Some Considerations on the Use of Digital Environments in Learning Numerical Sets

Maria Cristina Carrisi[a]

Dipartimento di Matematica e Informatica, Università degli Studi di Cagliari, Via Ospedale 72, Cagliari, Italy

Keywords: Numerical Sets, Arithmetic Operations, Digital Environments, Data Types.

Abstract: Teachers are nowadays encouraged to use technological tools to engage students more and transmit topics in a simpler way. In addition, information science has become part of many curricula as a separate topic or as a part of other subjects like mathematics. This article will discuss how choosing a particular technological environment can reinforce misconceptions or ideas and how important the role of the teacher is in building a learning environment effective and epistemologically relevant. In particular, the focus will be on the impact that some technological environments have on learning numerical sets and arithmetic operations such as division, because these are studied in all school systems and levels, even if with different approaches and degrees of detail. The topic will be contextualized referring to the Italian school system (degree 10), but conclusions are general.

1 INTRODUCTION

Since the second half of the 20th century, global society has been experiencing a digital revolution that has a strong impact on the productive, economic and social system. This transformation is accelerating and in this context the so called STEM skills are essential for countries and companies, in order to remain competitive, and for individual citizens, in their profession but also in everyday life, to be able to understand the complex economic and social processes in which we are all inevitably involved.

This led many countries, all over the word, to modify school curricula in order to introduce digital skills and computational thinking in the different grades of instruction. Nations are carrying out this process variously and substantial differences are observed also between European countries. In Italy Computer Science has been introduced in all grades, since 2013. Digital competences are addressed both as a separate subject and as a part of other subjects like Mathematics or Technologies, depending on the grade of instruction and, only for high schools, depending on the educational path. Moreover, in order to develop digital citizenship, teachers of all subjects should make students use digital environments, at school or at home, for the analysis of data, the production of reports, projects or presentations.

The relationship between the use of digital environments and learning effectiveness has been widely studied in the literature, as well as the knowledge interchange between Computer Science and the other STEM subjects, particularly mathematics. This paper aims to contribute to this debate by analysing some digital environments that process numerical data in order to study whether they can give rise, reinforce or solve misconceptions on mathematical topics and in particular on the division operation. We focus on the behaviour of the division into different numerical sets because this topic is of general interest, it is studied all over the world in the different degrees of education and has being recognized in the literature as one of the main mathematical concepts used in real life problems.

2 BACKGROUND AND MOTIVATION

We live in an increasingly digitalized world where technology is pervasive and knowledge is growing

[a] https://orcid.org/0000-0002-2837-3971

Carrisi, M.
Some Considerations on the Use of Digital Environments in Learning Numerical Sets.
DOI: 10.5220/0009566504800487
In *Proceedings of the 12th International Conference on Computer Supported Education (CSEDU 2020) - Volume 1*, pages 480-487
ISBN: 978-989-758-417-6

day by day. In this context, educational systems must necessarily modify their paradigms, in order to train at their best the citizens of the future. For this reason, in 2006 the European Commission released a document addressed to member states (European Commission, 2006) containing recommendations on educational systems and in which they identified eight key citizenship skills, among which *"Mathematical competence and basic competences in science and technology"* and *"Digital competence"*. Based on the results of international surveys such as the OECD (Organisation for Economic Cooperation and Development) PISA (Programme for International Student Assessment) and PIAAC (Programme for the International Assessment of Adult Competencies), which indicate that teenagers and adults still have insufficient basic skills and that the 44% of the Union population have low or no (19%) digital skills, in 2018 the European Commission released a new document (European Commission, 2018) highlighting the necessity to work more on issues like *"problem solving, critical thinking, ability to cooperate, creativity, computational thinking"* and to promote the acquisition of STEM and digital competences.

2.1 Digital Competences in Italian School

The different nations of the Union have implemented these indications with different time and modes (Heinz et al, 2016; Partanen et al, 2017).

