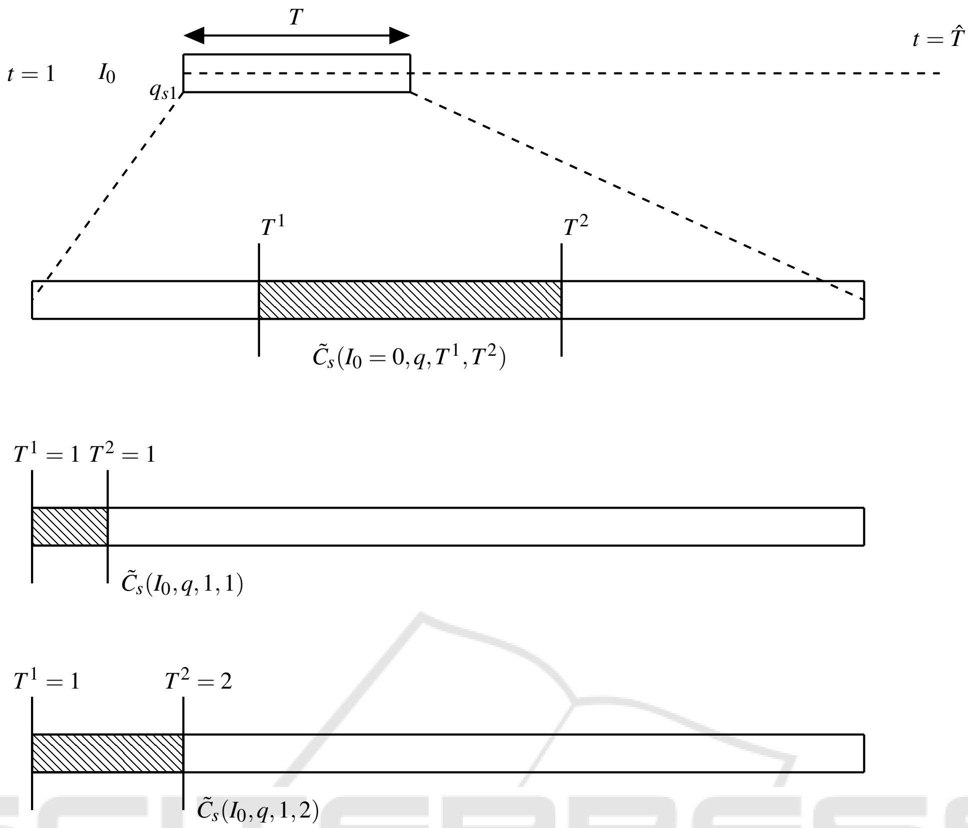

Figure 5: Illustration of an ordering mechanism and cost computation with a rolling horizon length T and planning horizon length $\hat{T}$. During each time period t order quantities are determined considering the opening inventory level and demand over a t to $t+T$ window. The black box stage give the optimal cost for all possible coverage periods T^1 to T^2.

4 NUMERICAL RESULTS

In this section, we present the experimental protocol and the detailed numerical results. To test the performance of our proposed method, we compare its cost with the optimal cost obtained with dynamic programming.

4.1 Experimental Protocol

In the previous section, we have explained our proposed two-stage method. In the first stage, we approximate the minimum expected costs for all possible coverage periods. We use these costs to formulate a dynamic programming approach to compute the minimum cost over the whole rolling horizon.

Our test-bed is as follows. At the beginning of time period $t = 1$, the demand forecasts upto period T are available. The decision maker uses this demand information, current inventory and cost parameters to compute the order quantity and places the ordered for the first period. The problem instances are presented in Table 3. In the beginning, we start with inventory $I_0 = 0$ and zero backorder. Any order placed is delivered immediately. A random demand following the same distribution as forecast is received and the corresponding excess inventory or backorder levels are updated. The period cost is computed as the sum of fixed order cost, inventory cost and backorder cost. The process is repeated until period T with demand information of period $\{2,3,...,T\}$, $\{3,4,...,T\}$ and so on upto $\{T\}$. The total cost is then computed as the sum of all period costs. We first obtain the optimal cost of the above process using dynamic programming. We conduct simulation to assess the expected cost with our proposed method. Since our method uses samples, we conduct 10^3 such simulations to estimate the expected cost of the planning horizon. We conduct tests for stationary demand that follow poisson distribution, with means equal to 5 and a planning horizon length of 20 periods. For the first set, inventory costs are $H = \{1, 0.5, 0.1\}$, the backorder cost $W = 20$ and for second set, the inventory cost is kept fixed at $H = 1$ and backorder costs are $W = \{30, 25, 15\}$. Our

