Efficient One-to-One Pair Matching for 2-D and 3-D Edge Detection Evaluation

Samuel Smith[a] and Ian Williams

Digital Media Technology Lab, Birmingham City University, Birmingham, U.K.

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Abstract: This paper introduces a novel efficient method of obtaining one to one correspondence matching for fast, accurate, performance evaluation of edge detectors. The proposed Efficient Pairing Strategy (EPS) overcomes the computational cost limitations of the Hungarian algorithm, enabling a fast and accurate evaluation of 3-D data and large 2-D data sets. In this work, the accuracy of the EPS method is measured against the optimal Hungarian method across a data set of 124240 images, and is shown to produce accurate results with a Pearson Pairwise Correlation coefficient of 0.99 . Additionally the efficiency of the EPS method is compared against the fast Closest Distance Match (CDM), the Cost Scaling Assignment (CSA), and the commonly applied Pratt figure of Merit (PFOM) methods. Analysis shows the EPS and CSA methods both produce cost scaling accuracy comparable to the Hungarian algorithm. However the EPS method outperforms the CSA method in computational efficiency, achieving linear computation time comparable to the efficient sub-optimal methods. More generally, we make recommendations for using one to one correspondence matching over other methods in order to produce reliable performance scores across 2-D and 3-D image data.

1 INTRODUCTION

Edge and surface segmentation is a prolific field of computer vision where the purpose is to filter 2-D and 3-D images in order to extract edges (2-D) or surfaces (3-D) to create a representative boundary map to support a higher level process (Williams et al., 2014). Any new advancements in detectors irrespective of the approach (morphological, topological, model based or machine learning) should be objectively evaluated to determine the valued improvement over prior methods and thus the potential contribution. Objective performance measurements allow for the systematic comparison of different algorithms in a repeatable, quantifiable manner often to find the optimal algorithm for a specific task (Lopez-Molina et al., 2013). While this is commonplace for 2-D edge detection evaluation, 3-D surface detection evaluation offers many barriers, notably the computational cost, which at present leaves 3-D performance evaluation largely unexplored.

Performance measures in edge detection can be broadly categorised into qualitative, quantitative or hybrid methods (Heath et al., 1996). Qualitative measures are analytical approaches which typically consist of multiple human assessors grading the output result images against specific criteria (i.e. object recognition, scene recognition etc.). However quantitative measures empirically assess the edge detection results in an automated or objective manner (Lopez-Molina et al., 2013) thus awarding a performance score to the results based on some similarity criteria. These objective performance methods aim to remove the subjective human element from the analysis, giving an unbiased evaluation, which can be applied rapidly to large data sets using both reference based analysis (i.e evaluating against a ground truth image) or non reference based (i.e. not requiring a ground truth image and evaluating the inherent properties of the edge map for example the edge connectivity)(Nercessian et al., 2009; Zhu, 1996; Kitchen and Rosenfeld, 1981). These methods provide a measure of edge or surface quality, which can be useful for determining how fit for purpose the outputs are for higher level operations for example region segmentation or object recognition.

Reference based performance analysis allows for the measurement of accuracy against the ground truth ideal image. The ground truth image is commonly a binary image containing all pixels labelled as either

[a] https://orcid.org/0000-0001-9276-0354

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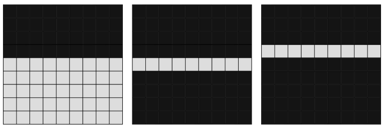


Figure 1: Displacement of edge. True edge fits between the 2 rows of pixels, however edge must conform to the pixel grid, therefore when maintaining single pixel edge thickness, this results in 2 correct edge locations, which needs to be accounted for during performance evaluation.

edge points or non edge points. The output binary edge map can be objectively compared against each of these points using a variety of methods (Savitzky and Golay, 1964; Abdou and Pratt, 1979; Bowyer et al., 2001; Prieto and Allen, 2003). The advantages of using a ground truth image for reference allows for the number of correctly detected edge points (True Positives) missed edge points (False Negatives), incorrect edge points (False Positives), to be measured. These values can used to grade the output through a variety of metrics and performance evaluation methods. Several objective performance algorithms such as the Pratt figure of Merit (PFOM) (Abdou and Pratt, 1979), Probabilistic Rand Index (PRI) of Savitzsky and Golay (Savitzky and Golay, 1964), Pixel Correspondence Metric (PCM) of Prieto and Allen (Prieto and Allen, 2003), Receiver Operating Characteristic (ROC) Curves of Bowyer et al citeBowyer2001, the Precision (P), Recall (R) and F-measures (F) of Martin et al (Martin et al., 2004), and also the Variation of Information (VI) measure by Meilă (Meilă, 2005). Each of these methods produce an objective metric which grades how accurately the result image correlates to the ground truth image.

