




A Genetic Algorithm Optimising Control Point Placement for Edge Bundling

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Abstract: This paper describes a novel approach of edge bundling that employs a genetic algorithm (GA) to optimise the placement of control points. Edge bundling is a useful technique to reduce visual clutter and a number of model-based edge bundling approaches have been introduced in the literature. However, these do not attempt to optimise aesthetic rules directly. Differently from them, our approach assumes that edge bundling is regarded as an optimisation problem for aesthetic rules. To solve this problem, we present an GA-based algorithm where gene representation defines control points of edges in order to allow flexibility and the fitness function is defined based on quantitative criteria for edge bundling. Experimental results using a visualisation of a Japanese airline map demonstrates the feasibility of our proposed method and its usability.

1 INTRODUCTION

Edge bundling is a method to decrease visual clutter and thus improve understanding the layout of edges by bundling edges based on certain rules.


Edge bundling is a well researched research topic. Most works in this area define a model to express edge bundling with one of the best known methods being Holten's work where they proposed Hierarchical Edge Bundling for a graph based on a tree structure (Holten, 2006).


Geometry-Based Edge Bundling (GBEB) proposed by Cui et al. (2008) realises edge bundling so as to bend edges based on meshes generated through a Delaunay triangulation, although this approach sometimes leads to some extreme bends. On the other hand, Holten et al. (2009) proposed Force-Directed Edge Bundling (FDEB) which performs bundling based on Hooke's law. Also, Selassie et al. (2011) introduced Divided Edge Bundling by improving FDEB to apply to directed graph, while


Hurter et al. (2012) proposed Kernel Density Estimation Edge Bundling based on image-based visualisation. Yamashita et al. (2017) presented a Line-Graph Based Edge Bundling that is based on the idea that clustered edges should be bundled with the clusters being detected by a line-graph.

In this paper, we propose an approach that differs from the above-mentioned ones. In particular, we propose a genetic algorithm (GA)-based approach for edge bundling. GA (Goldberg, 1989) is a well-known optimisation technique that is rooted in a model of evolution and the principle of survival of the fittest. A characteristic feature of our approach is that it allows for a flexible implementation and to easily modify parameters and fitness function.

Some recent related approaches also regard edge bundling as an optimisation problem. In particular, the work by Ferreira et al. (2018) formulates an optimisation problem where the number of edges including bundled edges is minimised based on the assumption that only edges sharing the same vertex

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should be bundled. They also use constraints on the bundled edges, in particular an angle threshold and compatibility constraints.

In this paper, we take quantitative criteria based on aesthetic rules into consideration and solve the optimisation problem using a genetic algorithm. For this, we adopt the control points approach used in FDEB and the criteria from (Sakamoto et al., 2019; Saga, 2016; Saga, 2018). As a result, we are able to overcome the shortcomings of Ferreira's model.

The main contributions of this paper are the following:

- It is the first approach of a genetic algorithm-based edge bundling algorithm optimising control points with regards to an aesthetic evaluation index.
- We show that edge bundling using a computational intelligence approach to optimisation yields a feasible method.
- We discuss the extensibility of our proposed method and its application in future work

2 GA-BASED EDGE BUNDLING

2.1 Genetic Algorithm

Genetic algorithms, which belong to the family of evolutionary algorithms, simulate Darwin's theory of evolution (Goldberg, 1989). GAs are employed to solve difficult, often NP-hard, optimisation problems. The genetic representation and fitness function depend on the problem and domain to solve. After these are defined, a GA proceeds iteratively through stages of selection, crossover, and mutation to improve a population of individuals that expresses candidate solutions to the problem.

2.2 Genetic Representation

In our approach, the genetic representation we choose is based on control-based approaches differently from Ferreira's. The approach employed in FDEB divides an edge uniformly by c control points. By moving these control points the edges can be controlled for edge bundling. In our algorithm, edges in the input graph are also divided based on c uniformly spaced points as shown in Figure 1. For each control point, we then store a displacement vector v (as (x,y) -coordinates) whose distance we limit. Thus, for n edges and using c control points per edge, we encode $2*n*c$ parameters.