In Italy, since September 2010, new high schools (educational paths) have been introduced. This promoted a reorganization of the old school curricula and the creation of new guidelines (MIUR, 2010; MIUR 2013; MIUR, 2018) with the aim to guide teachers in devising learning situations that allow students to acquire knowledge and skills, according to the European Commission recommendations. The use of digital environments and coding has been introduced into all levels' school programs:

Primary School: *"The conscious and motivated use of calculators and computers must be suitably encouraged from the early years of primary school, for example, in order to verify the correctness of mental and written calculations and to explore the world of numbers and shapes"*. Students must become able to *"search data to obtain information and build representations (tables and graphics) […] get information from data represented in tabular and graphic way"*.

Middle School: *"Whenever possible, students can be introduced to some particularly simple and versatile programming languages that bring them to develop a taste for ideation and realization of projects (interactive websites, exercises, games, utilities) and for understanding the relationship between source code and visible result"*. Students must become able to *"program and create simple instructions to control the behaviour of a robot"*.

Secondary School: It divides in Liceo, Technical and Professional schools. High school specializes and curricula diversify. Every teacher is encouraged to use digital environments and students are driven to increase their technological skills to search, organize and present information in an appealing way and to use specific application programs (Computer-Aided Design and Drafting, Image Manipulation Program, accounting software, specific database, ecc.), especially in technical schools. STEM teachers are more involved in this process and Math teachers in particular, because the Italian Ministry of Education, University and Research ruled that Math teaching includes also elements of computer science. For example, according to Liceo (MIUR, 2010) we read *"The acquisition of digital skills […] is certainly a theme developed in the first two years of each path within the Mathematics discipline"*. Math teaching is divided in Mathematics and Elements of Computer Science. Students are supposed to be *"able to use digital tools for geometric representation and calculus. […] The use of IT tools is an important resource that will be introduced critically, without creating the illusion that it can solve problems automatically and without compromising the necessary acquisition of mental calculation skills. […] A fundamental topic will be the concept of algorithm and the development of algorithmic resolution strategies for simple and easy modelling problems"*.

As in other European countries (Partanen, 2017), also in Italy the development of digital competences is largely entrusted to Math teachers, who have the heavy burden to choose the best way to transmit them together with the mathematical contents, so that each subject benefits from the other. Literature exhibits contradictory results on the successful integration between Mathematics and computer science, according to learning the specific contents of the two disciplines (Niess, 2005; OECD, 2015), and while it is clear that computer skills, particularly computer programming, benefit of a good understanding of math contents, the reverse is not so obvious and, in the author's knowledge, it seems not to have sufficiently discussed yet in the literature.

In my previous working experience as a high school teacher I had the opportunity to observe on the field students facing problems in the deep learning of mathematical concepts that arose while building an algorithm and writing a program. This led me to deepen the issue, in order to analyse if, and in which way, the most diffused digital environments in Italian schools interfere in learning mathematics.

In the following we present an overview on how some digital environments treat numerical data and the division operation, and which pros and cons there are in using such environment to treat a division with remainder problem. The choice of the mathematical topic, inspired by my observations during classroom activities and reinforced by the scientific debate on mathematics for the future (Gravemeijer, 2017; Dogan, 2019), will be properly motivated in section 3. The digital tools analysed in section 4 have been chosen referring to their presence in the most used Italian mathematics' textbooks and in pre-service and in-service teacher training courses.

3 WHICH MATHEMATICS IN THE DIGITAL ERA?

Scientific community is wondering about which direction mathematical education should take to prepare properly the citizens of the digital age. It is now commonly recognized that math teaching should focus on problem solving procedure more than computation, and this implies to develop the ability of analyse the domain of data, the presence of eventual constraints, and to interpret results. The blended teaching of Math and Computer Science surely help teachers to create learning situations appropriate to develop such skills. But what about mathematical contents?

In 2017 Gravemeijer et al. suggested that Math curricula should focus on topics that (1) are useful in everyday life, in order to be able to understand and interpret the data conveyed by the different information media about health, economy, etc.; (2) are required in workplaces; (3) complement computers' "abilities". In particular, the authors underlined the importance of numbering and quantifying (Gravemeijer, 2017).

Problems dealing with the division with remainder fall within this context. This type of problems has been widely studied in the literature (see for example Dogan, 2019) because, despite their apparent simplicity (they deal with natural numbers and arithmetic operations), they are very delicate. Indeed, they treat epistemologically relevant issues, like: the transition from the division with remainder to the decimal division, hence the passage from natural numbers to rational numbers; the ability to recognize when to operate in each of the two sets; the analysis of the opportunity and modalities of approximating partial results in order to obtain a final result compatible with the context of the problem.