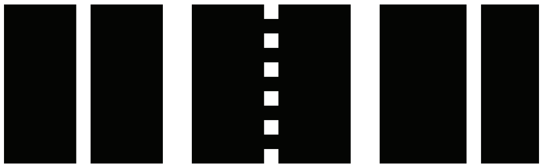
While suitable for reference based objective measures, the above techniques can result in errors between the overall performance score and the visual edge and surface detection results. Notably some objective measures do not account for a pixel (2-D) or voxel (3-D) shift between the detected interface and the ideal interface (Fig 1) or do not account for the fragmentation of the detected edges which can result in incorrect assumptions about the quality of the detection (Williams et al., 2008). Methods which aim to evaluate the fragmentation and displacement in detected edges often apply a one-to-many correspondence match whereby for each candidate pixel in the ground truth image, multiple candidates may be matched in the result image as is the case with the PFOM (Abdou and Pratt, 1979). This significantly affects the reliability of the performance method and leads to inaccuracies between the objective results and the visual results (Williams et al., 2008).

To solve this problem, one-to-one correspondence matching is required. In order to achieve optimal one-to-one correspondence, the Hungarian algorithm (Kuhn, 1955) for optimising solutions to the assignment problem can be employed. Although the Hungarian algorithm provides the optimal solution to reference based edge detection performance evaluation, the Hungarian algorithm introduces a high computational cost and is therefore impractical for large 2-D image data sets and 3-D edge detection evaluation such as Brats (et al., 2015). Acquiring more processing power to solve the assignment task is not always practical, thus an efficient method for performing accurate one-to-one performance evaluation is desirable, and such methods prior to this paper are lacking in the literature .

The rest of the paper is structured as follows, section 2 presents the overall problem associated with pair matching in image edge detection evaluation. Section 3 then presents an Efficient Paring Strategy (EPS) algorithm for pair matching, detailing the step by step functionality of this technique. Section 4 then presents a comparative analysis of this against three alternative approaches (CSA,CDM,PFOM). The accuracy of the performance measure with respect to optimal correspondence matching a Pearson pairwise comparison is given for each of the methods against the optimal Hungarian algorithm (Kuhn, 1955). We then present the cost efficiency of the EPS method compared against alternate methods, when applied to a large data set of both 2-D and 3-D images. Finally section 5 presents the overall conclusions and outcomes from this work and proposes the potential for the EPS to be applied in efficient edge and surface detection evaluation situations.

2 POINT CORRESPONDENCE IN PERFORMANCE MEASURES

Digital images are comprised of discrete data, thus the location of an edge (2-D) or surface (3-D) point is constrained by the pixel or voxel resolution of the image. Since these points are interfaces between regions the true position of a region cannot be accurately represented by a discrete pixel or voxel point in an image. Therefore an edge or surface detection algorithm must position the result in accordance with the discrete framework of the image and this introduces location error (see Fig. 1 and Fig. 2c). Therefore, when assessing the performance of these algorithms against a ground truth an allowance for displacement should be available to account for these localisation errors, since detected, connected boundaries even with a displacement are of value (Williams et al., 2008).



(a) Ground Truth (b) Fragmented edge (c) Displaced edge

Figure 2: Pratt figure of Merit. Does not adequately penalise fragmented edges. Fragmented edge 0.9545, Displaced edge 0.9000.

2.1 One-to-Many Correspondence

To account for a displacement in the detected edge points and a ground truth, one-to-many correspondence matching is commonly applied. This form of correspondence aims to determine the overall displacement of the detected points as a set compared to the desired points as a set. The displacement is then weighted depending on the displacement magnitude and influences the overall reference based performance metric.