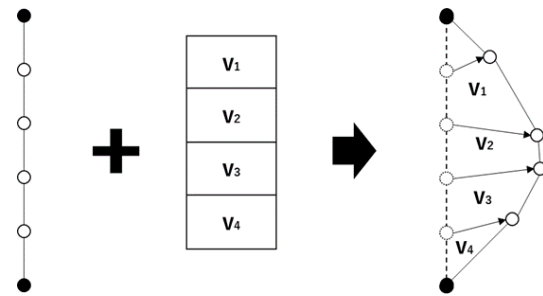


Figure 1: Genetic representation.

2.3 Fitness Function

An appropriate fitness function is key to a successful GA. Some investigations of graph layout using GA for visualisation design the fitness function based on aesthetics rules (Eloranta et al. 2001, Wang et al. 2005). In graph drawing, the following rules are generally accepted:

- (1) Uniform spatial distribution of vertices;
- (2) Minimise the total edge length on the pre-condition that the distance between any two vertices is no less than the given minimum value;
- (3) Uniform edge length;
- (4) Maximise the smallest angle between edges incident on the same vertex;
- (5) The angles between edges incident on the same vertex should be as uniform as possible;
- (6) Minimise the number of edge crossings;
- (7) Exhibit any existing symmetric feature.

For our problem at hand, it is necessary to quantify such aesthetics rules for edge bundling. Here, there are also some general accepted aesthetic rules like for the general graph drawing problem which have been introduced in the literature (Sakamoto et al., 2019). The data-ink ratio (Tufte 2001) is one of the most widely used ones to evaluate visualisation results quantitatively in all of visualization problems. It is based on the ink amount required for drawing a visualised figure. The path quality, proposed by Cui in GBEB, is also useful to evaluate the degree of zig-zag in edge bundling. Furthermore, Saga (2016, 2018) proposed three quantitative criteria to evaluate edge bundling which are formulated from the difference of edge length, area illustrated by edges (which is similar to data-ink ratio), and density of edges.

In our approach, we adopt these three criteria together with the path quality by Cui.

2.3.1 Mean Edge Length Difference

Mean Edge Length Difference (MELD) is a criterion to express the difference from the original edges after

edge bundling. A smaller change of edge lengths indicates superior edge bundling because of over-bundling, whereas a large change often leads to a loss of the meaning of the original network. MELD is calculated as

$$MELD = \frac{1}{n} \sum_{e \in E} |L'(e) - L(e)| \quad (1)$$

where n is the number of edges, E is the edge set, and $L(e)$ and $L'(e)$ are the lengths of edge e before and after edge bundling, respectively. Employing this criterion, we can prevent edges from over-bending and over-bundling. MELD can be normalised to $[0;1]$ by

$$MELD = \frac{1}{n} \sum_{e \in E} |1 - L'(e)/L(e)|$$

In our approach, we aim to minimise the MELD.

2.3.2 Mean of Occupation Area

Mean of Occupation Area (MOA) indicates the degree among the compressed areas before and after edge bundling. Based on the idea that better bundling can compress the area occupied by the edges, MOA is calculated as

$$MOA = \frac{1}{N} \left| \bigcup_{e \in E} O(e) \right| \quad (2)$$

where N is the number of total areas, $O(e)$ is the set of areas occupied by edge e based on an occupation degree (we use 5% of unit area), and $||$ indicates the number of elements contained by a set. Minimising the MOA is one of our optimisation goals.

2.3.3 Edge Density Distribution

Edge Density Distribution (EDD) is rooted in the idea that a better edge bundling method can gather edges within a unit area and that the density per unit is high. EDD is calculated from an image by

$$EDD = \frac{1}{n} \sum_{a \in A} |p(a) - p| \quad (3)$$

where A is a set of unit areas, $p(a)$ is the rate of the number of pixels, in which the edges pass in Area a , and p is a mean of $p(a)$. A variance-based measure, the EDD is higher when the values are concentrated on some ranges.

However, this calculation does not work well as it is calculated from an image and it is difficult to express the density correctly from an image. Also, EDD does not work well when edge spread in an area due to zig-zag although path quality mentioned later can address this.

Therefore, we redefine EDD to express the density more clearly by counting not the area but the number of edges per pixel and calculating the variance of edges as

$$EDD = \frac{1}{|P|} \sum_{p \in P} (H(p) - H)^2 \quad (4)$$

where P is a set of pixels, $H(p)$ is the number of edges pathing pixel p , and H is the average of $H(p)$. We aim to maximise the EDD.

