Learning Recursion: Insights from the ChiQat Intelligent Tutoring System

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Abstract: Recursion is a difficult concept to teach, and novice programmers struggle Learning it. The reasons include unfamiliarity with activities associated with analyzing recursion, such as visualizing program execution and difficulty understanding its back flow of control. In this paper we discuss approaches to teaching recursion that includes conceptual and program visualization methods. We also describe the recursion module of our ChiQat-Tutor system which relies on ideas from both approaches. We designed several activities that allow students to work on recursive problems: answering questions, animations, code tracing, validation, and construction tasks. We conducted four evaluation experiments at two different institutions, with a total of 89 students taking introductory Computer Science courses. We hypothesized that ChiQat-Tutor can help novice Computer Science students learn recursion, develop accurate mental models of recursion, and serve as an effective visualization tool with which hidden features of recursion can become evident. Our results showed some evidence that the animation, answering questions, code tracing, and validation tasks exhibit a trend towards significant learning gains.

1 INTRODUCTION

Recursion is an important concept in computer science, whether seen as a problem-solving approach, a way of expressing an algorithm, a mathematical concept, or a programming technique (McCauley et al., 2015; Hamouda et al., 2019). It is a powerful and essential computational problem solving technique that involves breaking down of a problem into smaller sub-problems of the same kind. Thus said, such decomposition is not easily comprehended by novice students learning recursion (Dann et al., 2001). Some computer science educators suggest that recursion is an inherently difficult concept to master, and it is one of the most difficult concepts to teach (Pirolli and Anderson, 1985). This is because it is a method to define subprogram, in which the subprograms being defined is applied within its own definition.

A key explanation why recursion is seen as a difficult concept because it is traditionally taught after students have developed preconceptions based on their learning of repetition (Turbak et al., 1999). Additionally, students sometimes have difficulty recognizing different invocations of the same function, and they get confused by the bookkeeping required for tracing each recursive call (Tessler et al., 2013). In particular, they struggle with unfamiliarity with activities associated with analyzing recursion, the visualization of the program execution, the back-flow of control after reaching the base case, comparison to loop structures, and the lack of everyday analogies for recursion.

It is acknowledged that in order to understand the process of recursion and be able to write recursive code, one must be able to visualize the nature of a problem and how solutions to smaller sub-problems are combined to solve the original problem (Bower, 1998). In a comprehensive review conducted by (McCauley et al., 2015), a number of successful approaches to teaching recursion were cited. It included; providing students with variety of analogies and examples, introducing repetition before recursion, tracing methods, teaching students problem solving tech-

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niques to break problems into small sub-problems, identifying base and recursive cases, and providing modifiable code templates.

The focus of this paper is about the strategies for teaching recursion which we implemented and evaluated in ChiQat-Tutor. We carried out four experiments in two locations: Carnegie Mellon University in Qatar (CMUQ), and the University of Illinois at Chicago (UIC), over a period of one year. We analyzed students' interactions with the recursion module of ChiQat-Tutor, looking for further insights on suitable approaches for recursion learning. In the next section we review some of the approaches to teaching recursion, discussing the advantages and limitations of each approach. We also introduce our approach to teaching recursion in ChiQat tutor.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 provides an overview of the ChiQat system. Section 4 explains our experimental protocol and data collection. Section 5 discusses our results; and Section 6 provides concluding remarks.

2 RELATED WORK

A number of approaches to teaching recursion was discussed in (Tessler et al., 2013). These include conceptual models of recursion and control flow, and the use of visual aids. Conceptual models are defined by teachers and are used as tools for understanding or teaching of a system. Conceptual models used for teaching recursion include mathematical induction, abstract and conceptual discussions of recursion, process tracing, and structure templates of recursive code (Wu et al., 1998). On the other hand, the visual aids approach relies on the use of algorithms animation, program visualizations using video games of animated characters.

