Self-organized Cognitive Algebraic Neural Network

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Abstract: This paper refers to author's patented invention that introduces a more efficient statistical (machine) learning method. Inspired by neuroscience, the paper combines the synaptic networks and graphs of quantum network to constitute interactions as information flow. Hitherto, several machine learning algorithms had some influence in business decision-making under uncertainty, however the dynamic cognitive states and differences thereof, at different timepoints, play an important role in transactional businesses to derive choice and choice-sets for decision-making at societal scale. In addition, deep neural functions that reflect the direction of information flow, the cliques and cavities, necessitate a new computational framework and deeper learning method. This paper introduces a proactive-retroactive learning technique - a quantified measurement of a multi-layered-multi-dimensional architecture based on a Self-Organized Cognitive Algebraic Neural Network (SCANN) integrated with Voronoi geometry – to deduce the optimal (cognitive) state, action, response and reward (pay-off) in more realistic imperfect and incomplete information conditions. This quantified measurement of SCANN produced an efficient and optimal learning results for individuals' transactional activities and for nearest-neighbor, as a group, for which the individual is a member. This paper also discusses and characterizes SCANN for those who handle decisions under conditions of uncertainty, juxtaposed between human and machine intelligence.

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1 INTRODUCTION

Human decision making routinely involves choice among temporally extended courses of *action*, *response* and *reward,* as pay-off, over a broad range of time scales depending on cognitive *state*. Consider a traveler deciding to undertake a journey to a distant city for work. To decide – go-no-go – the end-benefits in terms of *reward*, as pay-off, of the trip must be weighed against the cost. Having decided to go, choices must be made at each fragmented "smaller" decision e.g., whether the work is worth paying or not, whether to fly or to drive, whether arrange a local accommodation or stay with friends or relatives. With the brute force of computational processes and the better understanding of human intelligence – *how individuals go about solving their problem* – some of the existing learning technologies may train machines for the outcome. Here one would like to make a distinction between *precision engineering* and intelligence. One of the fundamental principles in precision engineering is that of determinism where systemic behavior is fully predictable, even to an individual's, or atomic-scale, activities. To do the job efficiently and correctly, one needs models and algorithms, where the basic idea is that machine follows a set of rules, cause and effect relationships, that are within human ability to understand and control and that there is nothing random or probabilistic about their behavior. Further, the causalities are not esoteric and uncontrollable, but can be explained in terms of familiar and precise engineering principles. Intelligence, on the other hand, as opposed to *fact*, is stochastic in nature. It finds optimal solutions, derives reasons, infers *actions*, recognizes patterns, comprehends ideas, solves problems and uses language to communicate, from (im)perfect and (in)complete information conditions.

However, some learning methods, where the result is the final *reward* or pay-off, are awfully hard to untangle the future information to foresee the sequence of *actions* that will benefit the user at some point in future. Some of these infrequent and delayed *rewards* or learnings limit decisions making process (Edward, Isbell, Takanishi, 2016). For some

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combinatorial problems, where all rules and information are known to all parties, one may set up intermediate positions for them to achieve the optimal results where, as in real-life conditions, such learning success depends on how well one would fragment a "major" decision or an objective into a series of multiple "smaller" decisions – the decision journey – and *actions* to measure their progress accurately. Unlike many statistical (machine) learning techniques, this approach has yielded both unreliability in the training process, and a general lack of understanding as to how the learning model converge, and if so, to what (Barnett 2018).

2 PROACTIVE-RETROACTIVE

Most humans or species do not learn by rote or by reinforcing the subject into the memory. In fact, the growth and maturation of a child's brain is an intricate process taking decades, in which the brain grows and adapts to the surrounding world (Aamodt, and Wang, 2008). The same research has shown that the developing brain has been shaped by thousands of generations of evolution to become the most sophisticated information-processing machine on earth. And, even more amazingly, it builds itself. The way the information is processed can be termed as *dynamic proactive-retroactive* learning wherein a human (or a system) proactively learn, either through instructions or through observations, and then waits for some kind of confirmation – either from nearest-neighbor or trusted source – which retroactively reinforce or modify in accordance with the subject matter. For example, when a child learns "A for Apple, B for Boy and C for Cat" from a book, he or she registers only an image of an apple, a boy or a cat. These images are retained in memory until, some point in future, when he or she physically observes the contextual appearance – *new information that connects the dots* – of an apple, or a boy or a cat, and confirmed by a trusted source, often parents, with the text – the name – associated with those physical images. Even most adult brains follow the same principle when they observe something new. At the time of observation, they retain this new information in their memory as postulations – "may be this is a peach" (language text) or "may be the boy is playful" (causal reasoning) or "may be this is Mr. Smith" (personality), until their postulations are confirmed by a trusted source, often in the nearest-neighbor. These observations and confirmations happen in two different time-points. And, sometimes the observed postulations are radically altered with the confirmation of new information – "oh no, this is an apricot, not peach" (language text) or "no, the boy is sarcastic, not playful" (causal reasoning) or "ah, this is Mr. David, not Mr. Smith" (personality) – at the time of confirmation. In this learning process, the former is *proactive* learning whereas, the latter – *retroactive* learning – changes the original postulations or replaces the deep-seated beliefs through new information connections, often either guided by experience or information from the nearest-neighbor or both (Sen, 2017).