Table 3: Problem instances.

H	W	K_s	R_s	MOQ
$Set-1$				
1	20	[20,20,20,20]	[10,9,8,7]	0
1	20	[20,40,80,150]	[10,10,10,10]	0
1	20	[20,40,80,150]	[10,9,8,7]	0
1	20	[20,40,80,150]	[10,8,7,5]	0
0.5	20	[20,20,20,20]	[10,9,8,7]	0
0.5	20	[20,40,80,150]	[10,10,10,10]	0
0.5	20	[20,40,80,150]	[10,9,8,7]	0
0.5	20	[20,40,80,150]	[10,8,7,5]	0
0.1	20	[20,20,20,20]	[10,9,8,7]	0
0.1	20	[20,40,80,150]	[10,10,10,10]	0
0.1	20	[20,40,80,150]	[10,9,8,7]	0
0.1	20	[20,40,80,150]	[10,8,7,5]	0
$Set-2$				
1	30	[20,20,20,20]	[10,9,8,7]	0
1	30	[20,20,40,80]	[10,10,10,10]	0
1	30	[20,20,40,80]	[10,9,8,7]	0
1	30	[20,20,40,80]	[10,8,7,5]	0
1	25	[20,20,20,20]	[10,9,8,7]	0
1	25	[20,20,40,80]	[10,10,10,10]	0
1	25	[20,20,40,80]	[10,9,8,7]	0
1	25	[20,20,40,80]	[10,8,7,5]	0
1	15	[20,20,20,20]	[10,9,8,7]	0
1	15	[20,20,40,80]	[10,10,10,10]	0
1	15	[20,20,40,80]	[10,9,8,7]	0
1	15	[20,20,40,80]	[10,8,7,5]	0
$Set-3$				
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[0,5,10,15]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[1,6,12,18]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[2,7,14,21]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[3,8,16,24]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[4,9,18,27]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[5,10,20,30]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[6,11,22,33]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[7,12,24,36]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[8,13,26,39]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[9,14,28,42]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[10,15,30,45]
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[11,16,32,48]

Table 4: Comparison of optimal expected costs of common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-1$. Backorder cost $W=20$, MOQ = None.

H	K_s	R_s	CS	DS(Gain %)
1	[20,20,20,20]	[10,9,8,7]	1032.0	1032.0(0.0)
1	[20,40,80,150]	[10,10,10,10]	1331.8	1331.8(0.0)
1	[20,40,80,150]	[10,9,8,7]	1331.8	1330.6(0.1)
1	[20,40,80,150]	[10,8,7,5]	1245.3	1242.9(0.2)
0.5	[20,20,20,20]	[10,9,8,7]	943.5	943.5(0.0)
0.5	[20,40,80,150]	[10,10,10,10]	1244.4	1244.4(0.0)
0.5	[20,40,80,150]	[10,9,8,7]	1230.2	1223.2(0.6)
0.5	[20,40,80,150]	[10,8,7,5]	1107.4	1090.4(1.5)
0.1	[20,20,20,20]	[10,9,8,7]	827.5	827.5(0.0)
0.1	[20,40,80,150]	[10,10,10,10]	1130.1	1130.1(0.0)
0.1	[20,40,80,150]	[10,9,8,7]	1032.0	1003.5(2.8)
0.1	[20,40,80,150]	[10,8,7,5]	825.7	806.0(2.4)

problem has 4 suppliers. Their unit price, fixed order costs are presented in respective result tables. We also test when minimum order quantity constraint is also present. The rolling horizon length is set at 20 periods.