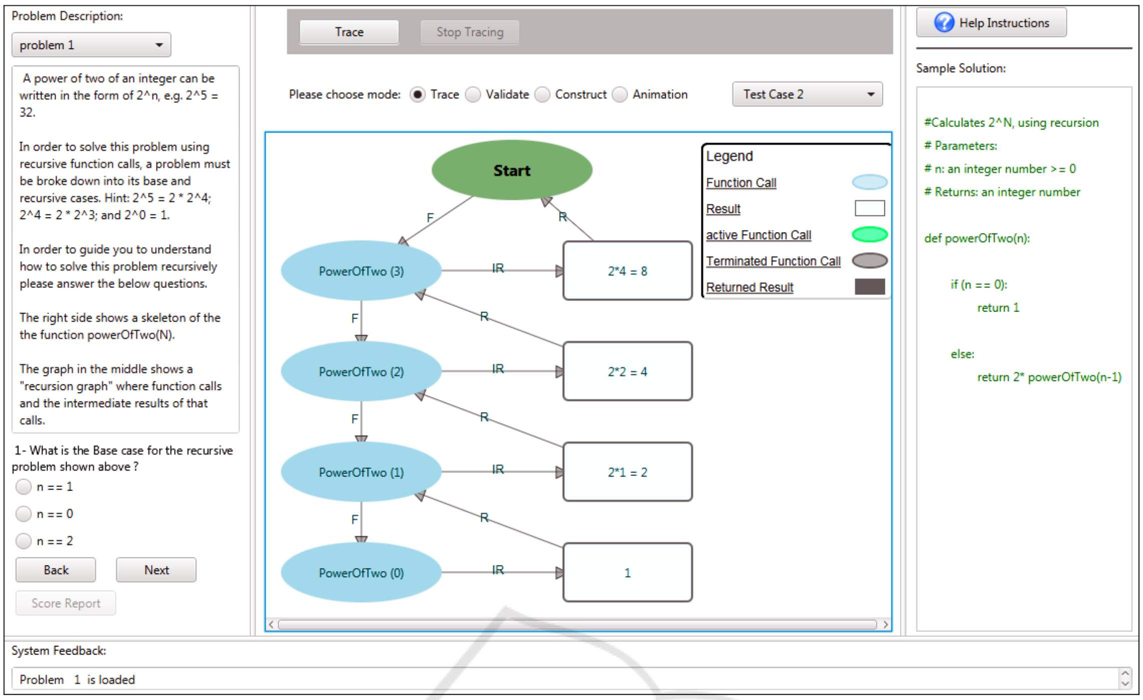
Some researchers advocated the use of program visualizations to introduce recursion. For example, (Dann et al., 2001) experimented with program visualization technique to introduce recursion to students. They utilized a software system named Alice which allowed students to gain intuitive sense and mathematical insight into the recursive process. However, (Edgington, 2007) cited a number of limitations with the Alice system.

Algorithm animation are also commonly used as an aid to teaching recursion. The instructor programs an animation for commonly used algorithms (e.g. quick sort, factorial). The student runs the prepared animation observing the behavior using different inputs. It is suggested that students should be able to manipulate the animations, not just watch them, in order to learn (Bower, 1998). It appears that there is no consensus on the benefits of algorithm animation as a learning aid. (Stasko et al., 1993) found no significant result suggesting that algorithm animators assist learning. They suggested that future research should focus on allowing students to construct their own animations. On the other hand, a meta-analysis of 24 experimental studies on the use of algorithm visualization (AV) as an aid for learning algorithms was conducted by (Hundhausen et al., 2002). Their results showed that the effectiveness of AV is determined by how students use the AV technology rather than what the AV technology shows them. Other research showed that animation seems to make a challenging algorithm more accessible, thus leading to enhanced learning (Kehoe et al., 2001).

Recursion Graphs (RGraph) is a visual tool used to aid the introduction of recursion to students (Sa and Hsin, 2010). RGraph is similar to recursion trees in that it shows the invocation sequence. However it adds the detailed calling sequences from one layer to another including the intermediate results for each recursive call. An RGraph is a directed graph with two sets of vertices (oval for a recursion call, and square for post/pre processing statements of recursion calls). The RGraph is built layer by layer from top to bottom (i.e. breadth-first) with directed edges indicating the processing sequence. One particular feature of an RGraph is that it is traceable as it shows the detailed invocation sequence from one layer to another. Results showed that the use of RGraphs helped improving student learning of recursion by providing flexibility in demonstration and more focused pedagogical interactions from students.