So, what happens to *state* of the information between *proactive* and *retroactive* – two different time points – in the learning cycle? The neuroscience research has shown that in early childhood, and again in the teens and subsequently at various stages of learning, brains go through bursts of refinement, forming and then optimizing the connections in the brain. Connections determine what the subject or object is, what does it do, and how does it do. Early childhood provides an incredible window of opportunity with neural connections forming and being refined at such an incredible rate, there isn't a certain time when babies are learning – they are always learning. Every moment, each experience translates into physical trace, a part of the brain's growing network (Bachleda and Thompson, 2018), One of the most powerful set of findings concerned with the learning process involves the brain's remarkable properties of "plasticity" – *to adapt, to grow in relation to experienced needs and practice, and to prune itself when parts become unnecessary* – which continues throughout the lifespan, including far further into old age than had previously been imagined (Skoe and Kraus (2012). The demands made on the human learning are key to the plasticity – *the more one learns, the more one can learn* – and, therefore required to be included in this architecture of artificial neural network for machine learning.

3 NEURAL NETWORK WITH VORONOI REGION

An effective method for designing neural network that derives the stages in-between *proactive* and *retroactive* learning in two different time points is to classify patterns in the multi-dimensional feature space. This deep learning architecture introduces a multi-dimensional feature space where the information *waits* in certain workspace – the *Voronoi region* – within the neural network based on distance to points in a specific subset of the plane. The

Voronoi diagram is derived over points in feature space which represents teachers' input in order to realize the desired classification. However, to reduce the size of the neural network and make the learning efficient, clustering procedure that enables the subject to manage a number of teachers in a lump is implemented (Kenji, Masakazu and Shigeru, 1999). Our approaches, however, only utilize point-wise cell-membership – *as new information* – by means of nearest-neighbor queries and do not utilize further geometric information about Voronoi cells since the computation of Voronoi diagrams is prohibitively expensive in high dimensions. Therefore, a Monte-Carlo-Markov-Chain integration-based approach (Polianskii and Pokorny, 2019) that computes a weighted integral over the boundaries of Voronoi cells, thus incorporates additional information – as *retroactive* confirmation – about the Voronoi cell structure is established. This dynamic *proactive-retroactive* learning method predicts and prescribes *an action* in "expected" *response* to an activity of human (or interchangeably a machine), depending on individual's *state*, for one or more end-*rewards*, or pay-offs at a given point in time.

Since most information related to immediate relevance including dynamic active cognitive *state* and/or active experiences, hence individuals apply a certain set of rules that are associated with either sequential monadic (e.g., individual's state from *a* to *á* as self-improvement) or paired-comparison (e.g., individual's state *x* compared with another individual's state *y*) with nearest neighbor or a group where individual is a member. This, in imperfect or asymmetric and incomplete information conditions, creates "hidden" multi-layered combinations on multi-dimensions – functional, non-functional, non-discriminating and discriminating – features to predict and determine the cognitive state (or "*state*"). The group, where individual is a member, may also apply a certain set of collective "hidden" information associated with either linear-non-linear (e.g., a race-car driver uses wind direction data while cornering at speeds more than 200 mph without informing the opponent) or paired comparison (e.g. race car the team analyzes data of other racers' degradation rates on the tires and of the health of various mechanical components, and recording the drivers' steering, braking and throttle inputs). In imperfect and incomplete information conditions, this generates aggregated "hidden" multi-layered combinations on multi-dimensions features to predict and determine a collective *state*. For example, a trading system analyzes data to predict if the *state* of any trading stock and its change with new features, conditions and functions – the underlying latent variables – affect the price, as an outcome, in the marketplace.

The hypotheses here are that the *dynamic proactive-retroactive* learning method would derive to be a better prediction on the individual's current *action* for future *reward*, as final pay-off, over a broad range of time and information scales, including (im)perfect and (in)complete information conditions. For example, if the trading system predicts that the *state* of the product (or service) and its change with the underlying latent variables affect price in the marketplace, then the expected *response* of the buyer may also likely to change (either to buy immediately or defer for the future price), thus may create a different *reward* or pay-off outcome (revenue or saving for the trader).

4 TRAINING DATA

A self-organized learning method, in accordance with the *dynamic proactive-retroactive* learning method is executed to segment a graph network data based on bounded diffusion of collective individual information interactions. The nearest-neighbor or group data is determined from grouping of individual transactional data for a group where individual is a member. After a certain upper-bound number of groups, the system applies a diffusion-limited aggregation ("DLA") – a formation process whereby individuals in a group, as particles, and their signals – defined as change or the first derivative in an individual's data – of a subject matter undergo a stochastic process for clustering together to different aggregates ("clusters") of such individuals. These signals and their changes – defined as the second derivative in an individual's data – are used for predicting the group's current *state*, as described above, and applied sheafing method, (or group theory) for "grouping" mechanism (Tennison, 2011) – depending on the geometry of the growth, for example, whether it be from a single point radially outward or from a plane or line – of clusters where the individual is a member, to determine the *state*.