The Pratt Figure of merit (PFOM)(Abdou and Pratt, 1979) is a common one-to-many performance metric for edge detection and is prolific in the literature. PFOM can also be used to evaluate surface detection via translation of this metric to 3-D data. Displacement within the PFOM is considered by measuring the Euclidean distance between edge detected points and the edge position in the ground truth, alongside the total number of detected edge points in the image and the total number in the ground truth. While PFOM offers a practical solution for edge and surface detection performance, in many cases it fails to produce an accurate result by allowing multiple-to-one and one-to-multiple correspondence between ground-truth and the algorithm result. Thus an imbalance can be demonstrated between displaced edges and fragmented edges where edge fragmentation can be awarded an higher performance than a connected edge with only a small displacement error (see Fig. 2a-c).

2.2 One-to-One Correspondence

To avoid the multiple-to-one and one-to-many problems illustrated with the PFOM, one-to-one correspondence should always be applied. To constrain the number of correctly detected points, each detected edge pixel needs to correspond to only a single edge pixel in the ground truth. Because of a need to tolerate a localisation error, a method of forcing one to one correspondence between ground truth and result image is required. It is therefore important to compute the correspondence in order to penalise multiple detections (Martin et al., 2004), since single detection is one of the three important criteria of edge detection laid out by Canny (Canny, 1986). Forbes and Draper (Forbes and Draper, 2000) paired farthest distance pixels within a tolerance zone of the ideal edge pixel, while Bowyer (Bowyer et al., 2001) opted for the closest match pair. This tolerance or allowance was labelled as the T_{match}. Whilst often used for objective performance, this general T_{match} allowance region does not offer the most consistent approach. Addressing this, Liu (Liu and Haralick, 2000) published a strategy for creating one to one correspondence by matching declared edge pixels to edge pixels in the ground truth image. Framing the task as the assignment problem (Kuhn, 1955), which is solved using the Hungarian algorithm.

For images, the Hungarian algorithm assignment case will possess many agents and tasks, it is therefore necessary to simplify the problem (Liu and Haralick, 2002). Using the T_{match} principle, Bowyer (Bowyer et al., 2001) was able to constrain the number of potential matches, and produced an optimal method for performance evaluation. This was further developed by Martin (Martin, 2003) using a Bipartite graph method to solve a constrained assignment problem for one to one correspondence, while Prieto (Prieto and Allen, 2003) used Weighted Matching in Bipartite Graphs to create the 2-D pixel correspondence metric.

While the optimal strategy for solving the assignment problem can be achieved using the Hungarian algorithm (Kuhn, 1955) and bipartite graph methods (Prieto and Allen, 2003), these are computationally expensive. Therefore using the Hungarian algorithm for the pairing strategy is feasible for 2-D images, or for small data sets only. However, when considering the 3-D images of surface detection (Smith and Williams, 2015), for example CT and MRI data, the increased complexity arising from more potential positions for a match makes this method computationally costly, resource intensive and often impractical. This complexity is compounded by the fact 3-D images typically contain a greater amount of voxels than a 2-D image contains pixels, making this strategy at current computational speeds impractical. Additionally for a reliable comprehensive analysis of surface detection methods, the performance measure needs to be applied to many thousands of result images, for example BRATS (et al., 2015). In order to undertake an evaluation in a reasonable time frame and maintain the most representative objective accuracy, an efficient method of solving the assignment problem for this case is required.

3 EFFICIENT PAIRING STRATEGY (EPS)

3.1 EPS Procedure

The EPS procedure aims to replace the Hungarian method for a fast solution to the assignment problem using a novel inflationary zone method within a defined T_{match} region. This method is an adaptation of the Closest Distance Match (CDM) method of Bowyer (Bowyer et al., 2001).However, the adaptation allows for a more consistent metric, which more closely replicates the correspondence matching of the optimal Hungarian method, additionally this is suitably fast for analysis of 3-D image volumes or fast computation of multiple 2-D images in large data sets.

The method utilises a concept of zones within a local neighbourhood window. Zones are defined to be regions within a local T_{match} neighbourhood window, which occupy the same Euclidean distance from the central pixel. Zones are ranked in levels from closest to farthest from the central pixel. 2-D and 3-D examples are presented in Fig 3.