Table 5: Comparison of optimal expected costs of common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-2$. Inventory holding cost $H=1$, MOQ = None.

W	K_s	R_s	CS	DS(Gain %)
30	[20,20,20,20]	[10,9,8,7]	1049.7	1049.7(0.00)
30	[20,40,80,150]	[10,10,10,10]	1352.9	1352.9(0.00)
30	[20,40,80,150]	[10,9,8,7]	1352.9	1350.7(0.17)
30	[20,40,80,150]	[10,8,7,5]	1265.7	1262.7(0.24)
25	[20,20,20,20]	[10,9,8,7]	1041.5	1041.5(0.00)
25	[20,40,80,150]	[10,10,10,10]	1343.1	1343.1(0.00)
25	[20,40,80,150]	[10,9,8,7]	1343.1	1341.3(0.13)
25	[20,40,80,150]	[10,8,7,5]	1257.6	1254.5(0.24)
15	[20,20,20,20]	[10,9,8,7]	1018.2	1018.2(0.00)
15	[20,40,80,150]	[10,10,10,10]	1314.7	1314.7(0.00)
15	[20,40,80,150]	[10,9,8,7]	1314.7	1313.4(0.10)
15	[20,40,80,150]	[10,8,7,5]	1228.9	1227.2(0.14)

Table 6: Comparison of optimal expected costs of common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-3$. Inventory holding cost $H=1$, backorder cost $W=20$, fixed order costs $K_s=[40,40,40,40]$, Purchase prices $R_s=[10,9,8,7]$.

MOQ	CS	DS	Gain (%)
$[0,5,10,15]$	1152.09	1149.01	0.27
$[1,6,12,18]$	1156.56	1150.89	0.49
$[2,7,14,21]$	1163.27	1154.51	0.76
$[3,8,16,24]$	1172.33	1159.80	1.08
$[4,9,18,27]$	1194.99	1171.65	1.99
$[5,10,20,30]$	1209.36	1183.54	2.18
$[6,11,22,33]$	1222.40	1195.97	2.21
$[7,12,24,36]$	1264.04	1215.25	4.01
$[8,13,26,39]$	1288.67	1235.14	4.33
$[9,14,28,42]$	1304.23	1253.52	4.05
$[10,15,30,45]$	1310.69	1271.93	3.05
$[11,16,32,48]$	1317.82	1289.63	2.19

4.2 Results and Discussion

The results are divided into two parts. Tables 4, 5 and 6 present the comparison between the common supplier selection and dynamic supplier selection. When the suppliers are equivalent in terms of fixed cost or unit purchase price, both approaches yield equal cost. Only when suppliers are different in both parameters, there is a cost difference. The results from Tables 4, 5 show that the cost difference is higher when the ratio of backorder cost to inventory holding cost is higher. When the minimum order quantity constraint is present, even with equal fixed cost for all suppliers, results show a cost different between both approaches. In each instance, dynamic supplier selection outperforms common supplier selection. The average runtime for common supplier selection is 2821 seconds on average.

In the second part, we present the results using our proposed approximate method in Tables 7, 8 and 9 for the same problem instances. We get major gain in terms of runtime. The average runtime of our method is 6 milliseconds. The average error for the problem

Table 7: Comparison of approximate expected costs using XDP for common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-1$. Inventory holding cost $W=20$, MOQ = None.