The self-organized learning method presents individual's data, for example, as stimulus, at some time t=0 and then presenting a response data at a variable time post stimulus on the group. The bounded diffusion in DLA, for example, may have one additional parameter, the position of the decision bound, say A. If at time t of the *state* data of the individual (or subject matter e.g., search for an item) is x, the distribution of the state at a future time may be s > t, hence the term "forward" diffusion. The backward diffusion, on the other hand, may be useful when the individual at a future time s has a particular behavior due to past decision, the distribution at time is t < s. This may impose a terminal condition, which is integrated backward in time, from s to t (hence the term "backward" is associated with this). Let g(x) be a bounded smooth (twice continuously differentiable having compact support) function, and let:

$$u(t,x) = E^{x,t}\big(g(X(T)\big) \equiv E\big(g(X(T))\big|X(T) = x \quad (1)$$

with the "terminal" condition u(T, x) = g(x). In addition, if X(t) has a density p(t, x), then for a probability density function μ(·), the probability densities satisfy the:

$$\frac{\partial}{\partial t}p(t,x) = (A^{*}p)(t,x) \quad (2)$$

where A* is the adjoint operator of A, defined as:

$$A^{*}v(t,y) = -\frac{\partial}{\partial y}\big(b(y)v(t,y)\big)\frac{1}{2}\frac{\partial^2}{\partial y^2}(\sigma^2(y)v(t,y) \quad (3)$$

This behavior may be described as fractal growth, as frequently observed in plants like ferns. The clusters may include formulating a group associated with the group's current activity as well as the nearest-neighbor for the individual where individual is a member.

These *state* data of individual are used to predict the group's current *action* where the individual is a member, to determine the choice clusters of likely *action*. The *action* data are further used to predict the group's expected *response* to formulate choice clusters of likely response. And, finally, these *response* data are used to predict the group's *reward* or pay-offs to derive choice clusters of the *reward* or pay-off in their decision journey.

5 COGNITIVE ALGEBRAIC NEURAL NETWORK

A multi-layered multi-dimensional *Self-Organized Cognitive Algebraic Neural Network* ("SCANN") learning method is formulated, in accordance with the *dynamic proactive-retroactive* learning method and self-organized learning method. This is required to arrange information and determine undefined rules based on a cognitive structure for the individual (or the subject matter). This may include choices and maximum likelihood estimation of each choice for the activity of the individual. A set of data in activity, for example, is determined for each individual (n) and more individuals are added to the activity content that form choices and different choice sets. The features (or attributes) of these choices and choice-sets may or may not be causal factors that influence a choice. A choice set attribute may comprise one or more attributes, for example, of the item such as combination of sensory attributes, (taste, looks, etc.), rational (price, ingredients, etc.) and emotional (feel good, lifestyle, etc.). In the formation of a group with different clusters, based on activity and/or factors thereof, each choice set becomes a function of activity and interactions within a group, where the individual is a member. One or more common contact individual and/or individual's activity content between individuals may exist in a group. Further, this indicates a "hub" contact with "cross" features and attributes for individual and/or individual's activity content between individuals in a group, thus forms a graph structure of the network.

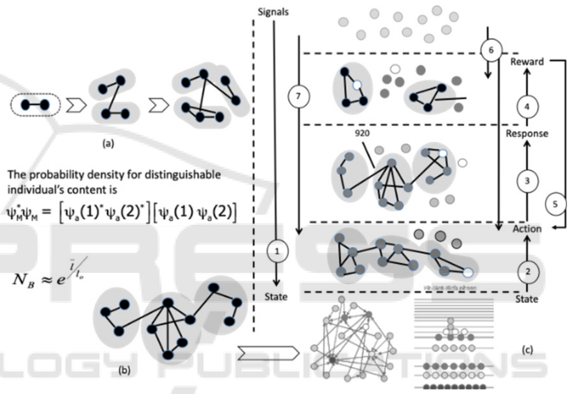

Figure 1: Multi-Layered Multi-dimensional Self-Organized Cognitive Algebraic Neural Network (SCANN).

The graph structure of the SCANN is a pair (*N*, **g**), where **g** is a network on the set of nodes *N*. A relationship between two nodes *i* and *j*, represented by $ij \in g$, is referred to as a *link* or *edge*. Thus, **g** will sometimes be an $n \times n$ adjacency matrix, with entry g_{ij} denoting whether *i* is linked to *j* and may also include the intensity of that relationship. The *neighbors* of a node *i* in a network (*N*, **g**) are denoted by N_i(**g**). The *degree* of a node i in a network (*N*, **g**) is the number of neighbors that *i* has in the network, so that $d_i(\mathbf{g}) = |N_i(\mathbf{g})|$. Many naturally occurring multi-layered multi-dimensional networks (Erdös, and Renyi, 1960), as represented in this Fig 1, explicitly incorporate multiple channels of connectivity and constitute the natural environment to describe systems interconnected through different categories of connections: each activity content module (signals, *states*, *actions*, *responses* and *rewards*) may be represented by a layer and the same node or entity may have different kinds of interactions (set of nearest-neighbors in each layer).