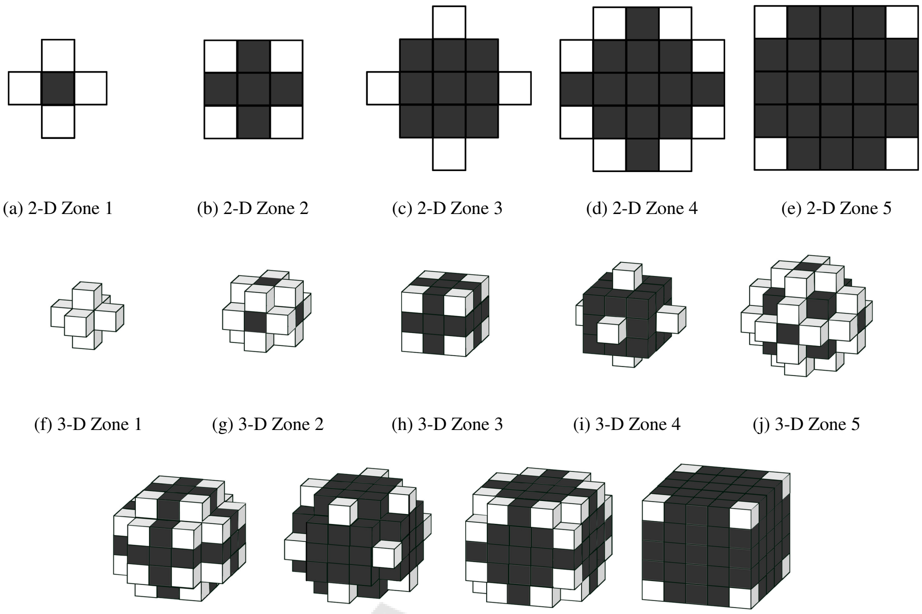
In order to compute one-to-one correspondence, and avoid creating multiple partners for each candidate, each match needs to be computed concurrently. For every false negative (FN) in the result edge and surface image, a set of 2-D or 3-D zones derived from a T_{match} scaled neighbourhood is established, centred on the FN location.

In each zone the number of potential candidates for a match are counted. Then in order to minimise the assignment cost, the closest match is preferential, thus pairings are made in the lowest level zone first. However, as some FNs share the same candidate match, FNs with the fewest candidates are assigned a match first. This technique produces a closest distance match correspondence, however, unlike the CDM method, the matches are optimised such that the maximum number of correspondences are produced. Once all candidates from the first zone are exhausted, if FNs remain, the procedure repeats with the next zone and so on until each zone in the T_{match} neighbourhood has been checked in its entirety. Any FNs without a pairing are labelled by the process as a missed response, while any FPs in the result without a pairing remain as a spurious response. From here a number of performance methods can be applied. Either Precision recall based such as in the work of Bowyer (Bowyer et al., 2001), but also a distance metric score can be applied such as in the work of Prieto (Prieto and Allen, 2003). A detailed step by step walk-through of the this procedure is given in the following section.

3.2 2-D Example Case with EPS

Presented here for clarity is a 2-D example case for matching of FN and FP responses for one to one correspondence within a designated T_{match} neighbourhood. For 3-D examples cubic T_{match} regions and inflationary zones can be used (Fig. 5f-n). Fig 4(a and b) shows an example of an edge detection result and a ground truth solution.

1. First label all the true positives (TP) then remove them from the response image (Fig 4(c).)
2. Next determine the location of all False Negatives. (Fig 4(d))
3. In the FN locations, establish a set of zones designated by the T_{match} neighbourhood (here 5×5).
4. In order to find which FN locations have the fewest number of candidate matches, in each zone, sum all potential correspondences between the FN locations and FP responses. These are signified by blue points in Fig 5.
5. Establish closest distance matches by pairing FN locations with FPs in the first available zone. Pairings are to be made starting with FN locations with the fewest available candidates in order to maximise the number of parings. Once a paring has been made, remove the T_{match} neighbourhood from that location and remove the FP response. In this example, Zone 1 has no potential matches.
6. When all matches from the previous zone have been established, repeat the process through each zone until all FN locations have been assigned a match or when the T_{match} neighbourhood has been exhausted. In the example case, Zone 2 FN1 and FN2 locations each have candidate matches. FN1 has 2(Fig 5f), FN2 has 1 (Fig 5g), while FN3 has zero (Fig 5h). Since FN2 has the fewest potential candidates (1) it is assigned first, then the remaining candidate FN1 is assigned. Zone 3 is processed next, where the final remaining FN (Fig 5i) is assigned a match and the pairing process is completed
7. Unmatched FNs remain as missing responses, while unmatched FPs remain as spurious responses.
8. Each pairing is assigned a cost which relates to the zone from which the paring was made, the cost is therefore the Euclidean distance, thus producing a distance function of the pairings. As 2 points have been matched from zone 2 and one from zone 3, this provides a distance function of [1.41,1.41,2] which can be used for a distance based metric.