On the other hand, conceptual models for teaching recursion are tools that aim at helping students formulate accurate mental models of the concept of recursion (Götschi et al., 2003; Wu et al., 1998). It is believed that if a person has a mental model of a process, s/he will then be able to make predictions about the behavior of that process, although perhaps inaccurately sometimes (Kahney, 1983). Moreover, possession of a model will allow a person to debug the model when s/he is faced with counter-examples.

Early research on teaching recursion embraced the use of the mathematical induction model (Ford, 1984). In this model students are taught recursion through the theory of recurrence relations (Wilcocks and Sanders, 1994). However, a possible limitation with this approach is that novice computer science students might not possess the necessary level of mathematical skills required to develop a clear understanding of recursion (Ford, 1984; Wilcocks and



Problem Description: problem 1

A power of two of an integer can be written in the form of 2^n, e.g. 2^5 = 32.

In order to solve this problem using recursive function calls, a problem must be broke down into its base and recursive cases. Hint: 2^5 = 2 * 2^4; 2^4 = 2 * 2^3; and 2^0 = 1.

In order to guide you to understand how to solve this problem recursively please answer the below questions.

The right side shows a skeleton of the the function powerOfTwo(N).

The graph in the middle shows a "recursion graph" where function calls and the intermediate results of that calls.

1- What is the Base case for the recursive problem shown above ?

- n == 1
- n == 0
- n == 2

Please choose mode: [✓ Trace] Validate Construct Animation

Test Case 2

Legend	
Function Call	⬭
Result	▭
active Function Call	🟢
Terminated Function Call	⬬
Returned Result	■

Sample Solution:

```
#Calculates 2^N, using recursion
# Parameters:
# n: an integer number >= 0
# Returns: an integer number

def powerOfTwo(n):

    if (n == 0):
        return 1

    else:
        return 2* powerOfTwo(n-1)
```

System Feedback:

Problem 1 is loaded

Figure 1: Interface of ChiQat-Tutor Recursion Module. Left: A Problem, Its Explanation, and Questions. Center: Task List and Recursion Graph. Right: Help Button and Recursive Code. Bottom: System's Feedback.

Sanders, 1994).

Some researchers believe that more emphasis should be put on the declarative, abstract level of problem decomposition rather than on the computational model and program visualization (Ford, 1984; Ginat and Shifroni, 1999). A divide-and-conquer strategy is applied at the problem level, regardless of the machine implementation. (Ginat and Shifroni, 1999) found that the use of such model significantly enhanced recursive programs formulation ability of students in comparison to establishing comprehension via understanding of the process of recursion execution. On the other hand, (Wu et al., 1998) found that concrete conceptual model, which provide an appropriate level of details of the process of recursion, is better than abstract conceptual models for teaching recursion.

A number of mental models of recursion possessed by both novices and expert programmers in the context of SOLO programming was identified (Kahney, 1983). These include; 1) The Loop model: novices are hypothesized to possess this model. They view recursive procedures as a single object instead of a series of new instantiations. 2) The Odd model: students acquired the notion that the flow of control statement, rather than the results of pattern matching, acts as the stopping condition for recursion. 3) The Syntactic "Magic" model: a student is able to match on syntactic elements and their positions, and make predictions about the behavior of the recursive programs on this basis, but has no clear understanding of how recursion works. 4) The Copies model: students view recursive instantiations of a recursive procedure as copies, as opposed to a single object, as in the Loop model. It is interesting point that novices have the Loop mental model of recursion. Some research suggested that it is important to teach recursion before loops not vice versa (Turbak et al., 1999).

The work of (Kahney, 1983) was extended by (Wilcocks and Sanders, 1994). A number of conceptual models to teaching recursion was evaluated. These include the mathematical, tree, copies, analogies, and graphical recursive structure models. They found that the Copies Model is more viable than the other models. It is believed that this Copies Model is what expert programmers have of recursion (Kahney, 1983). However, (Kahney, 1983) cautioned that having acquired the Copies mental model is not sufficient to determining what students really know about recursion. Since students can make predictions about recursive procedures behavior without fully understanding recursion.