The latent feature structure, as depicted in Fig. 1 above, is abstracted from variables to render microstate probabilities of each (dis)satisfied individual's choice-set attributes and latent causal variables, accessible by mere combinatorial, (im)perfect and (in)complete information conditions much in the same way as graph probabilities, become accessible in random graph. At an atomic level, for each individual, the structure finds the optimal choice-set of latent variables that has causal effect on the expected outcome or *reward* or pay-off (Sen, 2015). The interaction variables that are available for individuals to exercise preference, or any variable involving an interaction of the individual for a good or service. The coefficients are predetermined and represented a diminishing level of satisfaction, for example, over time. In addition, the latent learning represents that, in cognitive decision, despite their non-equilibrium and irreversible nature, the evolving network is mapped into an equilibrium Bose-Einstein ("BE") condensation nodes corresponding to energy levels, and links representing the individual's activity contents, as particles (Bianconi and Barabási, 2001). The existence of a *state* transition, phase to a BE condensate, the outcome distribution $g(\epsilon) = C \in^{\theta}$ where θ is a free parameter and the energies were chosen from $\epsilon \in (0, \epsilon_{max})$ with normalization $C = \theta + 1/(\epsilon_{max}^{\theta+1})$. For this class of distributions, the cognitive state for a Bose condensation is determined as:

$$\frac{\theta+1}{(\beta\epsilon_{max})^{\theta+1}} \int_{\beta\epsilon_{min}(t)}^{\beta\epsilon_{max}} dx \frac{x^{\theta}}{e^{x}-1} < 1 \quad (4)$$

The active strand of the study in this direction is to study individualized ensembles with fixed degree sequences, or degree distributions following, for instance, a power-law. This is the probability that a randomly chosen node in the network has exactly l links, is proportional to l^{-y} for some y$\in (2, \infty)$.

The choices for individuals (or interchangeably machines) in *N* have *action* spaces A_i. Let $A = A_1, \dots A_n$ at every stage in their decision journey. In this, the *action* spaces are finite sets or subsets of a Hilbert space. Generally, decision making is not necessarily associated with a choice of just one *action* among several simple given options, but it involved a choice between several complex options for *actions*. The *elementary prospect* (e_n) is the conjunction of the chosen *modes*, one for each *action* from the *intended* action. To each elementary prospect e_n, there corresponds the basic state $|e_n\rangle$, which is a complex function $A^N \rightarrow C$, and its Hermitian conjugate $\langle e_n|$. The structure of a basic state is

$$\langle e_n| = \otimes_{i=1}^{N} |A_{iv_i} \quad (5)$$

The cognitive or *mind space* is the closed linear envelope

$$M \equiv \text{span}\{|e_n\rangle\} = \bigotimes_{i=1}^{N} M_i \quad (6)$$

To each *prospect* πj, there corresponds a *state* $|\pi_j\rangle \in M$ that is a member of the *mind space*. $|\pi_j\rangle = \sum_n a_{jn}|e_n\rangle$. This applies a quantum decision theory as an intrinsically probabilistic procedure. The first step consists in evaluating, consciously and/or subconsciously, the probabilities of choosing different prospects from the point of view of their usefulness and/or appeal to the choosing agent. If the mapping from a *state* parameter *w* to the conditional probability density p(y|x, *w*) is one-to-one, then the model is identifiable, i.e. if the product in service is in its lowest *state* then the likelihood of that product to fail is significantly high. Otherwise, it is non-identifiable. In other words, this model is identifiable if and only if its parameter is uniquely determined from its *state* and/or cognitive behavior.

However, in non-identifiable cases, as depicted in Fig. 1, *actions* are more dynamic and remain in active workspace of the individual as they "wait" – *the Voronoi region* – for more signals in transactional data to make the connection for *action* (best matching nearest-neighborhood *action* cells). For these non-identifiable cases of *actions*, a new set of information is required, as new cell, C, and a local counter variable τ_{C} that constrains the number of input signals for which the *action* has best-matching unit (Fukushima, 2013). Further, introduction of a new signal data, as a new cell, C, with a local counter variable τ_{C} and since the cells are slightly moving around, more recent signals may be weighted stronger than previous ones. An adaptation step, for example, may be formulated as: a) choose an input signal data according to the probability distribution $P(\xi)$, b) locate the best matching unit $c = \emptyset_w(\xi)$; c) increase matching for c and its direct topological neighbors $\Delta w_c = \varepsilon_b(\xi - w_c)$; d) Increment the signal counter of c, as new signal data gets added, either via another activity, e.g., a call from a friend, or an 'autonomous' message: "how about going out for lunch" : $\Delta\tau_c = 1$; e) decrease all signal counters by a fraction α : $\Delta\tau_{\mathsf{C}} = -\alpha\tau_{\mathsf{C}}$ (not shown in the diagram) which is uniquely determines the change in *action* due to new signal data that influenced its *state* and/or cognitive behavior. The relative signal frequency of a cell C is: $h_c = \tau_c / \sum_{j \in A} \tau_j$. A high value of h_c, therefore, indicates a good position to insert a new latent variable, as cell, because the new latent variable, cell, is likely to reduce this high value of *action* to a certain degree. The insertion of new cells leads to a new Voronoi region, F, in the input space. At the same time, the *Voronoi regions* of the topological neighbors of C are diminished. This change is reflected by an according redistribution of the counter variables τ_C.

