

# A Novel Dispersion Covariance-guided One-Class Support Vector Machines

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**Abstract:** In order to handle spherically distributed data, in a proper manner, we intend to exploit the subclass information. In one class classification process, many recently proposed methods try to incorporate subclass information in the standard optimization problem. We presume that we should minimize the within-class variance, instead of minimizing the global variance, with respect to subclass information. Covariance-guided One-Class Support Vector Machine (COSVM) emphasizes the low variance direction of the training dataset which results in higher accuracy. However, COSVM does not handle multi-modal target class data. More precisely, it does not take advantage of target class subclass information. Therefore, to reduce the dispersion of the target data with respect to newly obtained subclass information, we express the within class dispersion and we incorporate it in the optimization problem of the COSVM. So, we introduce a novel variant of the COSVM classifier, namely Dispersion COSVM, that exploits subclass information in the kernel space, in order to jointly minimize the dispersion within and between subclasses and improve classification performance. A comparison of our method to contemporary one-class classifiers on numerous real data sets demonstrate clearly its superiority in terms of classification performance.

## 1 INTRODUCTION

The important motivation of one-class classification (OCC) has been studied under three main frameworks. First, generally, it is assumed that information from normal operation (targets) are easy to collect during a training process, but most faults (outliers) are not available or very costly to measure. For instance, it is possible to measure the necessary features for a nuclear power plant operating under normal circumstances. But, in case of accident, it is too dangerous or impossible to measure the same features. Second, outliers are badly represented and poorly distributed for training. This appears mainly in tumour detection or rare medical diseases, where a limited number of outliers are available during the training process. Third, for many learning tasks, many objects are unlabeled and few labeled examples are always available, but they are badly represented with unknown prior and ill-defined distributions. For these reasons, OCC problem can be found in many practical applications, such as, medical analysis (Gardner et al., 2006), anomaly detection, face recognition

(Zeng et al., 2006) and web page classification (Qi and Davison, 2009).

In OCC problems, to classify future data points as targets or outliers, three different categories of OCC method can be used: density-based methods, boundary-based methods and reconstruction-based methods. Density-based methods, like Parzen density estimator (Muto and Hamamoto, 2001) and Gaussian distribution (Parra et al., 1996), are based on the estimation of the probability density function (PDF) of the target class. In boundary-based classifiers, only the boundary points around the target class are used to classify data (Vapnik, 1998). The Support Vector Machine (SVM) (Cristianini and Shawe-Taylor, 2000) is a popular two-class classification method based on this philosophy. It aims to maximize the distance margin between the two considered classes using support vectors. It also found its application in OCC problem as One-Class SVM (OSVM) (Schölkopf et al., 2001) and Support Vector Data Description (SVDD) (Sadeghi and Hamidzadeh, 2018). Reconstruction-based classifiers like k-means clustering (Ahmad and Dey, 2011), have been introduced to model the data

rather than resolving classification problem. During classification, a reconstruction error for the incoming data point is calculated. The less the error, the more accurate is the model. The main problems with these three categories of one-class classification methods are that none of them consider the full scale of information available for classification. For instance, the density-based methods focus only on high density area and neglect areas with lower training data density. In boundary-based methods, the solutions are only calculated based on the points near the decision boundary, regardless the spread of the remaining data. A more reasonable method would be to simultaneously make use of the maximum margin criterion (Cristianini and Shawe-Taylor, 2000), while controlling the spread of data. Besides, unlike multi-class classification problems, the low variance directions of the target class distribution are crucial for OCC. In (Kwak and Oh, 2009), it has been shown that projecting the data in the high variance directions (like PCA) will result in higher error (bias), while retaining the low variance directions will lower the total error. Boundary-based methods privilege separating data along large variance directions and do not put special emphasis on low variance directions (Shivaswamy and Jebara, 2010). Moreover, we need to reduce the estimation error by taking projections along some variance directions and the estimated covariance is not accurate due to the limited number of training samples.