There has been some previous research on using structured templates and worked-out examples for teaching recursion in the context of LISP programming (Pirolli and Anderson, 1985; Hamouda et al., 2019). It was found that learning is facilitated by using abstract representations of the structure of recursion examples to guide initial coding attempts of students. The type of information exploited by this mechanism is an example or demonstration supplied by some source. However, (Rinderknecht, 2014) mentioned that although examples can be used to develop analogical problem solving skills, care should be taken not to rely on them too early. This is in order to prevent students from developing the magic or syntactic model, identified by (Kahney, 1983).

3 SYSTEM OVERVIEW

ChiQat-Tutor is a modular tutoring system whose goal is to facilitate learning of core CS data structures (e.g., linked lists, trees, stacks) and algorithmic strategies (e.g., recursion). The interface of ChiQat is shown in Figure 1. The use of RGraph is extended in our system by implementing several interactive tasks; described in Table 1. Students are able to interact with RGraph representation of different recursive problems (e.g., factorial, palindrome). The RGraph-based interactive tasks can help students identify the recursive structure of problems, understand recursive processes and identify critical features of recursive solutions to these problems. These tasks were designed with different levels of difficulty, which will allow users to progress from one task to another.

In ChiQat-Tutor we developed an approach to teaching recursion that is based on the visual model of RGraph; which is a clever visual representation of recursive execution. It also uses ideas from the conceptual model of teaching recursion in the form of code structured templates. We believe that our approach, mixing ideas from both the conceptual models and visual aids approaches will help students learn recursion more effectively. Capitalizing on the advantages of both approaches, and avoiding misconceptions presented by either of them. A similar approach is followed by RecTutor (Hamouda et al., 2019). It is found that student performance in learning recursion was enhanced by doing more tracing and writing exercises practice that addresses common recursion misconceptions.

Previous research has shown the importance of using graphical representations, in the form of diagrammatic traces and animations of recursive problems, which allowed novice programmers develop correct mental models of recursion (Wilcocks and Sanders, 1994). The tasks that can be carried on RGraphs in ChiQat combine characteristics from different conceptual models of teaching recursion. These include Stack Simulation, the Copies Model, and Tree Model (Wu et al., 1998). For example, the animation task represents a stack simulation approach to teaching recursion. Tracing, validating, and constructing RGrahs tasks can enforce the Copies and tree mental models of learning recursion. This allows students to look at recursion solutions from different angles. We believe that the use of RGraphs coupled with the different tasks that students can perform on them can help students develop the Copies Model that expert programmers are hypothesized to possess.
Our approach for teaching recursion also embodies a much widely used strategy of using structure templates of recursive code and worked-out examples as an instructional material (Pirolli and Anderson, 1985; Hamouda et al., 2019). The cognitive processes required to learn from a solved example are well understood. The learner must: (a) store in procedural memory the steps in the example; (b) interpolate the missing steps, since solved examples are necessarily incomplete to some extent; (c) infer the purpose of at least the main steps in the example; and (d) generalize over the specifics of the example. Working through examples is pervasive in instructional situations, and much appreciated by students. The ChiQat-Tutor will serve as an effective visualization tool with which hidden effects of nested function calls would become evident when applied to a whole range of problems that can be solved recursively.

4 DATA AND METHODS

The recursion module was evaluated using multiple experiments conducted at both UIC and CMUQ. Ethical approvals were obtained from IRB offices at CMUQ and UIC prior to the conducting of experiments. Table 1 provides a summary of number of students who participated in these experiments. First, a general description of the experiments is given. Second,experiments at both CMUQ and UIC experiments are discussed separately. Finally, an analysis of the two groups of students which relate to the correlations between tasks performed with learning gain is presented.

4.1 Experimental Protocol

All students were asked to complete a consent form. Then they were asked to complete a pre-test. The test included problems that involved recursive problem decomposition, utilizing RGraphs, as tutored in ChiQat. The consent form and pre-test time was capped at 10 minutes. Then, they were asked to solve problems using ChiQat-Tutor, the allotted time was 30 minutes. For each of the problems, all of the

Table 1: Task Description.