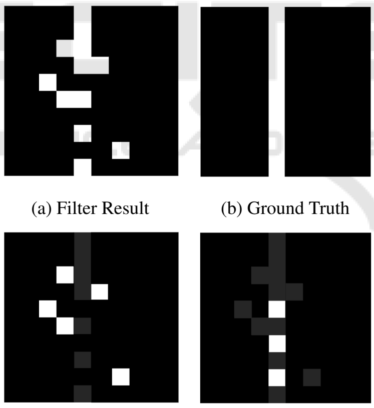


(a) 2-D Zone 1 (b) 2-D Zone 2 (c) 2-D Zone 3 (d) 2-D Zone 4 (e) 2-D Zone 5

(f) 3-D Zone 1 (g) 3-D Zone 2 (h) 3-D Zone 3 (i) 3-D Zone 4 (j) 3-D Zone 5

(k) 3-D Zone 6 (l) 3-D Zone 7 (m) 3-D Zone 8 (n) 3-D Zone 9

Figure 3: A set of 2-D and 3-D Zones for a T_{match} value of 5. Each zone level possesses a cost function equal to the Euclidean distance to the centre of the missing edge point (FN). Locations for each level are defined by a white box, while previously evaluated levels are shaded grey. A local neighbourhood with a T_{match} of 5 results in 5 Zones in 2-D and 9 zones in 3-D.

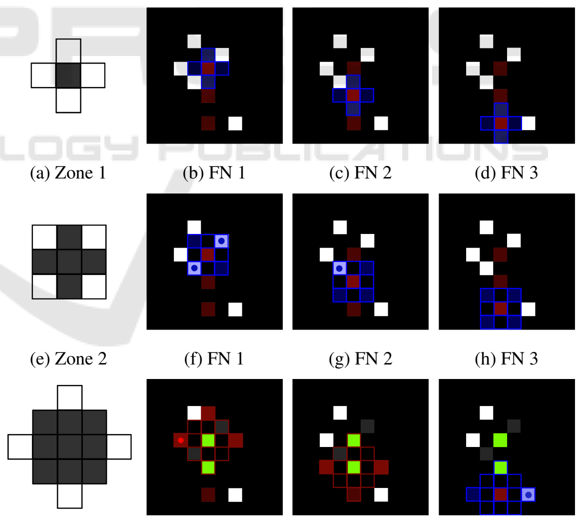


(a) Filter Result (b) Ground Truth

(c) Count and remove TPs (d) Locate FNs

Figure 4: (a) Ground truth solution. (b) 2-D filter result. (c) Count and remove TPs. (d)Locate FNs.

To produce a performance score, a number of approaches can be taken. One method is to use the one-to-one correspondence matching for ROC or PR analysis as outlined by Bowyer (Bowyer et al., 2001). Additionally Liu and Haralick (Liu and Haralick, 2000) made recommendations to use one-to-one correspondence to produce a distance based metric, this can be achieved by using the one-to-one distance cost function of the matches with the PFOM calculation 1)