3.1 Numerical Sets in Italian School

From the analysis of the indications given by the Italian Ministry of Education (MIUR, 2010; MIUR, 2013; MIUR, 2018), we see that in Italy numbers and operations are studied in all the three educational cycles, with various approaches and different levels of detail. It represents what institutionally is asked to students, regardless of the differences that clearly arise in the scholastic activity.

Primary School: Students face natural numbers and they learn how to operate with them. Pupils become able to perform division, finding quotient and remainder. Decimal numbers and fractions are introduced immediately after. From this point on, the remainder seams to disappear from students' background and they consider the division with a decimal result the right way to operate, that replaces the "wrong" or "outdated" method of the division with remainder.

Middle School: The work on Integers is focused on factorization and on the research of multiples and submultiples, therefore only on cases in which the division gives zero remainder. Students use to say that the division gives no reminder. This linguistic inaccuracy may hide a misconception: the absence of the remainder, in many students' mind, means that such operation isn't a division with remainder but it is another type of operation.

High Schools: Numerical sets are studied in terms of their algebraic structure. Arithmetic is given for known and acquired and students are considered to be capable to understand to which numerical set the solution of a problem belongs. The focus is on real numbers and their approximation and this may lead students to see Integers or Rational numbers only as an approximation of Real numbers, without an independent essence.

Almost all problems presented in middle and high schoolbooks deal with Real solutions, reinforcing such kind of approach. Students are not induced to analyse the domain of data and of expected solutions, even if it is a fundamental step in problem solving. However, there are problems, like that involving the division with remainder, that can be solved only in the context of Integers. Arithmetic and Integers have ever been of great interest for mathematicians and today have a new life, been fundamental for many computer security methods (see for example Rivest et al., 1987), but it seems that their treatment, in Italian Math curricula, is unappropriated to overcome those critical points that makes learning effective.

3.2 A Case Study

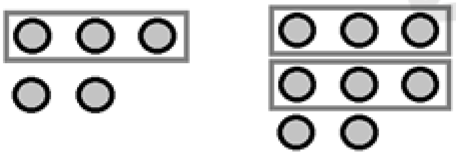
A classroom observation has been conducted in academic years 2017-2018 and 2018-2019 in 5 second classes (degree 10) of a “Liceo Scientifico opzione Scienze Applicate”, in which I observed about ninety 15-16 years old students facing the following problem:

In a supermarket, the following sales promotion on a certain product is active: if you buy three products of the same kind, only two have to be paid. If the number of product bought and the price of the single product are known, what is the total price of the purchase?

The above is an example of a problem framed almost entirely in the context of Integers (data and operations). Only the price of the product could have a Rational value and this data has a secondary role in the solving process. Such kind of problems can be proposed in different classes and in all school levels, having the foresight to adapt it to students’ level of competence.

In primary schools it is appropriate to fix the value of N, the number of the purchased products. Students may proceed by trials and errors, starting from particular cases and using a graphical approach to find the solution, like the following:

Case **N=5** Case **N=8**



Using a graphical representation and different cases the teacher should bring the students to generalize the method by using the division with the remainder, conveying to a more formal approach.

In high school the graphical approach can be also used but in addition we expect that students are able to consider the number N as a variable and that they reach by themselves an algebraic formulation, like the following:

*Let N be the number of products bought and p be the price of the single product. Let’s divide N times 3 obtaining the quotient q and the remainder r. The quotient represents the quantity of groups of three products. For each group, only two products have to be paid. The number of remaining products, identified by the remainder, have to be paid entirely. Thereby, the final price, considering the sale promotion, is (2*q + r)*p.*

During the observation the above problem has been presented to students, asking them to create the resolution algorithm and subsequently a program in C++ language. The objective of the observation wasn’t to collect data about the number of students that succeed or fail, but rather to observe which strategies and prior mathematical knowledge they would have used to face the problem resolution. Moreover, I was interested in students’ reactions when using a digital environment that treats data differently from their previous experiences. At the time of observation, students were able to use spreadsheets and to search autonomously for suitable predefined functions, almost all them learned Scratch in middle school, and they were experiencing for the first few time C++ language (declaration of variables of different type and arithmetical operations).