2.3.4 Path Quality

Path Quality (PQ) expresses the degree of zig-zag. The lower the PQ, the better the edge bundling. PQ is calculated by the summation of angle differences between neighbours as

$$PQ = \sum_{e \in E} (-\sum_{i=3}^m \gamma_i |\Delta_i|) \quad (5)$$

with

$$\Delta_i = \begin{cases} A_i - A_{i-1} & \text{if } -\pi < |A_i - A_{i-1}| < \pi \\ |A_i - A_{i-1}| - 2\pi & \text{if } |A_i - A_{i-1}| > \pi \\ 2\pi + |A_i - A_{i-1}| & \text{if } |A_i - A_{i-1}| < -\pi \end{cases} \quad (6)$$

and

$$\gamma_i = \begin{cases} 0 & \text{if } \text{sign}(\Delta_i) = \text{sign}(\Delta_{i-1}) \\ 1 & \text{if } \text{sign}(\Delta_i) \neq \text{sign}(\Delta_{i-1}) \end{cases} \quad (7)$$

, where m is the number of segments divided by control points+1, and A_i is the angle between the original edge and the segment edge. In our GA, we try to maximise PQ.

We use the above four criteria separately and perform multi-objective optimisation.

2.4 Genetic Operations

We employ a standard genetic algorithm. We perform random initialisation, use uniform crossover and uniform mutation, while we update the population based on an elitist strategy. Note that, this problem is solved as a multi-objective optimisation problem, so that in our elitist strategy, pareto solutions are regarded as elite and inherited to the next generation while the remaining individuals are selected randomly from parents and offsprings.

3 EXPERIMENTS

3.1 Goal, Dataset, and Parameters

To confirm the usability of our proposed method, we perform a set of experiments using a Japanese airline map with 79 nodes and 233 edges. Figure 2 shows the map as well as the result obtained by FDEB.

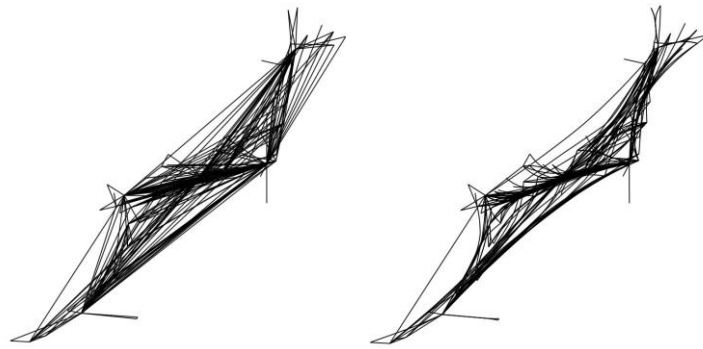


Figure 2: The original Japanese flight map and FDEB result.

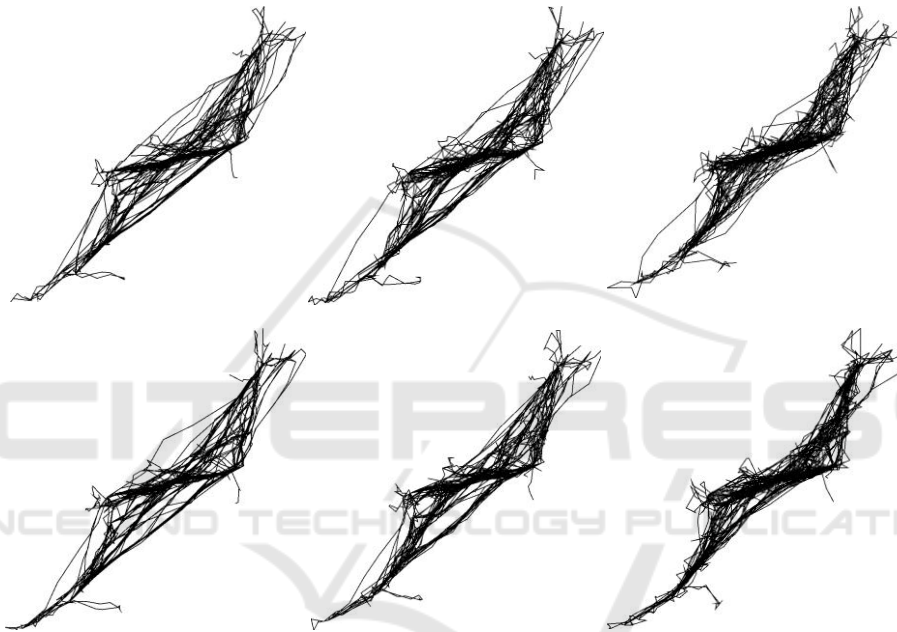


Figure 3: The results of our proposed method (First row: population:200 max: 300, Second row: population: 500, max:750).