However, taking these projections before training leads to an important loss of characteristics. Some powerful classifiers have been proposed to take the overall structural information of the training set into account through the incorporation of the covariance matrix into the objective OSVM function then we can mention the most relevant among them: The Mahalanobis One-class SVM (MOSVM) (Tsang et al., 2006), the Relative Margin Machines (RMM) (Shivaswamy and Jebara, 2010) and the Discriminant Analysis via Support Vectors (SVDA) (Gu et al., 2010). In the one-class domain, the most relevant work is the Covariance-guided One-class Support Vector Machine (COSVM) (Khan et al., 2014). The principal motivation behind COSVM method is to put more emphasis on the low variance directions by incorporating the covariance matrix into object function of the OSVM (Schölkopf et al., 2001). In fact, before training, we want to keep all data characteristics and use the maximum margin based solution, while taking projections in specific directions. In terms of classification performance, COSVM was shown to be very competitive with SVDD, OSVM and MOSVM.

However, there are still some difficulties associated with COSVM application in real case problems, where data are highly dispersed and the target class can be divided into subclasses. In order to handle spherically distributed data, in a proper manner, we intend to exploit the subclass information. In one class classification process, many recently proposed methods try to incorporate subclass information in the standard optimization problem. We can mention among them: The Subclass One-Class Support Vector machine (SOC-SVM) (Mygdalis et al., 2015) and the Kernel Support Vector Description (KSVDD) (Mygdalis et al., 2016). The basic principle of the SOC-SVM method is to introduce a novel variant of the OSVM classifier that exploits subclass information, in order to minimize the data dispersion within each subclass and determine the optimal decision function. Experimental results denote that (SOC-SVM) approach is able to outperform OSVM in video segments selection. On the other hand, KSVDD method modifies the standard SVDD optimization process and extends the proposed method to work in feature spaces of arbitrary dimensionality. Comparative results of KSVDD with the OSVM, the standard SVDD and the minimum variance SVDD (MV-SVDD) (Zafeiriou and Laskaris, 2008) demonstrate the superiority of KSVDD. We presume that we should minimize the within-class variance, instead of minimizing the global variance, with respect to subclass information. Thus, a clustering step is achieved in order to estimate the existing subclasses into the target class. Furthermore, It has been shown in (Zhu and Martinez, 2006) that the clustering does not have a major impact in the classification accuracy. Hence, any clustering algorithm can work in this approach. Then, to reduce the dispersion of the target data with respect to newly obtained subclass information, we express the within class dispersion and we incorporate it in the optimization problem of the COSVM.

In this paper, we propose a novel Dispersion COSVM (DCOSVM), which incorporates a dispersion matrix into the objective function of the COSVM, in order to reduce the dispersion of the target data with respect to newly obtained subclass information and improve classification accuracy. Unlike the SOC-SVM and the KSVDD methods, the DCOSVM has the advantage of minimizing not only the data dispersion within each subclass, but also data dispersion between subclasses, in order to improve classification performance. Moreover, the DCOSVM utilizes a trade off controlling parameter to fine-tune the effect of the dispersion matrix on the classification accuracy. The proposed method is still based on a convex optimization problem, where a global op-

timal solution could be estimated easily using existing numerical methods. The rest of the paper is organized as follows: The next section describes in details a novel subclass method based on COSVM. Section 3 presents a comparative evaluation of our method to other state of the art relevant one-class classifiers, on several common datasets. Finally, Section 4 contains some concluding remarks.

## 2 THE NOVEL DISPERSION COSVM

In this section we describe in details our proposed method. First, we present the COSVM method which provides more importance towards the low variance directions by incorporating the estimated covariance matrix of target class.

### 2.1 The COSVM Method

The estimated covariance matrix of the training data contains all projectional directions, from high variance to low variance. Thus, to keep the robustness of the OSVM classifier intact while emphasizing the small variance directions, (Khan et al., 2014) incorporate the kernel covariance matrix into the objective function of the OSVM optimization problem. So, using the kernel trick, the convex optimization problem of COSVM method can be described as follows:

$$\min_{\alpha} \alpha^T (\eta \mathbf{Q} + (1 - \eta) \Delta) \alpha \quad (1)$$

$$s.t. \quad 0 \leq \alpha_i \leq \frac{1}{vN}, \quad \sum_{i=1}^N \alpha_i = 1,$$