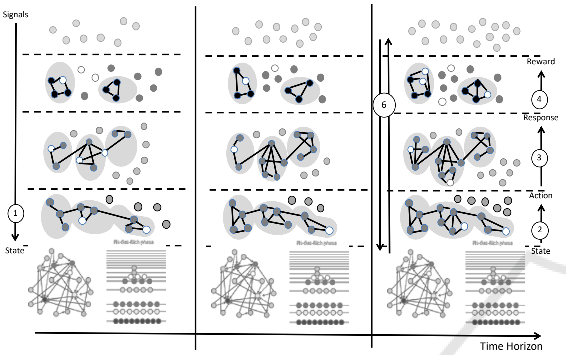

Figure 2: Self-Organized Cognitive Algebraic Neural Network (SCANN) With Voronoi Region.

There are also many conditions where the individual and/or individuals in a group choose *actions* without fully knowing with whom they will interact and what would be their *response*. Instead of a fixed network, individuals are now unsure about the network that will be in place in the future, but have some idea of the number of interactions that they will have. To fix ideas, the individual and/or a group where individual is a member and their *action* data may choose to find expected *response* that is only useful in interactions with other individuals who has the same product as well, but without being sure of with whom one will interact in the future. In particular, the set of individuals *N* is fixed, but the network (*N*; **g**) is unknown when individuals choose their *actions*. An individual *i* knows his or her own degree d_i, when choosing an *action*, but does not yet know the realized network. Individuals choose *actions* in {0,1}, individual *i* has a cost of choosing *action*$_1$, denoted c_i. Individual *i*'s payoff from *action*$_1$ when *i* has d_i neighbors and expects them each independently to choose 1 with a probability *x* is: $v\,(d_i, x) - c_i$ and so *action*$_1$ is an expected *response* for individual *i* if and only if $c_i \leq v(d_i, x)$. The payoff to the individual from taking *action*$_1$ compared to *action*$_0$ depends on the number of neighbors who choose action$_1$, so that

$$sign\left(u_i(1, a_{N_i(\mathbf{g})}) - u_i(0, a_{N_i(\mathbf{g})})\right) = sign\left(\sum_{j \in N_i(\mathbf{g})} a_j - \sum_{j \in N_i(\mathbf{g})} 1 - a_j\right) \quad (7)$$

If more than one half of i's neighbors choose *action*$_1$, for example, then it is best for individual *i* to choose 1, and if fewer than one half of *i*'s neighbors choose *action*$_1$ then it is best for individual *i* to choose *action*$_0$. There may be multiple equilibria in this situation. In non-identifiable cases of expected *response* may be dynamic and/or in active workspace, as the expected *response* data of the individual *"wait"* for more signal data or *action* data to make connection for expected *response* or lack of confidence (best matching neighborhood *action* cells) on the existing signal data. The prospect probability may be defined as: $p(\pi_j, \tau) = \mathrm{T}\, r_{AB\hat{\rho}AB}(\tau)\hat{P}(\pi_j)$. The interaction of the decision maker with the group may ensure that the individual keeps distinct identity and personality while, at the same time, possibly changing *state* of mind. In other words, the surrounding group does influence the individual's *state*, but does so in a way that does not suppress the person making own decisions. This corresponds to the behavior of a subsystem that is part of a larger system that changes the subsystem properties, while the subsystem is not destroyed and retains its typical features.

Introduction of a new signal data, as in Fig 2, or *action* data as a "new" cell, C, with a local counter variable τ_C and since the cells are slightly moving around, more recent signals may be weighted stronger than previous ones. Here, the changes of the signal and *action* counters as redistribution of the counter variables may be seen as ascribing to the new cell. This new cell is connected to the existing expected *response* cells in such a way that may again a structure consisting only of k-dimensional simplices:

$$\Delta\tau_C = \frac{\left|F_C^{New} - F_C^{old}\right|}{\left|F_C^{old}\right|} \quad (8)$$