I focused on the following questions:

- Students deal with division with remainder problems with ease?
- High school students are able to understand when to operate with quotient and remainder or with decimal results? They show the misconception of the division between integers?
- The prior knowledge of digital environments eases or complicates the resolution of such kind of problems?
- The most used digital instruments may route informally mathematical notions?
- It is possible to identify digital tools that behave properly with respect to above question?

I briefly summarize my observations. Almost all students approached the problem by considering a particular case and fixing the number of products N and the price of the single product.; they hold the calculator and tried to divide N by 3, finding, obviously, a decimal number. Most of the students realized that the decimal result was not correct but, except for some attempts to obtain the right solution by approximation, rounding up the result, they were unable to proceed. I suggested to represent graphically the problem this helped students to realize that truncation was a better approach rather than rounding up, but something still was missing. Students searched it in the decimal part of the result, vainly trying to convert it in an Integer number. The remainder of the division, necessary to obtain the right solution, was still lacking because in their mind the division always produces a decimal result. When I tried to suggest to use the division with remainder they *say “That thing that we did in lower school?”*. It is evident that students conceive that knowledge confined in space and time. This suggestive is strengthened by the analysis of curricula in the previous section and by the functioning of the main hardware and software instruments the students deal with, starting from the calculator.

Subsequently I proposed to students to implement the algorithm in C++ language. Although the functioning of the division operator had already been explained, strongly highlighting the different behaviour of such operator with respect to the type of data on which it works, and it had recently been brought back to memory in the context of binary numbering, students expected that the division between integers would have produced a decimal result and they inferred that computer was showing an approximated result. This make us understand how deeply rooted such misunderstanding is and how, over time, even involuntarily, it has been enforced during their schooling.

4 ANALYSIS OF SOME DIGITAL ENVIRONMENTS

Any modern digital instrument is programmed in order to perform different types of operations on different types of data. Data are represented through variables whose values belong to a particular domain and in this set operations are defined. Regardless of the particular instrument or programming language, according to numerical data it is important to underline that, because of its space limits, any device can store a finite number of digits and therefore it always works with a limited range of numbers that can be represented with a finite number of decimal digits. Nowadays, programming languages use mainly two ways to represent numerical data (Rodriguez, 2008), that corresponds to different data types:

- Two's complement, that allows to represent a subset of mathematical Integers;
- Floating point, that allows to represent a subset of Rational numbers including 0.

Data type defines the set of possible values assumed by the variable, the operations that can be done on data and their result, the meaning of the data, and the way values of that type will be stored into the hardware.

A strongly typed programming language has strict typing rules in variables declaration and assignment, in return values and function calling, and this implies that errors and exceptions may appears in compiling and executing (Liskov et al, 1974). The implicit type conversion at runtime is not allowed and it prevents the possibility of producing unpredictable results. In a strongly typed programming languages the programmer has to declare explicitly data type before using a variable and such type can't change unexpectedly during execution. Such kind of languages force the programmer to take continuously into account data type and its implication on the operations' results.

In weakly typed programming languages and in informal environments this doesn't happen. To simplify users' use, only floating point are generally usable in a simple and immediate way, and arithmetic operations are defined in this numerical set.

In school activities teachers are free to choose the digital environment they consider more suitable, depending to the school level, to the type of school, to the subject or to the particular topic they want to address. This choice is not simple at all because of the large amount of factors involved in the decision. In high schools the choice could be inspired by the desire to offer students an instrument immediately expendable in the work market or during the academic studies. In this context languages like C/C++ or Python take place. In lower degrees, or when the digital skill is used to support the acquisition of other subjects' knowledge, informal environments are generally preferred, like spreadsheets or Scratch (Resnick et al, 2009). They are widespread in Italian schools and are proposed in most school books because of their strengths, like the possibility to focus on the algorithm building process, being released from the syntax of a particular language and for this reason they can be used for different subjects and in all school levels. But to achieve this simplicity, such environments must keep hidden important aspects of the management of variables, like the types of data (variables domain) and the functioning of operations, relations and functions, all relevant aspects from mathematical point of view. Teachers have to be aware of this in order to make their choice appropriate (Chevallard, 1985) and intervene in order to prevent misconceptions (Shaughnessy, 1985; Maurer, 1987; Zan, 2000; Zan, 2007).