where

$$\Delta = \mathbf{Q}(I - 1_N)\mathbf{Q}^T. \quad (2)$$

For clarity, we have used the vectorized form of  $\alpha = (\alpha_1, \dots, \alpha_N)$  and  $v \in (0, 1]$  is the key parameter that controls the fraction of outliers and that of support vectors (SVs).  $\mathbf{Q}$  is the kernel matrix as defined in Eq. (3):

$$\mathbf{Q}(i, j) = \mathcal{K}(x_i, x_j), \quad (3)$$

$$i = 1, \dots, N; \quad j = 1, \dots, N.$$

$I$  is the identity matrix and  $1_N$  is a matrix with all entries  $\frac{1}{N}$ , and  $\eta$  is the tradeoff parameter that controls the balance between the kernel matrix  $\mathbf{Q}$  and the dual kernel covariance matrix  $\Delta$ . According to (Khan et al., 2014), by controlling the value of  $v$  in the training phase, one can control the confidence on the training dataset directly. If the training dataset is very reliable,

$v$  can be set to a low value so that the whole training dataset is considered. On the other hand, if it is not known whether or not the training dataset truly represents the target class,  $v$  can be set to some higher value.

### 2.2 Derivation of Dispersion COSVM (DCOSVM)

The DCOSVM takes into account the subclass distribution in order to provide more efficient and robust solutions than standard COSVM. The whole idea is based onto projecting the training data set of  $N$  samples,  $\mathcal{X} = \{x_i\}_{i=1}^N$  to a higher dimensional feature space  $\mathcal{F} = \{\Phi(x_i)\}_{i=1}^N$  by the function  $\Phi$ , where linear classification might be achieved. In practice,  $\mathcal{F}$  is not calculated directly. The *kernel trick* (Vapnik, 1998) is used to calculate the mapping, where a kernel function  $\mathcal{K}$  calculates the inner products of the higher dimensional data samples:  $\mathcal{K}(x_i, x_j) = \langle \Phi(x_i), \Phi(x_j) \rangle, \forall i, j \in \{1, 2, \dots, N\}$ . After mapping to feature space, since the entire training set belongs to one class only, we cluster all the training vectors in order to determine  $K$  clusters  $\{C_d\}_{d=1}^K$ , where  $|C_d| = N_d, \forall d \in \{1, 2, \dots, K\}$ . Let  $m_{\Phi}^d$  denotes the mean of the cluster  $C_d$  samples calculated in feature space:

$$m_{\Phi}^d = \frac{1}{N_d} \sum_{i=1}^{N_d} \Phi(x_i). \quad (4)$$

We used  $m_{\Phi}^d$  to calculate the kernel covariance matrix  $\Sigma_{\Phi}^d$  of the training cluster  $C_d$ :

$$\Sigma_{\Phi}^d = \sum_{i=1}^{N_d} (\Phi(x_i) - m_{\Phi}^d)(\Phi(x_i) - m_{\Phi}^d)^T. \quad (5)$$

Considering the case where  $K$  subclasses are formed within the target class, the within subclass scatter matrix (dispersion of the training vectors) can be expressed as follows:

$$S_{\Phi}^w = \sum_{d=1}^K \left( \frac{N_d}{N} \Sigma_{\Phi}^d \right). \quad (6)$$

Where  $\frac{N_d}{N}$  is the prior probability of the  $d$ -th subclass. The between scatter matrix can be defined as follows:

$$S_{\Phi}^B = \sum_{d=1}^K \sum_{b=1, b \neq d}^K (m_{\Phi}^d - m_{\Phi}^b)(m_{\Phi}^d - m_{\Phi}^b)^T. \quad (7)$$