(a) Zone 1 (b) FN 1 (c) FN 2 (d) FN 3

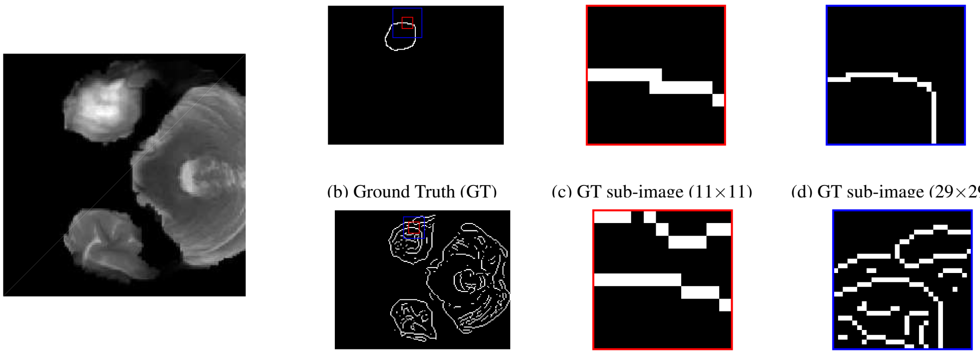
(e) Zone 2 (f) FN 1 (g) FN 2 (h) FN 3

(i) Zone 3 (j) FN 1 (k) FN 2 (l) FN 3

Figure 5: An example case of one to one correspondence matching using a set of 2-D zones. Here each zone is signified by a blue grid, unmatched FNs are red, unmatched FPs are white. While matched FNs are green and matched FPs are shaded grey. (T_{match}: 5×5).

$$FOM = \frac{1}{\max(N_I, N_B)} \sum_{i=1}^{N_B} \frac{1}{1 + \alpha \times d_i^2} \qquad (1)$$

Where N_I are the number of points in the ground truth and N_B is the sum of TP and FP points in the



(a) MRI layer (b) Ground Truth (GT) (c) GT sub-image (11×11) (d) GT sub-image (29×29) (e) Test Image (f) Test sub-image (11×11) (g) Test sub-image (29×29)

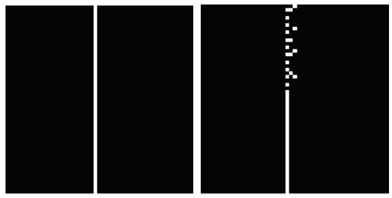
Figure 6: Example cases for analysis at different sub-image scales.

result image. d_i is the distance cost function of one to one correspondence matches. While α is a calibration constant set at $\alpha = 1/9$, a value established by Pratt (Abdou and Pratt, 1979).

4 COMPARISON OF PERFORMANCE METRICS

Different performance measures can possess different performance characterisation. In terms of one to one correspondence between reference and result image, the Hungarian algorithm is considered optimal since it solves the assignment problem with the smallest aggregate cost function. Thus the accuracy of alternate sub-optimal methods for one to one correspondence require direct comparison against the Hungarian algorithm. The EPS method of performance measure presented in this work is measured for accuracy against a metric score obtained using Hungarian algorithm to solve the assignment problem in a series of example cases for obtaining one to one correspondence. In addition, the CSA assignment method of Goldberg (Goldberg and Kennedy, 1995), and the Closest Distance Match method are also assessed for comparison. The commonly applied non-correspondence Pratt figure of merit metric is also included.

In order to assess the accuracy of the measures, the performance evaluation algorithms were applied to a series of 2-D sub-images and compared against a ground truth solution. The sub-images are regions selected from the binary outputs of edge filters (Williams et al., 2014; Smith and Williams, 2015) from a set of 5 MRI image volumes. The region locations are derived from the positions of edge points in the ground truth solutions provided in the BRATS data sets (et al., 2015). The sub-image sizes range from 11×11 and increase in odd increments through



(a) 51×51 Ground Truth (b) 51×51 Test Image

Figure 7: Performance scores, visual example with fragmented edge. Hungarian 0.8308. CSA 0.7811, EPS 0.8308, CDM 0.8950, PFOM 0.9014.

to 29×29, and are neighbourhoods surrounding a ground truth edge point located at the sub-image centre. An example is shown in Fig 6. For each sub-image size, 12424 sub-images were used for a total of 124240 images, one for each ground truth edge point across the data sets. The performance measure scores are measured for accuracy against the optimal Hungarian algorithm using Pearson's pairwise correlation, the results of which are presented in table 1.