A new *Voronoi region* exists now. As much input signals and/or *actions* as it would have received if it had existed since the beginning of the process. In the same way the reduction of the counter variables of its neighbors may be motivated by making more information available to all. In such network interactions the possible outcomes of the D and C to two basis vectors $|D\rangle$ and $|C\rangle$ in the space of a two-*state* condition, e.g., either coffee (A) or juice (B), the *state* of the situation may be described by a vector in the product space which could be spanned by the basis $|CC\rangle$,$|CD\rangle$,$|DC\rangle$ and $|DD\rangle$, where the first and second entries refer to A's and B's *states*, respectively. This may denote the *responses* initial *state* by $|\psi_0\rangle = \hat{J}|CC\rangle$, where $\hat{J}$ is a unitary operator which may be known to both individuals. For fair *response*, $\hat{J}$ must be symmetric with respect to the interchange of the two individuals. The strategies are executed on the distributed pair of *state* situations in the *state* $|\psi_0\rangle$. Strategic moves of two individuals, for example, A and B are associated with unitary operators $\widehat{U}_A$ and $\widehat{U}_B$, respectively, which are chosen from a strategic space S. The independence of the individuals dictates that $\widehat{U}_A$ and $\widehat{U}_B$ operate exclusively on the *states* in A's and B's possession, respectively. The strategic space S may therefore be identified with some subset of the group of unitary 2 x 2 matrices. Having executed their moves, which leaves the situation in a *state* $(\widehat{U}_A \otimes \widehat{U}_B)\hat{J}|CC\rangle$, A and B forward their *states* for the final measurement which determines their payoff. The only *strategic* notion of a payoff may be the *expected* payoff. A's expected payoff may be given by

$$\$_A = rP_{CC} + pP_{DD} + tP_{DC} + sP_{CD} \quad (9)$$

where $P_{\sigma\sigma'} = |\langle\sigma\sigma'|\psi_f\rangle|$ is the joint probability that the channels σ and σ'. A's expected payoff $\$_A$ not only depends on her choice of strategy $\widehat{U}_A$, but also on B's choice $\widehat{U}_B$.

Individual *i*'s *reward* or payoff function may be denoted u_i: $A \times G(N) \rightarrow \mathbb{R}$. A given individual's payoff depends on the group where the individual is a member or other individuals' *actions*, but only on those to whom the individual is (directly) linked in the network. In fact, without loss of generality the network may be taken to indicate the payoff interactions in the group. More formally, individual's payoff may depend on a_i and $\{a_j\}_{j \in N_{i(\mathbf{g})}}$ so that for any i, a_i, and $\mathbf{g}$: $u_i(a_i, \mathbf{a}_{-i}, \mathbf{g}) = u_i(a_i, \acute{\mathbf{a}}_{-i}, \mathbf{g})$ whenever $\mathbf{a}_j = \acute{\mathbf{a}}_j$ for all $j \in N_i(\mathbf{g})$. Unless otherwise indicated the equilibrium, may be a pure strategy Nash equilibrium: a profile of *actions* $\mathbf{a} \in A = A_1 \times \ldots A_n$, such that $u_i(a_i, \mathbf{a}_{-i}, \mathbf{g}) \geq u_i(\acute{a}_i, \mathbf{a}_{-i}, \mathbf{g})$ for all $\acute{a}_i \in A_i$. In the case with large fluctuations in input of expected *response* with large-scale networks, however, the weights increase without limits due to the diffusion effect if weight constraints are absent. Nevertheless, the choice probability of a network with diverging weights asymptotically approaches matching behavior. A weight-normalization constraint may be imposed for the diffusion effect to become more evident than in cases without normalization.

However, in non-identifiable cases of *reward* may be in dynamic and/or active workspace, as the *reward* data of the individual *"waits"* for more signal data or *action* data or expected *response* data to make connection for *reward* or lack of confidence (best matching neighborhood *action* cells) on the existing signal data.

Introduction of a new signal data, or *action* data or expected *response* data as a new cell, C, with a local counter variable τ_{C} and since the cells are slightly moving around, more recent signals may be weighted stronger than previous ones. Here the main characteristic of the model could be that several adaptation steps may sometimes be followed by a single insertion. One may note the following feedback relation between the two types of *action*: a) every adaptation step may increase the signal, *action* and *response* counters of the best-matching unit and thereby increases the chance that another cell will be inserted near this cell; b) insertion near a cell C decreases both the size of its Voronoi field F_{C} and the value of the signal or *action* or expected *response* counter. The reduction of the Voronoi field makes it less probable that C will be best-matching unit for future input signals.

Networks are then analyzed in terms of groups of nodes that are all-to-all connected, termed as *cliques*. The number of neurons in a clique determines its size, or more formally, its dimension. In directed graphs it is natural to consider directed cliques, which are cliques containing a single source neuron and a single sink neuron and reflecting a specific motif of connectivity (Song, Sjöström, Reigl, Nelson and Chklovskii, 2005), wherein the flow of information through a group of neurons has an unambiguous direction. The manner in which directed cliques bind together can be represented geometrically. When directed cliques bind appropriately by sharing neurons, and without forming a larger clique due to missing connections, they form, termed as, *cavities* ("gaps," "voids" or "unknowns") in this geometric representation, with high-dimensional cavities forming when high-dimensional (large) cliques bind together. Directed cliques describe the flow of information in the network at the local level, while cavities provide a global measure of information flow in the whole network. Using these naturally arising structures, we established a direct relationship between the structural graph and the emergent flow of information in response to stimuli, as captured through time series of functional graphs (Reimann, Nolte, Scolamiero, Turner, Perin, Chindemi, Dlotko, Levi, Hess and Markram, 2017).