4.1 Calculators

Calculator is the first digital instrument students deal with and in such sense it is the most important according to what knowledge it can implicitly route. In Italy, its use is promoted since from the early years of primary school (see section 3.1), when Math studying is focused on Natural numbers but calculators operate on floating point data. The division between two integers always produces a decimal number. In order to obtain the quotient of the division students must truncate the decimal result and only few calculators provide for an operator to calculate the remainder of a division. The early use of an instrument quick and easy to use, but that gives results different from that obtained by students using the classical division algorithm is certainly destabilizing as observed in the classroom experience described in section 3.2.

4.2 Spreadsheet

Spreadsheet allows to treat various types of data but regarding numbers it makes available only floating point numbers. All arithmetical operations are allowed and they are represented trough the classical symbols + - * and /. Integers can be represented as decimal numbers without decimal ciphers, activating an approximation that rounds up or down, as appropriate, and not simply truncates. It is possible to properly represent integers only using the predefined INT function that converts any decimal number in its integer part by truncating the decimal part of the number, but this implies a more advanced competence in the use of this digital environment. According to division, spreadsheets furnish the operator / that gives the decimal result of a division, and the functions QUOTIENT and REMAINDER that should receive two integers and return an integer but they work also when the dividend, the divisor, or both are decimal without returning any error message, therefore, don't behaving exactly like the corresponding mathematical operations.

C	D	E	F
DIVIDEND	DIVISOR	QUOTIENT	REMAINDER
6,5	5	1	1,5
6	2,3	2	1,4

Figure 1: Functions Quotient and Remainder working on rational values.

In order to solve properly the problem discussed in sec. 3.2, teachers should encourage students to use the INT function (see Figure 2) in order to cast the data and make the QUOTIENT and REMAINDER functions work properly even in case of incorrect data entry. In such way a reflection is induced on the domain of data and results, allowing to give to the activity a deeper meaning from the mathematical point of view. Despite Figure 2 shows a very compact formulation, used for opportunity reasons, during the work in classroom it is preferable to use auxiliary cells to receive the results of the INT, QUOTIENT and REMAINDER functions so that students can clearly see the various steps of the solving procedure.

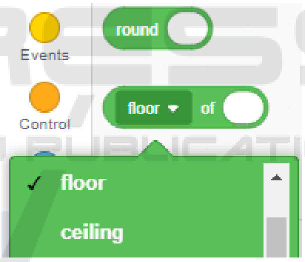
=(QUOZIENTE(INT(E1);3)*2+RESTO(INT(E1);3))*E2

D	E	F
Number of products	20	
Price of the single product	1	
Total price	**14**	

Figure 2: Solution of the problem described in sec. 3.2.

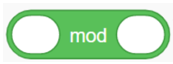
4.3 Scratch

According to numbers, Scratch behave as spreadsheet and calculators, allowing to treat only floating point numbers. The only way to obtain an integer number is by approximation. Scratch allows to approximate in three different ways by using blocks in the "Operators" category (green colour): *'Floor'* performs a truncation, *'ceiling'* an upper approximation, *'round'* behave as *'floor'* when the decimal part of the number is less than 0.5 and as *'ceiling'* otherwise.



The "Operators" category provides blocks for arithmetical operations

defined in floating point set, and to calculate the remainder of a division via the *'mod'* operator



Unfortunately, as in spreadsheets, it works differently from the division defined in Integers, giving a decimal result when operating on rational data.

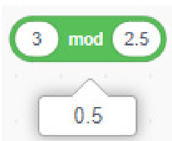


Figure 3: Scratch *'mod'* block working on rational data.

Scratch blocks are organized in sections and, in each section they are grouped, generally by scope. It happens for arithmetical operations, for relations (<, >, =) but also for blocks '*mod*' and '*round*'. This induce the user to think that they are logically related and referred respectively to remainder and quotient of a division but it is not correct. Instead it is necessary to act as in the use of calculators, namely truncating the decimal result of a division, by using the *'floor'* block.