Task Name	Description
Answering questions	Students answer multiple choice questions related to the current recursive problem. This task is designed to test learners' understanding of the recursive problem and recursion in general.
Animating	Learners play a prepared animation and observe the execution order of the recursive code. The node's color changes as the animation progresses; green indicates an active function call; gray indicates a terminated call or an intermediate result, which is explained in a legend next to the RGraph.
Tracing	Users click on the nodes of the RGraph and follow the right order of execution. The nodes' color will change as users make progress.
Validating	Students work on two types of RGraph: an incomplete RGraph, and an RGraph that contains errors. Given a sample code, students are required to fill the partial RGraph, and then validate their solution. Similarly, they are required to correct the errors in the flawed RGraph, and then validate their solution.
Constructing	Learners build an RGraph for a given recursive code. The first few nodes of the RGraph are provided to them. Students need to validate their solution after finishing constructing the RGraph.

tasks described in Table 1 were presented. Students were asked to complete the three recursion problems *Power-of-two, Factorial, and is-Palindrome*, but they were allowed to peruse them in whatever order they wanted , and no mechanism was used to enforce completion of any of the problems, or of the activities. Students were then asked to complete a post-test (identical to the pre-test) after finishing working on the problems. The time allotted for the post-test was also 10 minutes.

Each question of the pre and post tests was graded by two graders on a scale from 0 to 5 following a written guideline. There were two questions per test, which brings the total of each test to 10 points. Scores were then rescaled between 0 and 1. Hake's normalized learning gain (g) was then computed according to the formula in equation 1. It is defined as the ratio of the difference in total score to the maximal possible increase in score (Hake, 1998). We also computed and used of absolute learning gain as defined in equation 2.

$$g = \frac{post - pre}{1 - pre} \tag{1}$$

$$g = post - pre \tag{2}$$

4.2 Experiments at CMU Qatar

The students from CMUQ were a mix of Information Systems, Biology, and Business majors. They were exposed to recursion while taking the 15-112 class Fundamentals of Programming and Computer Science, which is an introductory Python programming course. This class devoted two weeks to recursion, and the experiments were conducted during the second week, when the students were still being exposed to these concepts. Both CMUQ experiments took place in two single lab sessions. The difference between pre-test ($\mu = 5.27$, $\sigma = 1.87$) and post-test ($\mu = 6.26$, $\sigma = 1.79$) is significant ($t = 3.9, df = 36, p < .001$) for the two combined CMUQ students experiments. However, analyzing the two CMUQ experiments separately, no significant difference between pre- and post-test for Fall 14 is observed. On the other hand, there is a significant difference between pre- and post-test for the Fall 15 experiments ($p = 0.004, t = 3.29$).

Table 2: Recursion Experiments - Student Distribution.

Semester & School	No. of Students
Fall 14, CMUQ	16
Fall 15, CMUQ	21
Fall 14, UIC	37
Spring 15, UIC	21
Total	95

4.3 Experiments at UIC

Recursion is introduced to students at UIC while they enrolled in the CS151 Mathematical Foundations of Computing class. The CS151 class is required of CS and Computer Engineering majors. The CS151 class devoted four weeks to induction and recursion. In Fall 2014, the experiments took place about two weeks after the end of the lectures on recursion. But in Spring 2015, they took place about seven weeks later towards the end of the semester (15 weeks semester).

In Fall 2014, the experiment took place in two lab sessions during the same day. The difference between pre-test ($\mu = 6.74$, $\sigma = 2.59$) and post-test ($\mu = 7.19$, $\sigma = 3.01$) is only marginally insignificant ($t = 1.98, df = 36, p < .055$). In Fall 2015, the experiment was conducted in three laboratory sessions, and no significant difference between pre-test and post-test scores can observed. There is also no significant difference between pre-test and post-test scores for combined populations of both UIC experiments.

Table 3: Recursion Experiments Test Scores and Learning Gains.

Semester and school	Pre-test		Post-test		Absolute Gain		Normalized Gain	
	μ	σ	μ	σ	μ	σ	μ	σ
Fall 14, CMUQ	5.43	1.71	6.03	1.33	0.6	1.43	0.09	0.3
Fall 15, CMUQ	5.14	2.01	6.43	2.09	1.29	1.75	0.21	0.41
Fall 14, UIC	6.74	2.59	7.46	1.90	0.72	2.20	0.01	0.54
Spring 15, UIC	7.19	3.01	7.05	2.16	-0.14	2.89	-0.28	1.28

Table 4: Tasks - Derived Variables.