Using this definition, we incorporate the within subclass and between subclass scatter matrices as an additional  $\mathbf{w}^T S_{\Phi}^w \mathbf{w}$  and  $\mathbf{w}^T S_{\Phi}^B \mathbf{w}$ , respectively, into the

objective function of the optimization problem of COSVM Eq. (1). In fact, the term  $\mathbf{w}^T S_{\Phi}^W \mathbf{w}$  is used to minimize the dispersion within subclasses, whereas the term  $\mathbf{w}^T S_{\Phi}^B \mathbf{w}$  has the advantage of minimizing the dispersion between subclasses. However, the dual problem is the one that is solved through some optimization algorithm for COSVM, not the primal one. Thus, it is more appropriate to incorporate the subclass scatter matrix directly in the dual problem. Therefore, we have to use the kernel trick to represent the additional term  $\mathbf{w}^T S_{\Phi}^W$  in terms of dot products only. From the theory of reproducing kernels, we know that any solution  $\mathbf{w}$  must lie in the span of all training samples. Hence, we can find an expansion of  $\mathbf{w}$  of the form:

$$\mathbf{w} = \sum_{i=1}^N \alpha_i \Phi(x_i). \quad (8)$$

By using the definitions of  $\Sigma_{\Phi}^d$  Eq. (5),  $m_{\Phi}^d$  Eq. (4) and the kernel function  $\mathcal{K}(x_i, x_j) = \langle \Phi(x_i), \Phi(x_j) \rangle$ ,  $\forall i, j \in \{1, 2, \dots, N\}$ , we derive the dot product form in Eq. (9), where  $\mathbf{Q}$  is the kernel matrix as defined in Eq. (3).  $I$  is the identity matrix and  $1_N$  is a matrix with all entries  $\frac{1}{N}$ .

$$\begin{aligned} \mathbf{w}^T S_{\Phi}^W \mathbf{w} &= \left( \sum_{i=1}^N \alpha_i \Phi^T(x_i) \right) \left( \sum_{d=1}^K \left( \frac{N_d}{N} \Sigma_{\Phi}^d \right) \right) \left( \sum_{k=1}^N \alpha_k \Phi(x_k) \right) \quad (9) \\ &= \sum_{i=1}^N \sum_{k=1}^N \sum_{d=1}^K \sum_{j=1}^{N_d} \frac{N_d}{N} \alpha_i \Phi^T(x_i) (\Phi(x_j) - m_{\Phi}^d) (\Phi(x_j) - m_{\Phi}^d)^T \alpha_k \Phi(x_k) \\ &= \sum_{i=1}^N \sum_{k=1}^N \sum_{d=1}^K \sum_{j=1}^{N_d} \frac{N_d}{N} \left( \alpha_i^d \mathcal{K}(x_i, x_j) - \frac{1}{N_d} \sum_{l=1}^{N_d} \alpha_i^d \mathcal{K}(x_i, x_l) \right) \left( \alpha_k^d \mathcal{K}(x_k, x_j) - \frac{1}{N_d} \sum_{m=1}^{N_d} \alpha_k^d \mathcal{K}(x_k, x_m) \right) \\ &= \sum_{i=1}^N \sum_{k=1}^N \sum_{d=1}^K \sum_{j=1}^{N_d} \frac{N_d}{N} \left( \alpha_i^d \alpha_k^d \mathcal{K}(x_i, x_j) \mathcal{K}(x_k, x_j) - \frac{2\alpha_i^d \alpha_k^d}{N_d} \sum_{l=1}^{N_d} \mathcal{K}(x_i, x_j) \mathcal{K}(x_k, x_l) + \frac{\alpha_i^d \alpha_k^d}{N_d^2} \sum_{l=1}^{N_d} \sum_{m=1}^{N_d} \mathcal{K}(x_i, x_l) \mathcal{K}(x_k, x_m) \right) \\ &= \sum_{i=1}^N \sum_{k=1}^N \sum_{d=1}^K \sum_{j=1}^{N_d} \frac{N_d}{N} \left( \alpha_i^d \alpha_k^d \mathcal{K}(x_i, x_j) \mathcal{K}(x_k, x_j) - \frac{\alpha_i^d \alpha_k^d}{N_d} \sum_{l=1}^{N_d} \mathcal{K}(x_i, x_j) \mathcal{K}(x_k, x_l) \right) \\ &= \sum_{d=1}^K \frac{N_d}{N} \alpha^{dT} \mathbf{Q}^{dT} \mathbf{Q}^d \alpha^d - \alpha^{dT} \mathbf{Q}^{dT} 1_{N_d} \mathbf{Q}^d \alpha^d \\ &= \sum_{d=1}^K \frac{N_d}{N} \alpha^{dT} \mathbf{Q}^{dT} (I - 1_{N_d}) \mathbf{Q}^d \alpha^d \\ &= \alpha^T \sum_{d=1}^K \frac{N_d}{N} \Delta_d \alpha \end{aligned}$$