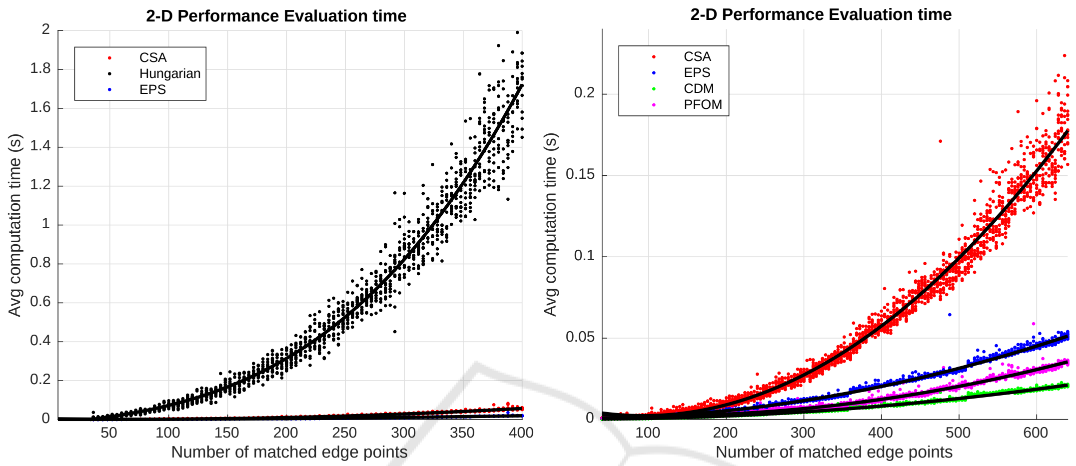
The results of the comparison show that the presented EPS is strongly correlated to the Hungarian algorithm solution over a range of different sub image sizes with a coefficient of 0.99. The CSA method is also strongly correlated to the Hungarian solution achieving coefficient scores ranging from 0.99-1.00. This indicates that for optimal paring the CSA or EPS method are highly preferred over the other methods, offering greater accuracy, and therefore greater reliability when compared against the CDM and PFOM methods,

4.1 Time Analysis

An analysis of computational efficiency through timing the algorithms was undertaken both in 2-D and 3-D. The procedure for measuring the efficiency of the methods required running the performance measures

Table 1: Pearson Pairwise Correlation between optimal Hungarian method and other sub-optimal methods. 11503 example images at odd sub-image sizes from 11×11 to 29×29. Pvals for all results were 0.

Metric	11×11	13×13	15×15	17×17	19×19	21×21	23×23	25×25	27×27	29×29
EPS	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
CSA	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CDM	0.86	0.91	0.94	0.95	0.96	0.97	0.97	0.97	0.97	0.97
PFOM	0.80	0.87	0.91	0.93	0.95	0.96	0.96	0.97	0.97	0.97



(a) 2-D Hungarian v Alternatives

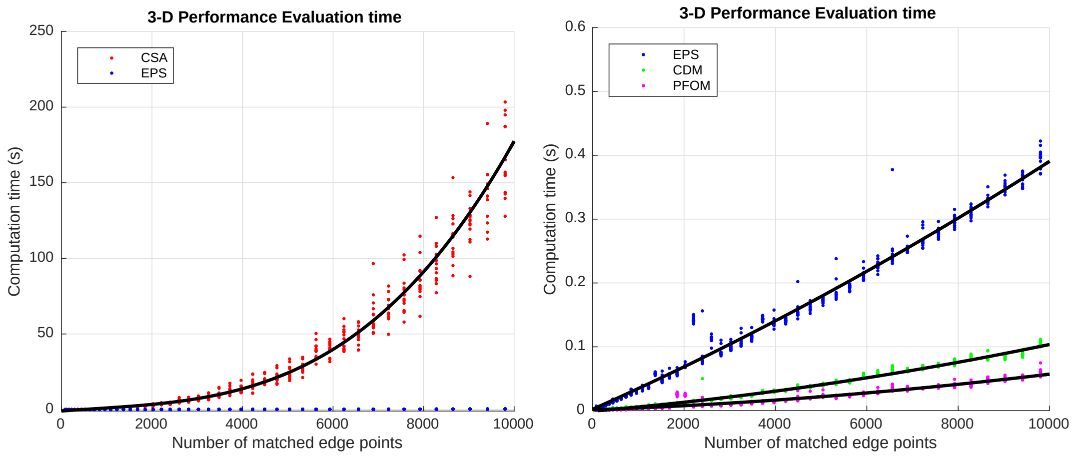
(b) 2-D Alternatives

Figure 8: (a) Computational time analysis of 2-D performance measures, including Hungarian (Kuhn, 1955), CSA (Goldberg and Kennedy, 1995) and the proposed EPS method. (b) Computational time analysis of alternative performance measures, including CSA, EPS, CDM and PFOM.