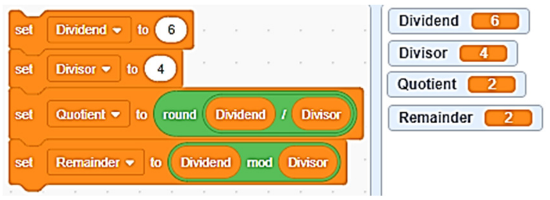


Figure 4: Incorrect use of Scratch blocks *'mod'* and *'round'* to calculate a division with remainder.

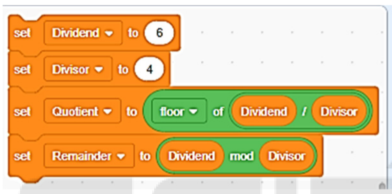


Figure 5: Correct use of Scratch blocks *'mod'* and *'floor'* to calculate a division with remainder.

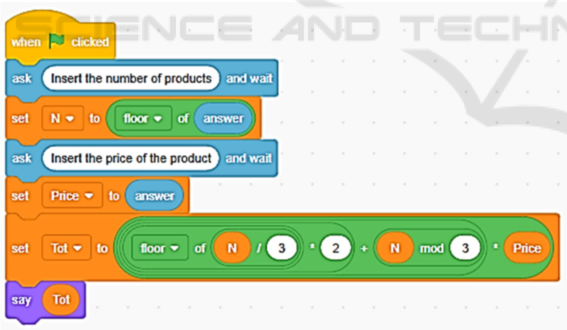


Figure 6: Scratch solution of the problem presented in sec. 3.2.

In view of the above, the Scratch solution of the problem presented in sect. 3.2 is presented in Figure 6. Also in this case it is necessary to cast the data to prevent that incorrect data entry makes the functions work improperly. This makes the development rather cumbersome, despite of the apparent simplicity of the environment.

4.4 C/C++ Language

C/C++ is a strongly typed programming language. Variables must be declared before use, providing name and type and they cannot unexpectedly change type during execution. Integers and floating point numbers are allowed and the programming language provides 5 arithmetic operations: + - * / %. All operations are closed with respect to the set they act on. In particular, the division adapts, working differently depending on operands' type. When it works on at least one floating point number it returns a floating point result. When it acts on two Integer numbers it returns an integer result, i.e. the quotient of the division. There is no way to obtain a decimal result from a division between integers, except by explicitly forcing the interpretation of one of the operands as a floating point number. The fifth arithmetical operation, indicated by the symbol %, is defined only on integers and returns the integer remainder of a division. In case of incorrect use an error is reported during program compiling, for example if the programmer tries to use the operator % on floating point numbers.

According to C++ language the development of the problem presented in section 3.2 is:

```
int N;
float p,tot;
cin >> N >> p;
tot =((N/3)*2+N%3)*p;
cout << tot;
```

where only the content of the main function is reported. The development is more compact and extremely more adherent to the algebraic formulation that students should have found. Incorrect data entry is prevented by typing, in fact even if the user inserts a decimal number it will be automatically truncated and the algorithm will work properly. It is however evident that such solution is well suited only for high school paths and unthinkable in primary school.

5 DISCUSSION AND FUTURE WORK

In this paper some of the most popular digital environments used in Italian schools have been analysed, with respect to their functioning about the division between Integer numbers. It has been shown that, behind the friendly and ready to use interface, environments, like Scratch or spreadsheets, hide delicate aspects in the management of data and their operations, that can be properly faced only by a shrewd and qualified user. On the other hand, more rigorous environments seem to be not affordable for lower level students. This doesn't mean that such environments mustn't be used in school but rather that teachers have to choose them carefully, depending on the activity they propose to students and to the grade of instruction. Math teachers can take advantage on the above mentioned weakness, in order to make the activity epistemologically relevant from the mathematical point, aimed at overcoming the misconception on the division between Integers.

Who produce or promote digital environments for learning, especially if they are addressed for elementary or middle school students, who are building their linguistic and mathematical knowledge, should be careful not to introduce excessive, and sometimes only apparent, simplifications that may strengthen misconceptions. It may be interesting, as a future work, to build a calculator for elementary students or a personalized version of Scratch, using its extension BYOB (Harvey & Monig, 2010) that can overcome the problems underlined in the present paper.