$$\begin{aligned} \mathbf{w}^T S_{\Phi}^B \mathbf{w} &= \left( \sum_{i=1}^N \alpha_i \Phi^T(x_i) \right) \left( \sum_{d=1}^K \sum_{b=1, b \neq d}^K (m_{\Phi}^d - m_{\Phi}^b) (m_{\Phi}^d - m_{\Phi}^b)^T \right) \left( \sum_{k=1}^N \alpha_k \Phi(x_k) \right) \quad (12) \\ &= \alpha^T \sum_{d=1}^K \sum_{b=1, b \neq d}^K (M_d - M_b) (M_d - M_b)^T \alpha. \end{aligned}$$

$\Delta_d$  is the transformed version of  $\Sigma_{\Phi}^d$  to be used in the dual form:

$$\Delta_d = \mathbf{Q}_d (I - 1_N) \mathbf{Q}_d^T. \quad (10)$$

This form of kernel covariance matrix  $\Delta_d$  is only in terms of the kernel function and can be calculated easily using the kernel trick. Let  $M_d$  is the ‘‘kernel mean of cluster  $C_d$ ’’, which is an  $N_d$  dimensional vector. Each component of  $M_d$  is defined as:

$$(M_d)_j = \frac{1}{N} \sum_{i=1}^{N_d} \mathcal{K}(x_i, x_j), \quad \forall j = 1, \dots, N. \quad (11)$$

Using this definition, the between scatter matrix  $S_{\Phi}^B$  is defined in Eq. (12).

Hence, our target term to incorporate into the COSVM dual problem is:

$$\alpha^T \left( \sum_{d=1}^K \frac{N_d}{N} \Delta_d + \sum_{d=1}^K \sum_{b=1, b \neq d}^K (M_d - M_b) (M_d - M_b)^T \right) \alpha. \quad (13)$$

With this replacement, our proposed Dispersion COSVM method can be described by the optimization problem defined in Eq. (14).

$$\min_{\alpha} \alpha^T \eta \mathbf{Q} \alpha + \alpha^T (1 - \eta) \left( \sum_{d=1}^K \frac{N_d}{N} \Delta_d + \sum_{d=1}^N \sum_{b=1, b \neq d}^K (M_d - M_b)(M_d - M_b)^T \right) \alpha \quad (14)$$

$$s.t. \quad 0 \leq \alpha_i \leq \frac{1}{vN}, \quad \sum_{i=1}^N \alpha_i = 1.$$

The proposed method still results in a convex optimization problem since both the kernel matrix  $\mathbf{Q}$  and the covariance matrix  $\Delta$  are positive definite (Michelli, 1986; Horn and Charles, 1990). As a result, the solution to this optimization problem will have one global optimum solution and can be solved efficiently using numerical methods.

However, we control the balance between the dispersion matrix and the Kernel matrix through our control parameter  $\eta$ .

### 2.3 The Impact of the Tradeoff Parameter $\eta$

One important step for achieving better classification with DCOSVM is finding the appropriate value for  $\eta$ . The contribution of our kernel matrix  $\mathbf{Q}$ , the between scatter matrix  $S_{\Phi}^B$  and the within subclass scatter matrix  $S_{\Phi}^w$  is controlled using the parameter  $\eta$ . **Figure 1** shows the case where the optimal decision hyperplane for the example target data is on the same direction as the high variance. The control parameter  $\eta$  is set equal to 1 which means the low variance directions will not be given any special consideration in this case. On the other hand, **Figure 2** is the case when the direction optimal of decision hyperplane and the low variance are parallel. In this case,  $\eta$  can be set to 0. However, in real world cases ( $0 < \eta < 1$ ), the optimal decision hyperplane is highly unlikely to be entirely parallel to the direction of low variance or high variance. For this reason, the value of  $\eta$  needs to be tuned so that we have less overlap between the linear projections of the target data and the outlier data. We use an indirect approach to optimize  $\eta$  which will be explained in detail in the next section.