H	W	K_s	R_s	CS	Error(%)	DS	Error (%)
1	20	[20,20,20,20]	[10,9,8,7]	1033.40	0.14	1033.40	0.14
1	20	[20,40,80,150]	[10,10,10,10]	1347.88	1.20	1347.88	1.20
1	20	[20,40,80,150]	[10,9,8,7]	1335.87	0.30	1332.54	0.14
1	20	[20,40,80,150]	[10,8,7,5]	1246.06	0.06	1245.53	0.21
0.5	20	[20,20,20,20]	[10,9,8,7]	948.54	0.53	948.54	0.53
0.5	20	[20,40,80,150]	[10,10,10,10]	1260.60	1.30	1260.60	1.30
0.5	20	[20,40,80,150]	[10,9,8,7]	1232.75	0.20	1226.30	0.26
0.5	20	[20,40,80,150]	[10,8,7,5]	1117.98	0.95	1108.49	1.66
0.1	20	[20,20,20,20]	[10,9,8,7]	836.30	1.07	836.30	1.07
0.1	20	[20,40,80,150]	[10,10,10,10]	1133.79	0.32	1133.79	0.32
0.1	20	[20,40,80,150]	[10,9,8,7]	1041.51	0.92	1010.19	0.67
0.1	20	[20,40,80,150]	[10,8,7,5]	830.07	0.53	811.55	0.69
Average					0.63		0.68

Table 8: Comparison of approximate expected costs using XDP for common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-2$. Backorder cost $H=1$, MOQ = None.

H	W	K_s	R_s	CS	Error(%)	DS	Error (%)
1	30	[20,20,20,20]	[10,9,8,7]	1 059.74	0.96	1059.74	0.96
1	30	[20,20,40,80]	[10,10,10,10]	1 383.74	2.28	1383.74	2.28
1	30	[20,20,40,80]	[10,9,8,7]	1 381.00	2.07	1378.49	2.06
1	30	[20,20,40,80]	[10,8,7,5]	1 266.37	0.05	1263.33	0.05
1	25	[20,20,20,20]	[10,9,8,7]	1 051.27	0.93	1051.27	0.93
1	25	[20,20,40,80]	[10,10,10,10]	1 353.66	0.79	1353.66	0.79
1	25	[20,20,40,80]	[10,9,8,7]	1 383.64	3.02	1365.56	1.81
1	25	[20,20,40,80]	[10,8,7,5]	1 261.35	0.30	1259.22	0.37
1	15	[20,20,20,20]	[10,9,8,7]	1 030.42	1.20	1030.42	1.20
1	15	[20,20,40,80]	[10,10,10,10]	1 369.00	4.13	1369.00	4.13
1	15	[20,20,40,80]	[10,9,8,7]	1 349.40	2.64	1319.11	0.44
1	15	[20,20,40,80]	[10,8,7,5]	1 235.79	0.56	1229.82	0.21
Average					1.58		1.26

Table 9: Comparison of approximate expected costs using XDP for common supplier selection (CS) and dynamic supplier selection (DS) approaches for input data $Set-3$. Inventory holding cost $H=1$, backorder cost $W=20$, fixed order costs $K_s=[40,40,40,40]$, Purchase prices $R_s=[10,9,8,7]$.

H	W	K_s	R_s	MOQ	CS	Error(%)	DS	Error (%)
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[0,5,10,15]	1192.09	3.47	1178.31	2.55
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[1,6,12,18]	1168.56	1.04	1154.84	0.34
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[2,7,14,21]	1174.27	0.95	1172.75	1.58
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[3,8,16,24]	1212.33	3.41	1162.70	0.25
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[4,9,18,27]	1203.99	0.75	1187.47	1.35
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[5,10,20,30]	1226.36	1.41	1217.51	2.87
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[6,11,22,33]	1238.40	1.31	1214.87	1.58
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[7,12,24,36]	1272.04	0.63	1224.24	0.74
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[8,13,26,39]	1306.67	1.40	1259.84	2.00
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[9,14,28,42]	1320.23	1.23	1270.82	1.38
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[10,15,30,45]	1327.69	1.30	1274.98	0.24
1	20	[40, 40, 40, 40]	[10, 9, 8, 7]	[11,16,32,48]	1341.82	1.82	1317.23	2.14
Average						1.56		1.42

problem instance is less than 2% for both common supplier selection and dynamic supplier selection

5 CONCLUSIONS

In this article, we address the integrated replenishment planning and supplier selection problem. This problem can be formulated as a multi-stage stochastic problem. Due to the curse of dimensionality, it is intractable for medium to large size problems. For its practical importance and complexity, it has received considerable attention in the literature post 2003, however, mostly as a measure of risk mitigation. Nowadays, multi-brand retailers face the problem during their day to day operation. This gives rise to the need of its study as a economic option and the development of faster optimization method. We first conduct the financial benefit analysis of dynamic supplier selection versus selecting a common supplier for the planning horizon. Then we propose an approximation framework for the dynamic supplier selection problem.