on images of increasing complexity. Complexity was increased by increasing the number of potential correspondences in each case. For this test, the number of edge points in the ideal image was made to precisely match the number of edge points in the result image to ensure conditions which allow for one to one correspondence of all edge points. The edge pixels in the test images were pseudo randomly generated such that declared edge pixels were located within the tolerance zone (T_{match}) of an ideal edge (Fig. 10. Here T_{match} was set to accommodate a 5×5 window around the ideal points. The same experiment was repeated using 3-D one to one correspondence measures and 3-D image volumes. Here T_{match} was set to accommodate a 5×5×5 neighbourhood. Complexity was again increased by increasing the number of potential correspondence matches.

The performance measures were compared first in 2-D against the Hungarian(Kuhn, 1955) Munkres algorithm. The results are shown in Fig 8a), here it can be seen that for 2-D performance measures, as the complexity of the analysis increases through increasing the number of correspondences, the Munkres Hungarian completion time increases exponentially and is slower than than the CSA and EPS methods. 2-D analysis indicates that the fastest method is the PFOM and the CDM method was the most efficient of the one-to-one correspondence methods (Fig 8b) however, the more accurate EPS method offered similar linear computational efficiency in 2-D. The accurate CSA assignment method finishes in a time we believe to be adequate for 2-D performance evaluation. However, in the case of the CSA assignment method the additional computational complexity introduced by 3-D data and surface information, leads to a significant increase in computational time for 3-D performance evaluation, when compared against the EPS method (Fig 9a). For surface evaluation in 3-D, using the CSA algorithm for correspondence matching becomes impractical as the number potential correspondences increases.

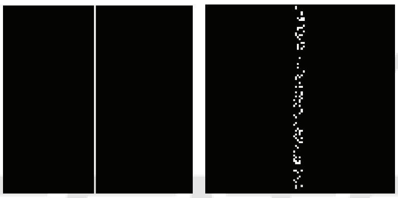
While the CSA method achieves exponential time complexity in relation to the number of potential correspondences (Fig 9a), Fig 9b shows that that in the context of 3-D, the time complexity of the problem remains linear for the EPS, CDM and PFOM methods. The EPS method offers similar accuracy to the CSA and Hungarian method, while maintaining the computational efficiency similar to that of the sub-optimal CDM and PFOM methods.



(a) 3-D CSA v EPS

(b) 3-D EPS v linear time methods

Figure 9: (a) Computational time analysis of optimal 3-D performance measures, CSA (Goldberg and Kennedy, 1995) and the proposed EPS method. Allowance 0. (b) Computational time analysis of fast sub-optimal 3-D performance measures, including EPS, CDM and PFOM.



(a) Ground Truth Example

(b) Test Image Example

Figure 10: Example of test images and ground truth images for time analysis with 100 potential correspondences. Edge points are created within T_{match} neighbourhood such that the number of edge points in both ground truth and test image are equal in order to allow for correspondence matching of all points.

5 CONCLUSIONS

This paper presented a novel method for efficient one-to-one correspondence matching for 2-D and 3-D edge performance evaluation. The Efficient Pairing Strategy offers increased accuracy over existing performance methods, notably the commonly applied one to many correspondence PFOM technique, and reliably presents an objective measure that more closely reflects the visual image results by adequately penalising fragmented edges and surfaces (Fig. 7. The EPS results are shown to be consistently accurate, with a 0.99 Pearson correlation against test assignment cases solved by the Hungarian algorithm, improving over the existing CDM and PFOM methods for one to one correspondence matching (Table 1). Furthermore the EPS method was shown to provide results comparable to the Hungarian and CSA methods in terms of accuracy of correspondence matching with less computational cost. Finally, it should be noted that the EPS in this form does not offer a general solution for the assignment problem. However, the EPS does provide a fast and accurate alternative to the Hungarian and CSA algorithms in the context of performance evaluation for edge and surface detection, finally allowing for practical fast one to one correspondence matching which is suitable for large 2-D image data sets and 3-D data.

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