### 2.4 Schematic Depictions

In this section, **Figure 3** presents schematic depictions to show the advantage of our DCOSVM method over the unimodal COSVM.

## 3 EXPERIMENTAL RESULTS

This section presents the detailed experimental analysis and results for our proposed method, performed on both artificial and benchmark real-world one-class

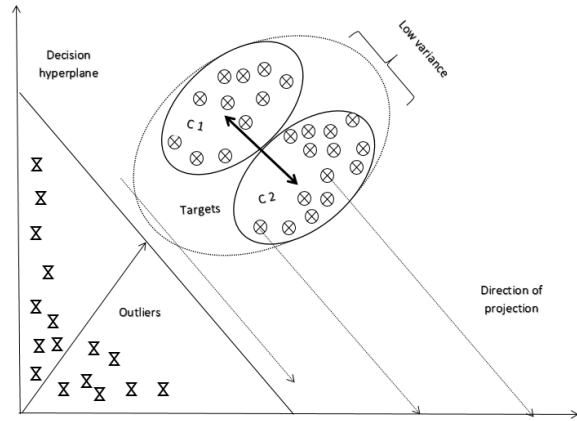


Figure 1: Case 1: Schematic depiction of the decision hyperplane for DCOSVM when the optimal linear projection would be along the direction of high variance. In this case, the optimal control parameter value for DCOSVM is  $\eta = 1$ .

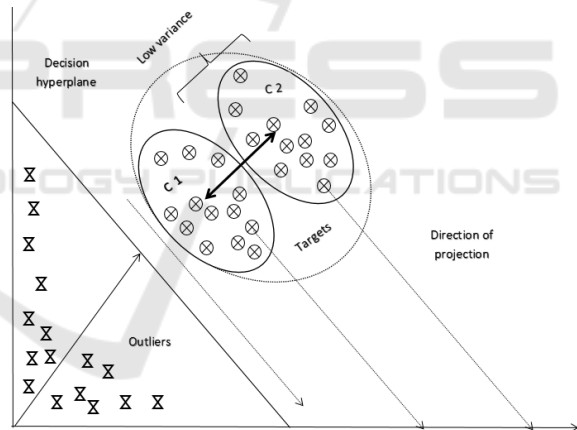


Figure 2: Case 2: Schematic depiction of the decision hyperplane for DCOSVM when the optimal linear projection would be along the direction of low variance. In this case, the optimal control parameter value for DCOSVM is  $\eta = 0$ .

datasets, compared against contemporary one-class classifiers.

### 3.1 Data Sets Used

In our experiments, to test the robustness of our proposed method in different scenarios, we have used both artificially generated datasets and real world datasets. For the experiments on artificially generated data, we have created several sets of 2D four-class data drawn from two different sets of distribu-

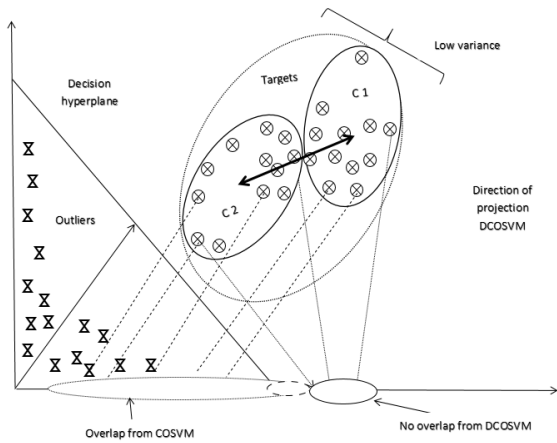


Figure 3: Comparison between COSVM and DCOSVM: The value of the tradeoff parameter is set equal to 0 ( $\eta = 0$ ), to only consider the dispersion term.