These structural graphs are analyzed at different timepoints. As time progresses, for example, the parameters in rules associated with active experience or historical or neither may change and/or eliminated, and thereby change prediction and prescription that determine the *action* data for the *action* indicator. As time progresses, at each step of determining the *state* data, the *action* data, the expected *response* data, and the *reward* data also change optimal controls for the individual and groups. The Voronoi region F_C, for example, by an n-dimensional hypercube with a side length equal to the mean length $\bar{l}_C$ of the edges emanating from C with $\bar{l}_C$ computed by

$$\bar{l}_C = {}^{1}\!/_{card\ (N_C)} \sum_{i \in N_C} \|w_C - w_i\| \tag{10}$$

From the above, it is evident that it would be very helpful to know the true dimensionality of the data, meaning the smallest dimensionality t, such, that a t-dimensional sub-manifold of V may be found containing all (or most) input data. Then t-dimensional hyper-cubes may be used to estimate the size of the Voronoi regions. However, to figure out the value of t, especially because the mentioned sub-manifold may not have to be linear but could be randomly twisted. Therefore, even analyses of the signal, *state*, expected *response* and *reward* data may not, in general, reveal their true dimensionality and remain "unaided", but gives only (or at least) an upper bound. However, the method of training for machine to learn and, therefore, gives some general rules for choosing such an estimate that do work well for all activities that may be encountered subsequently.

Moreover, as time progresses, the learning system may accelerate or decelerate the speed of information flow between signal and *state* and *action*, and expected *response* and *reward*. This may support the two structural update operations: a) insertion of a cell, as a neuron; b) deletion of a cell, as a neuron. These operations may be performed such that the resulting structure consists exclusively of multi-dimensional structure $\mathcal{H}$. Although such a data structure may already be sufficient in this example, a considerable search effort may be needed to make consistent update operations. The removal of a cell may also require other neurons and connections are removed to make the structure consistent again. Simple heuristics as, for example, to remove a node remove all neighboring connections and the node itself may not work properly. For this purpose, a tracking mechanism of all the $\mathcal{H}$ may be introduced in the current network. Technically, a new data type simplex may be created, an instance of which contains the set of all nodes belonging to a certain $\mathcal{H}$. Furthermore, with every node associated to the set of those $\mathcal{H}$ the node may be part of. The two update operations can now be formulated as: a) a new node r may be inserted by splitting an existing edge qf. The node r may be connected with q, f, and with all common neighbors of q and f. Each $\mathcal{H}$ containing both q and f (in other words, the edge being split) may be replaced by two $\mathcal{H}$ each containing the same set of nodes as $\mathcal{H}$ except that q respectively f may be replaced by the new node r. Finally, the original edge qf may be removed. The new $\mathcal{H}$ may be inserted in the sets associated with their participating nodes. b) to delete a node, it may be necessary and sufficient to delete all $\mathcal{H}$ the node may be part of. This may be done by removing the $\mathcal{H}$ from the sets associated with their nodes. The same may be done with nodes having no more edges. This strategy may lead to structures with every edge belonging to at least one $\mathcal{H}$ and every node to at least one edge. Therefore, the resulting k-dimensional structures may be consistent, that is, contain only k-dimensional $\mathcal{H}$.

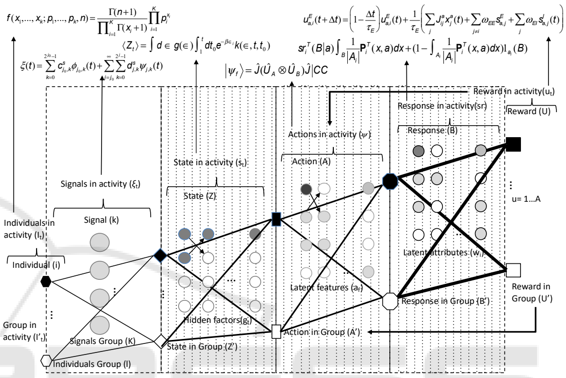

Figure 3: Optimal Learning of SCANN With Voronoi Regions Derive Choice-sets.

Fig. 3 above illustrates an optimization method of multiple-layered multi-dimensional with dynamic expansion and contraction of SCANN structure – *the plasticity* – where the individual activity content, as structured in the learning system, are optimized with dynamic programming method to minimize statistical errors. A relational clique $c \in c$ is constructed of a clique over all activities at various locations on a trajectory, which has an activity of one or more individuals. Each clique C is associated with a potential function $\phi_c(v_c)$ that maps a tuple (values of decisions or aggregations). These evolutionary structures may establish a relationship between the structural graph and the emergent flow of information in *response* to activity content, as captured over time of functional graphs. The activity content and likelihood of (K-1) dimensional simplex S_k in the network structure may find multi-nomial distribution, which could be denoted as Mult$(p_1, \dots p_K;\, n)$, in a discrete distribution over K dimensional non-negative integer vectors $\mathbf{x} \in \mathbb{Z}_+^K$ where $\sum_{i=1}^{K} x_i = n$. Here, $\boldsymbol{p} = (p_1; \dots; p_K)$ in an element of S_K and $n > 1$. Together they may provide a) activity-content, b) probability mass function as expressed,

$$f(x_1, \ldots, x_k;\ p_1, \ldots p_k, n) = \frac{\Gamma(n+1)}{\prod_{i=1}^{k} \Gamma(x_i + 1)} \prod_{i=1}^{K} p_i^{x_i} \quad (11)$$

to optimize activity content for the individual with minimized errors. For example, the buyer's system in a buyer-seller-trader network optimizes the product information workspace that may wait for additional information to formalize specific rules, say "predict", and minimize errors.