Task	Derived Variable	Description
Answering Qs	No of Answering Qs	Number of questions attempted
	Answering Correct	Number of correct answers out of all answers given
Animating	No of animation	Number of Animations attempted
Tracing	No of Tracing	Number of tracing tasks attempted
	Tracing Correct	Number of tracing tasks correctly completed
	Node Tracing Correct	Number of nodes students clicked in correct order
	Node Tracing Errors	Number of nodes students clicked in incorrect order
Validating	No of Validating	Number of Validating tasks attempted
	Validating Correct	Number of validating tasks correctly completed
Constructing	No of Constructing	Number of Constructing tasks attempted
	Constructing Correct	Number of constructing tasks correctly completed
	Constructing Validation Attempts	Number of validation attempts while constructing RGraphs

However, if all students scores from both CMUQ and UIC are combined, a significant difference between pre- ($\mu = 6.27$, $\sigma = 2.55$) and post-test ($\mu = 6.9$, $\sigma = 1.97$) can be observed ($p < 0.006, t = 2.79$). The next section will analyze these experiments in terms of the correlation of activities students performed and their learning gains.

5 RESULTS AND DISCUSSION

The focus now is on analyzing features of interaction of students with the ChiQat system. This is in order to shed light on which tasks are conductive to learning. For this analysis, few subjects (4 from the UIC Fall 14 group, and 2 from the CMU Fall 15 group) were excluded. These students took pre- and post-test, and did use the system, but for some unforeseen reason, their logs could not be matched. This leaves us with 89 students in total, of which 35 at CMUQ and 54 at UIC.

The pre-tests of the two groups of students at CMUQ ($\mu = 0.52$, $\sigma = 0.19$) and UIC ($\mu = 0.68$, $\sigma = 0.28$) still significantly differ (unpaired t-test, $p < 0.0023, t = 7.5050, df = 87$); however the two groups do not differ as gains, or normalized gains are concerned.

We decided to examine the correlation between the activities the students perform in the system and learning, to inform future research on which activities should be emphasized in the system. In this type of analysis, new variables are added to well-known variables that often have an effect on learning such as time (because of the practice effect) and pre-test (previous knowledge). Indeed, through a simple linear regression, we first analyzed the effect of time on task on learning (this is the total time students interacted with the system.

The reason time is explored as a predictive variable is that a significant difference is found in time spent on system between students at CMUQ ($\mu = 25.30$, $\sigma = 4.06$) and UIC ($\mu = 14.26$, $\sigma = 6.01$) (unpaired t-test, $p < 0.0001, t = 9.1412, df = 87$). Students at both CMU and UIC were given 30 minutes to work on the system, however, the different experimental settings clearly had an effect. The CMU experiments took place during formal lectures of 75 minutes long. While the UIC experiments took place during laboratory sessions which are 50 minutes long, exactly the length of the whole experiment (10 minutes pre-test + 30 minutes system usage + 10 minutes post-test). The time on system did not correlate with learning gains, or normalized learning gains. A linear

Table 5: Regression Models: Absolute Gain as Dependent Variable.

Students	M	Predictor	β	R^2	P
All	A	Time	0.003	0.421	*ns*
		Pre-test	−0.573		< 0.001
	B	Time	0.019	0.421	*ns*
		Pre-test	−0.599		< 0.001
		Answering questions	0.033		*ns*
		Animations	−0.219		0.051
		Tracing	−0.233		*ns*
		Node Tracing Correct	0.020		*ns*
		Node Tracing Errors	0.056		*ns*
		Validating	−0.096		*ns*
		Validating Correct	−0.096		*ns*
CMUQ	A	Time	0.052	0.212	*ns*
		Pre-test	−0.439		0.004
	B	Time	0.055	0.223	*ns*
		Pre-test	−0.411		0.004
		Answering questions	−0.058		*ns*
UIC	A	Time	0.043	0.496	*ns*
		Pre-test	−0.653		< 0.001
	B	Time	0.049	0.564	*ns*
		Pre-test	−0.675		< 0.001
		Answering questions	0.154		0.014
		Animations	−0.218		*ns*
		Tracing	−0.083		*ns*
		Node Tracing Errors	0.063		*ns*
		Validating	−0.160		0.086
		Validating Correct	0.312		*ns*

Table 6: Regression Models: Normalized Gain as Dependent Variable.