A common supplier for the whole planning horizon is a practically more appealing feature. However, the dynamic supplier selection results in higher economic benefits. Both of the aforementioned problems are multi-stage stochastic optimization problems. However, the latter one is relatively more complex due to its increased number of possible actions. Numerical analysis suggest that the dynamic supplier supplier selection approach always outperforms the approach with one common supplier, especially when the inventory holding costs and the backorder costs are very different, and when the suppliers impose a minimum order quantity constraints. Finding the optimal solutions of any of the above approaches is time consuming. Hence, we develop an approximation framework based on dynamic programming.

The framework works in two stages. The first stage is a general black box which gives the optimal order quantity and cost for discrete coverage period. We then end up with $\frac{T(T+1)}{2}$ different costs for a rolling horizon of length T. Afterwards, a dynamic programming approach optimizes the total cost for the rolling horizon. We conduct numerical analysis to attest the performance of our proposed method. For the synthetic instances the approach provides near optimal solution at a fraction of the computation time. On average the optimality gap is 1.18%. The average computation time is 6 milliseconds.

Future research aspects are to test the method in lost sale environments and when the suppliers give quantity discounts which are some of the common practices nowadays in retail. Also, deeper analysis can be done to suggest when the economic benefits of a dynamic supplier selection problems outweighs the practical benefits of selecting a common supplier.

REFERENCES

Berling, P. and Martínez-de Albéniz, V. (2015). Dynamic speed optimization in supply chains with stochastic demand. *Transportation Science*, 50(3):1114–1127.

Berling, P. and Martínez-de Albéniz, V. (2016). A characterization of optimal base-stock levels for a multistage serial supply chain. *Naval Research Logistics (NRL)*, 63(1):32–46.

Cheaitou, A. and Van Delft, C. (2013). Finite horizon stochastic inventory problem with dual sourcing: Near myopic and heuristics bounds. *International Journal of Production Economics*, 143(2):371–378.

Defourny, B., Ernst, D., and Wehenkel, L. (2012). Multistage stochastic programming: A scenario tree based approach to planning under uncertainty. In *Decision theory models for applications in artificial intelligence: concepts and solutions*, pages 97–143. IGI Global.

Homem-de Mello, T. and Bayraksan, G. (2014). Monte carlo sampling-based methods for stochastic optimization. *Surveys in Operations Research and Management Science*, 19(1):56–85.

Özen, U., Doğru, M. K., and Tarim, S. A. (2012). Static-dynamic uncertainty strategy for a single-item stochastic inventory control problem. *Omega*, 40(3):348–357.

Puterman, M. L. (2014). *Markov Decision Processes.: Discrete Stochastic Dynamic Programming*. John Wiley & Sons.

Rahdar, M., Wang, L., and Hu, G. (2018). A tri-level optimization model for inventory control with uncertain demand and lead time. *International Journal of Production Economics*, 195:96–105.

Rossi, R., Kilic, O. A., and Tarim, S. A. (2015). Piecewise linear approximations for the static–dynamic uncertainty strategy in stochastic lot-sizing. *Omega*, 50:126–140.

Sahu, R. K., Dhaenens, C., Veerapen, N., and Davy, M. (2019). A sample-based optimization approach for the stochastic replenishment planning. *Submitted*.

Scarf, H. (1959). The optimality of (s, S) policies in the dynamic inventory problem. Technical report, Stanford Univ Calif Applied Mathematics and Statistics Labs.

Yao, M. and Minner, S. (2017). Review of multi-supplier inventory models in supply chain management: An update. *Available at SSRN 2995134*.