Any new or update on activity content may initiate signals in activity, which may form the maximum likelihood estimate ("MLE") of the signal and noise (e.g. data not immediate relevance) for imperfect or incomplete information condition parameters may train machine to learn as a signal, as well as the MLE of the noise parameters may be trained to be learned as noise. The ratio of these two quantities may be taken and compared with upper and lower thresholds until a decision may be made, based on two properties desirable in a continuous sequential detection which may have no analogue in fixed-sample detection, or even in sequential detection, and optimized content as in

$$\xi(t) = \sum_{k=0}^{2^{j_0}-1} c_{j_0,k}^{s} \emptyset_{j_0,k}(t) + \sum_{j=j_0}^{\infty} \sum_{k=0}^{2^{j}-1} d_{j,k}^{s}\, \psi_{j,k}(t) \quad (12)$$

First, the likelihood ratio could be a continuous function of the length of the observation interval for fixed parameter estimates; second, the MLEs could also be continuous functions of the observation interval.

Each individual signal data, as quantum candidate, are aggregated into groups as a function of one or more of attributes and features including time, location, transition and constraints. The grouping included an aggregation of each individual's decisions into groups, based on sheafing methods used earlier for the aggregation into groups for systematically tracking each individual's signal data, with various attributes and features, attached to open sets of a topological space. We fix a set Λ of values for a latent variable. A latent-variable model h over Λ assigns, for each $\lambda \in \Lambda$ and $C \in \mathcal{M}$, a distribution $h_C^{\lambda} \in \mathcal{D}_{\mathcal{R}}\mathcal{E}(C)$. It also assigns a distribution $h_{\Lambda} \in \mathcal{D}_{\mathcal{R}}(\Lambda)$ on the latent variables. This may obtain the map $\mathcal{E}(X) \longrightarrow \prod_{C \in \mathcal{M}} \mathcal{P}(\mathcal{E}(C))$. We may use the isomorphism

$$\prod_{i \epsilon I} \mathcal{P}(X_i) \cong \mathcal{P}(\coprod_{i \epsilon I} X_i) \quad (13)$$

which may take the limit of the cohomology groups of the neural network system as

$$H^0(\{U_i \to U\}, F) = \ker\, (\mathrm{Hom}(\oplus_i Z_{U_i}, F) \rightrightarrows \mathrm{Hom}(\oplus_{i,j} Z_{U_{i,j}}, F) = \mathrm{Hom}(Z_{\{U_i - U\}}, F) \quad (14)$$

The groups determined by grouping methods use prediction activities and optimization of content operation for the groups. Based on the predictions for the group, an optimal set of choices may be determined for the group. For example, in the trading system of buyer-seller-trader network optimizes the product information workspace for the aggregated group to "forecast" price of nearest-neighbor, predict maximum likelihood of forecasted price of the nearest-neighbor and minimize errors to formalize specific rules and optimal policies for various features and attributes that drive forecast.

This abstraction of dynamic and active workspace, as layer, created for each optimized signal data including *"wait"* data and *"new cell"* data, as explained above, parallel connections between any cliques and cavities as described above, as sigma cell in the layer (*l*) and the output of any data, as neuron, in the layer (l_{-1}) may be generated. The number of these parallel connections is equal to the number of activation functions in the layer (*l*). Therefore, in the layer (*l*) an activation function along with all sigma cells or equivalently the sigma blocks are considered as a single multi-dimensional data or neuron, as shown by dashed line in Fig. 3.

6 CONCLUSIONS

Multi-layered and multi-dimensional SCANN networks explicitly incorporate multiple channels of connectivity and constitute the natural environment to describe decision-making system interconnected through different categories of connections: each channel (relationship, activity, category) is represented by a layer and the same node or entity may have different kinds of interactions (different set of neighbors in each layer).

In addition, when SCANN is used, a smaller error rate of about 0.32% can be acquired with a much smaller number of reference vectors, if the SCANN is combined with tune-up Voronoi region (Vr). The computational cost of this method is smaller not only for deep learning but also for the pattern recognition due to smaller number of reference vectors.

The future research will study the interaction structures of economic or knowledge networks accounts for cognitive intelligence, if any, that require SCANN methods. The study will emphasize the properties of perfect and complete information; the interaction of potential use of SCANN; and the *exponentiality* of the deeper neural network.

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