Students	M	Predictor	β	R^2	P
All	A	Time	0.007	0.137	*ns*
		Pre-test	−0.105		< 0.001
	B	Time	0.010	0.155	*ns*
		Pre-test	−0.111		< 0.001
		Answering questions	0.022		*ns*
		Animations	−0.043		*ns*
CMUQ	A	Time	0.023	0.191	0.069
		Pre-test	−0.075		0.023
	B	Time	0.009	0.269	*ns*
		Pre-test	−0.093		0.008
		Answering questions	−0.007		*ns*
		Animations	−0.061		0.046
		Tracing	−0.026		*ns*
		Node Tracing Correct	0.014		0.033
UIC	A	Time	0.007	0.103	*ns*
		Pre-test	−0.118		0.023
	B	Time	−0.014	0.212	*ns*
		Pre-test	−0.145		0.002
		Answering questions	0.073		0.012
		Animations	−0.032		*ns*
		Tracing	−0.015		*ns*
		Node Tracing Errors	0.027		*ns*
		Validating	−0.058		*ns*
		Validating Correct	0.237		0.012

regression analysis between time on system and learning gain/normalized learning gain was not significant.

After ascertaining that time was not explanatory, a systematic multiple linear regressions was run. It included time, pre-test, and number and type of tasks attempted by students. The tasks are defined in Table 1. We also define derived variables of the original tasks which are listed in Table 4. In this regression analysis, collinear variable with VIF (Variance Inflation Factor) value greater than 5 was excluded.

Tables 5 and 6 report the results of our regression analyses. We report the smallest significant model (A) which always includes time on task and pre-test. Then the best significant model (B) that includes other variables, in addition to the pre-test, that are significantly correlated with absolute, or normalized, learning gain. Models B have higher adjusted R^2 (other than for absolute gain, for all students in Table 5) and highlight which specific tasks are significantly correlated with learning. For absolute gains, the alternative models B are not significantly different from the simpler models A, whereas for normalized learning gains, models B are always significantly better than their A counterparts, for all groups of students (all together, only CMUQ, only UIC). The two tables 5 and 6 allow us to draw some tentative conclusions on which tasks may be effective for learning. The conclusions are tentative because they do not generalize across the two groups of students (CMUQ and UIC) and because the β coefficients are small.

First, pre-test is always included, as a measure of previous knowledge. It may be surprising to see that pre-test is included in the analysis of normalized gain, since the definition itself of normalized gain takes pre-test into account. However, in this particular case, pre-test functions as a negative confounder, or suppressor, that hides the (weak) effect of other independent variables on learning. As concerns the tasks that appear in the model we reported, the constructing task never appears in any of the models. This may be due to the lengthy nature of the construction task itself. Students needed more time working on this task, and consequently small number of students managed to attempt an complete the task. If we look at all students together, the only task that trends towards significance is the number of animations; this task is negatively correlated with absolute gains, but not with normalized gains.

For CMUQ students, no tasks correlate with absolute gains, but as concerns normalized gains, the number of animations negatively correlates, and the number of tracing tasks correctly completed positively correlates with gains. The models for UIC students, for absolute gains, have the most explanatory power, with an adjusted $R^2 = 0.564$. For this model, answering questions positively correlates with learning gains; the number of validating tasks exhibits a trend towards significance, but a negative correlation with gains. For UIC students, for normalized gains, the model is not as strong, but still answering questions is positively correlated with gains, and so is the number of correctly completed validating tasks.

6 CONCLUSIONS

We discussed the importance and difficulty of teaching recursion to novice computer science students. Different approaches to teaching recursion were also discussed, with a focus on conceptual and visual models. Our approach to teaching recursion in ChiQat-Tutor was then introduced. Results from our experiments showed that answering questions, animation, tracing tasks have significant correlation with learning. The validating task has a trend towards significant. The constructing task didn't have a significant effect on learning, and this may due the small number of students who managed to attempt the task. These results suggest that interactive visual representation of recursion algorithms may help students learn. More work is needed in order to help students develop accurate mental models of recursion. Future work will be focused on adding structured templates for writing code, scripting capabilities, intelligent feedback, and student modeling.

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