tions: 1) Gaussian distributions with different covariance matrices. 2) Gamma distributions with different shape and scale parameters. For each distribution, two different sets were created, one with low overlap and the other with high overlap. **Figure 4** shows the plots of these generated data sets. We had a total 4 classes, one of the class is designated target and the other ones as outlier. For the real world case, most of these datasets were originally collected from the UCI machine learning repository (Blake and Merz, 1999). We have primarily focused on one of the important fields of one-class classification, such as, medical diagnosis (Fazli and Nadirkhanlou, 2013). Since these datasets are originally multi-class, one of the class is designated target and the other ones as outlier. Some of the target and outlier sets were too trivial to classify. We have omitted those sets from our results. We have also used varying size and dimensions of data sets to test the robustness of our method against different feature sizes. As we can see from **Table 1**, the dimensions vary from 3 to 300, while the training set sizes vary from 21 to 288.

### 3.2 Experimental Protocol

The performance of the DCOSVM has been compared to Covariance guided One-class SVM (COSVM), One-class SVM (OSVM), Support Vector Data Description (SVDD),  $K$  Nearest Neighbors (K-NN), Parzen, Gaussian. The classifiers are implemented with the help of DDtools (Tax, 2012), and the radial basis kernel was used for kernelization. This kernel is calculated as  $\mathcal{K}(x_i, x_j) = e^{-\|x_i - x_j\|^2 / \sigma}$ , where  $\sigma$  represents the positive “width” parameter. For all data sets used, we set the number of clusters  $C_{min} = 2$  and  $C_{max} = 10$  with the assumption that each data sets

Table 1: Description of Real Data Sets.

Data set Name	Number of Targets	Number of Outliers	Number of Features	Number of clusters
Haberman’s Survival	81	225	3	2
Biomedical (diseased)	67	127	5	4
Biomedical (healthy)	127	67	5	4
SPECT Images (normal)	95	254	44	2
Balance-scale left	288	337	4	5
Balance-scale right	288	337	4	4
waveform	21	600	300	3

target has a minimum of 2 clusters (sub-class) to a maximum of 10 clusters. The number of subclasses is determined by applying a clustering technique and the validity index proposed in (Bouguessa et al., 2006), on the samples belonging to each of the classes independently. Second, we used 10-fold stratified cross validation. In fact, we added 10% randomly selected data to the outliers for testing, and the remaining was used as the training data. To build different training and testing sets, this approach was repeated 10 times. The final result was achieved by averaging over these 10 models. This ensures that the achieved results were not a coincidence. Besides, to evaluate the methods, we have used the Area Under the ROC Curve (AUC) (Fawcett, 2006) produced by the ROC curves, and we presented them in the results Table2. Consequently, the AUC criterion must be maximized in order to obtain a good separation between targets and outliers.

### 3.3 Results and Discussion

**Table 2** and **Table 4** contain the average AUC values obtained for the classifiers on the artificial and real data sets. As we can see, the DCOSVM is superior to all the other classifiers and provides best results on almost data sets, in terms of the obtained unbiased AUC values by averaging over 10 different models. In fact, the DCOSVM has the advantage of minimizing dispersion within and between subclasses of the target class, thereby reducing overlapping and improving classification accuracy. In general, we see that for real-world datasets (Table 4), the performance of k-NN, Gaussian and Parzen classifiers are poor when compared to the SVM-based classifiers (SVDD, OSVM, COSVM). This is because of the limitations inherent in these classifiers. Since k-NN classifies a data point solely based on its neighbors, it is sensitive to outliers (Jiang and Zhou, 2004). The Gaussian classifier has some obvious limitations from the as-

assumption that the underlying distribution is Gaussian, which is not always the case for real datasets. The Parzen classifier is prone to degraded performance in case of high-dimensional data or small sample size (Muto and Hamamoto, 2001). We can easily see this limitation of the Parzen classifier from the poor results on the Gene Expression datasets which has a very high dimension. The SVM-based classifiers are free from all these assumptions and, hence, leads to better result in majority of the cases. However, in case of the artificial datasets (Table 2), we see that these three classifiers (i.e. k-NN, Gaussian and Parzen) are competitive with the SVM-based methods, sometimes even better. This is because the artificial datasets are generated from a pre-defined regular distribution. We see that the Gaussian method performs well for the cases where the dataset was generated from a Gaussian distribution, which is expected. It also performs comparatively well for the datasets generated from the banana distribution. This is because the banana distribution is actually generated by superimposing an underlying Gaussian distribution on a banana shape. However, in case of Gamma distribution, it performs poorly since the distribution does not match with the assumption of the classifier.