For our algorithm, we used the following parameter settings: number of generations: 750, population: (initial 200, maximum 300) and (initial 500, maximum 700); mutation probability: 0.01, crossover probability: 0.7, s of MOA: 5; c (the number of control points): 4; v : 20, 30, and 50.

3.2 Results

Figure 3 shows one of the pareto solutions for each of the tested values for v and population sizes.

From these, we see that all results have areas where edges are successfully bundled. For $v=20$ and $v=30$, mainly, edges in the area where the edge density is low in the original graph (for example, around Sado Island) were separated without being bundled well. This is probably due to the fact that the edges cannot be deformed to an appropriate bundle

position due to the number of control points and their limits of displacement distance.

Overall, edges tend to be less smooth as the displacement distance is increased. This is likely caused by control point moving more than necessary given the wider range of flexibility.

We can also compare the difference of the results between population configurations. From Figure 3, we can see that a larger population leads to an improved visualisation.

Interestingly, our proposed method is able to separate the route from Tokyo to Okinawa (in the bottom-left area of the graph) clearly for $v=30$ and 50, whereas FDEB is unable to do so.

We notice that in our results the edges still show some zig-zag appearance, this is not unexpected since the path quality is only one of the four criteria we employ.

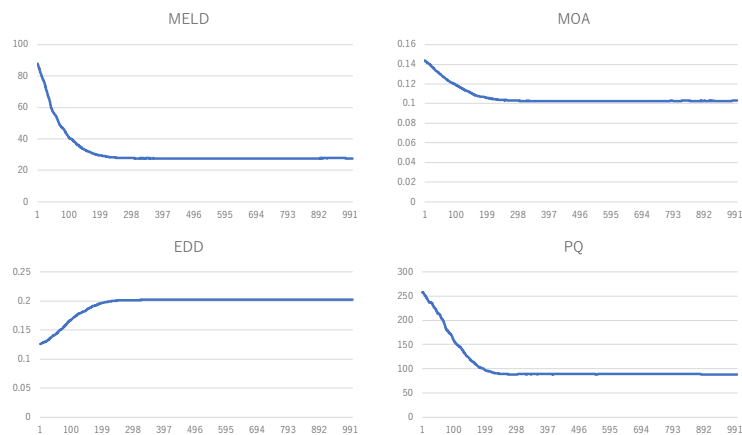


Figure 4: Fitness Function (x: generation, y: criteria).

Figure 4 plots the four criteria of the fitness function as the GA progresses through the generations. From there figure, we can be seen that the values converge and the evolution has stopped. Therefore, it is speculated that this result has fallen into local optimization, and it is speculated that this will be an issue. In other words, there is room to improve the quality when we can prevent the algorithm from falling into local optimization.

4 CONCLUSIONS

In this paper, we have proposed a genetic algorithm-based edge bundling methods for visualisation applications. We employ control point information that is encoded in the GA together with a fitness function that optimises several aesthetic rules. The obtained results on a Japanese air route map confirm the applicability and usability of our proposed algorithm. We conclude with some issues that we plan to investigate in future work.

4.1 Fitness Function

The employed fitness function can be modified or extended to consider also the possibility of *faithfulness* (Nguyen et al., 2013; Nguyen and Eades, 2017) and other indicators such as the ink-ratio.

4.2 Extensibility

In this approach presented here, the genetic representation is based on control points. Adding information on nodes and aesthetic rules of nodes would allow also edge bundling in consideration of the arrangement of nodes. Also, in this work, we have

employed only a simple standard GA whereas a large number of other, more advanced GA algorithm can be utilised.

4.3 Limitations

In this study, our aim is to highlight the potential of generating an acceptable edge map visualisation employing computational intelligence for edge bundling. There are of course still limitations. One is the computational complexity, which is a general drawback of black-box optimisation techniques such as GAs, in particular for large graphs. There is also a problem with visual encoding. Although we do not discuss visual encoding here, this extension can also be applied if information on Visual Encoding is added to a single locus.

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