In the future the author would like to extend the results of the present paper to other mathematical topics like the division between signed number, analysing the behaviour of the most used digital environments with respect to this.

REFERENCES

Chevallard, 1985. La transposition didactique. Du savoir savant au savoir enseigné. La Pensée Sauvage, Grenoble.

Dogan Coskun, S., Ev Cimen, E. (2019). Pre-service elementary teachers' difficulties in solving realistic division problems. Acta Didattica Napocensia, 12(2), 183-194. DOI:10.24193/and.12.2.14

European Commission, 2006. Recommendation of the European Parliament and of Council of 18 December 2006 on key competences for lifelong learning. Official Journal of the European Union, L 394/10 (https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32006H0962&from=IT accessed 18.3.2020)

European Commission, 2018. Council Recommendation of 22 May 2018 on key competences for lifelong learning. Official Journal of the European Union, C 189/1 (https://eur-lex.europa.eu/legal-content/ EN/ TXT/PDF/?uri=CELEX:32018H0604(01)&qid=1584484557047&from=IT accessed 18.3.2020)

Gravemeijer, K., Stephan, M., Julie, C., Lin, F., Ohtani, M., 2017. What Mathematics Education May prepare Students for the society of the Future?. Int. J. of Sci. and Math. Educ. Suppl 1, S105-S123.

Harvey, B., Monig, J., 2010. Bringing 'No Ceiling' to Scratch: Can One Language Serve Kids and Computer Scientists? In proceedings of Constructionism 2010.

Heintz, F., Mannila, L., Färnqvist, T., 2016. A Review of Models for Introducing Computational Thinking, Computer Science and Computing in K-12 Education. Frontiers in Education *October*.

Liskov, B, Zilles, S., 1974. Programming with abstract data types. doi:10.1145/942572.807045.

Maurer, S.B., 1987. New knowledge about errors and new views about learners: what they mean to educators and more educators would like to know. Cognitive Science and Mathematics Education, Lawrence Erlbaum Associates.

Ministero dell'Istruzione, Università e Ricerca, 2010. I Regolamenti - Le indicazioni Nazionali. https://archivio.pubblica.istruzione.it/riforma_superiori/nuovesuperiori/index.html accessed 18.3.2020.

Ministero dell'Istruzione Università e Ricerca, 2013. Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione, Gazzetta Ufficiale della Repubblica Italiana, serie generale n.30, 5-2-2013.

Ministero dell'istruzione, Università e Ricerca, 2018. Indicazioni Nazionali e Nuovi Scenari, https://www.miur.gov.it/documents/20182/0/Indicazioni+nazionali+e+nuovi+scenari/ accessed 18.3.2020

Niess, M. J., 2005. Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. Teaching and Teacher Education 21 (5), 509-523.

OECD, 2015. Students, Computers and Learning.

Partanen, T., Niemelä, P., Mannilla. L., Poranen, T., 2017. Educating Computer Science Educators Online-A Racket MOOC for Elementary Math Teachers of Finland, In Proceedings of the 9th International Conference on Computer Supported Education (CSEDU 2017).

Rivest, R., Shamir, A., Adleman, L., 1987. A method for obtaining digital signatures and public-key cryptosystems, Communications ACM, 21, 120-128.

Resnick, M., Maloney, J., Monroy-Hernández, A., Rusk, N., Eastmond, E., Brennan, K., Millner, A., Rosenbaum, E., Silver, J., Silverman, B., Kafai, Y., 2009. Scratch: Programming for All. Communications of ACM, 11. http://scratch.mit.edu/

Rodriguez, G., 2008. Algoritmi Numerici. Pitagora Editrice, Bologna. ISBN: 88-371-1714-0.

Shaughnessy, J.M., 1985. Problem-Solving Derailers: The Influence of misconceptions on Problem-Solving performance. Teaching and Learning Mathematical Problem Solving. Lawrence Erlbaum associates.

Zan R., 2000. 'Misconceptions' e difficoltà in Matematica, L'insegnamento della Matematica e delle scienze Integrate, vol. 23, Centro di ricerca in didattica Ugo Morin.

Zan, R., 2007, Difficoltà in matematica. Osservare, interpretare, intervenire. Springer.