Table 2: Average AUC of each method for the 4 artificial data sets (best method in **bold**).

Dataset	COSVM	DCOSVM
Gauss. (low overlap)	95.66	<b>96.88</b>
Gauss. (high overlap)	88.29	<b>91.10</b>
Gamma (low overlap)	91.27	<b>93.03</b>
Gamma (high overlap)	98.17	<b>98.69</b>

In terms of training computational complexity, the DCOSVM has almost the same complexity as COSVM. In fact, the computation of dual kernel covariance matrix can be done as part of the pre-processing and re-used throughout the training phase. The DCOSVM algorithm uses sequential minimal optimization to solve the quadratic programming problem, and therefore scales with is  $O(N^3)$ , where  $N$  is the number of training data points (Schölkopf et al., 2001). **Table 3** shows the average training times for both the artificial and the real-world datasets. As we

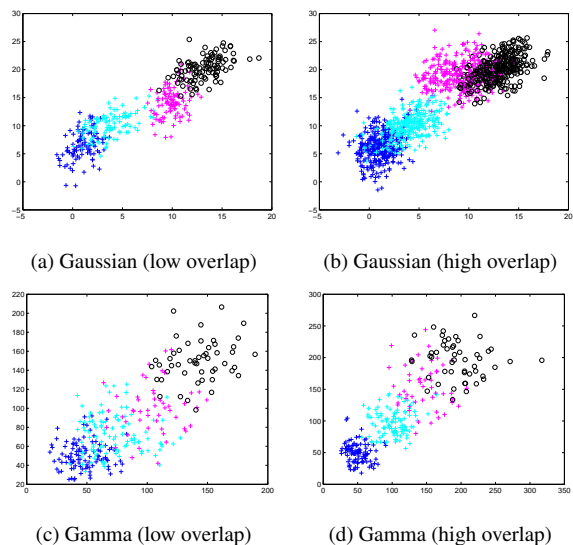


Figure 4: Four artificial four-class datasets used for comparison. The blue class represents the target class (in each subfigure caption).

Table 3: Average training times in *milliseconds* for COSVM and DCOSVM for the experiments on the artificial and real-world datasets.

Experiment	COSVM	DCOSVM
Artificial Datasets	7.4	8.5
Real-world Datasets	127.7	131.2

expect, DCOSVM has almost the same training time as COSVM.

## 4 CONCLUSIONS

In this paper, we have improved COSVM classification approach by taking advantage of target class subclass information. The novel variant of the COSVM classifier, namely, Dispersion COSVM (DCOSVM), minimizes the dispersion within and between subclasses and improves classification performance. We have compared our method against contemporary one-class classifiers on several artificial and real-world benchmark datasets. The results show the su-

Table 4: Average AUC of each method for the 7 real-world data sets (best method in **bold**, second best *emphasized*).

Dataset	k-NN	Parzen	Gaussian	SVDD	OSVM	COSVM	DCOSVM
Biomedical (healthy)	36.83	40.02	64.66	81.38	82.65	<i>85.80</i>	<b>89.81</b>
Biomedical (diseased)	89.42	90.09	89.60	90.28	91.04	<i>91.04</i>	<b>92.16</b>
SPECT (normal)	84.28	96.45	93.90	92.32	95.29	<i>96.79</i>	<b>97.81</b>
Balance-scale left	87.78	91.23	92.09	94.20	96.13	97.19	<b>98.51</b>
Balance-scale right	87.76	91.23	91.95	94.72	97.57	97.69	<b>98.82</b>
Haberman's Survival	66.49	67.97	60.09	68.87	69.62	<i>69.59</i>	<b>70.28</b>
waveform	87.78	97.02	92.10	97.20	97.19	<i>97.23</i>	<b>98.36</b>

priority of DCOSVM which provides significantly improved performance over the other classifiers. In future work, we will validate the proposed DCOSVM on security applications, such as, face recognition, anomaly detection, etc.

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