Minimising the User's Effort during Wheelchair Propulsion using an Optimal Control Problem

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Abstract: This paper proposes a study of the optimal control problem with state constraint, using two types of a power-assist wheelchair propulsion. The cost function is given by the metabolic function, which represented by a compromise between the work exerted by the joints muscles (mechanical effect) and an efficiency function that converts chemical into mechanical energy (biomechanical effect). The dynamic wheelchair is given by a simple model, which connects the push force to the wheelchair speed. An upper bound constraint is considered in order to limit the energy consumed by the motor. This study used an approach that calls the Pontryagin's maximum principle, the optimal solution varies with the parameters of the problem. Finally, a numerical comparison is enabled using two types of assistance: constant and proportional. This numerical comparison is based on the framework of the optimal control theory with two different costs. The first cost is given by the integral of the squared user's force and the second by the integral of the metabolic function. This Numerical results show that the user provides less effort with metabolic cost than with the energy user's force.

1 INTRODUCTION

In the long term, the manual wheelchair user can cause upper limb pain and degeneration. Most of the users have difficulty to avoid an obstacle or change direction. In order to decrease the human suffering caused by this activity, many working tasks are performed manual wheelchair level (motor, frame structure, etc.) (Pezzuti et al., 2006). In addition, many researches are made in biomechanical (Horiuchi et al., 2014) and (Luhtanen et al., 1987). This paper proposes a human-machine interaction and within this framework, a power assist manual wheelchair propulsion is achieved using an optimal control problem (Cooper et al., 2002) and (Cuerva et al., 2016).

Usually, the model of optimal control problem inspired from (Oukacha and Boizot, 2020), altering the energy of the motor vehicle by the effort required for the user in a manual wheelchair propulsion. The motor energy takes into account as an upper bound constraint. It is then a optimal control problem with state constraint.

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In order to reduce these efforts, a metabolic cost function, also called "biomechanical metabolic" is used. This metabolic function is the quantity of energy consumed by a person during a muscular activity (Horiuchi et al., 2014). Generally, this function measured the amount of oxygen used by the muscles. During the muscle contraction, the cells use the ATP as an energy source, which is produced by hydrolysis of ATP to ADP (chimical energy). In the literature, the metabolic cost function can be expressed mathematically, thanks to the tests made by different scientists and it has several forms. In reference to (Ardigo et al., 2005) and (Horiuchi et al., 2014), this function could be formulated as a second degree equation, which depend only of the linear speed of the wheelchair. As stated above, this metabolic function could be designated as the energy expended during a manual wheelchair propulsion. This function measures the mean of the Oxygen uptake and the carbon dioxide output (Yang et al., 2009). Since the effort provides by a person in a manual wheelchair propulsion, depends on his/her upper limb ability, this function may also be presented by all the joint moments of the shoulder, elbow and wrist (Rozendaal et al., 2003). Finally, the metabolic function can be generated under the efficiency function.

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In the works cited above, the metabolic function is achieved through the analysis of experimental data. In this article, the metabolic function value is calculated by solving an optimal control problem. Moreover, we interested more specifically in this latter function. Thus, the metabolic function is given by a compromise between the work of the push force and efficiency function, which is only related to the linear speed of the wheelchair (Cooper, 1990b). There are several different types of the efficiency: gross, net, true, etc. as described in (Hintzy et al., 2002) and (Luhtanen et al., 1987). In this work, we will focus on the gross efficiency, contrary of what is done in (Cooper, 1990a), where it studied an optimal control problem using the net efficiency that appears in the dynamical model. The advantage of using the gross efficiency is that it allows to take into account the energy expended at rest.

In this paper, we tackle an optimal control problem with two approaches: an analytical study and a numerical resolution. First, a general study is performed with the help of Pontryagin's Maximum Principle (PMP) (Pontryagin et al., 1962). The PMP is a very effective approach, where these theoretical results obtained that fits perfectly with the experimental tests. For example, the article (Berret et al., 2008) deals an optimal control problem in biomechanical, where the authors have shown that the optimal solution obtained by the PMP similar to the experimental results. In the second step, a numerical method is used to provide a solution to a complex optimal control problem. The numerical simulations uses real data, for example, the efficiency function model is retrieved in (Cooper et al., 2002). This function formulated through experimental tests and the curve efficiency profile according to (Ardigo et al., 2005).

In addition, a comparison is made between the optimal control problem given in (Cuerva et al., 2016) and a new problem which is will be presented. In the previous paper, a comparison of three different types of power-assist wheelchair propulsion is made using an optimal control problem.

The section 2 is devoted to a more detailed description of the optimal control problem, the model dynamic wheelchair and the running cost. Section 3 is dedicated to the presentation of the approach solving the optimal control problem. In the section 4, we present the numerical results of each power-assist wheelchair propulsion. The sections 5 discusses the obtained results. Finally, the section 6 presents the conclusion and perspectives of this study.

2 MODEL OF THE OPTIMAL CONTROL PROBLEM

In this optimal control problem with state constraint, we will focus particularly on the metabolic cost function which is generalized by an efficiency function. The dynamic wheelchair-user system is given by the Netwon's second law of motion.

2.1 Dynamic Wheelchair Propulsion

The equations of the motion wheelchair-user model are determined by a first order dynamic system. Assume that air resistance is negligible and the rolling resistance of the wheelchair-user system propelled at linear speed in a straight line. Following the Figure 1, the movement of a person assisted by a manual wheelchair is given:

$$\ddot{x} = \frac{1}{M}\left(F_p + F_m - \text{sign}(\dot{x})F_r - C\,\dot{x}\right) \tag{1}$$

where, x, $\dot{x}$, $\ddot{x}$ are the longitudinal position, the linear velocity, the linear acceleration of wheelchair respectively, F_p is the user's force, F_m is the motor force, F_r is the rolling resistance force, C is the viscous damping coefficient and M is the total mass (user + wheelchair). The motor force is proportional to the user's force, since it acts of a power-assist wheelchair. The control strategy is given by the F_p and $F_m = F_m(F_p)$.

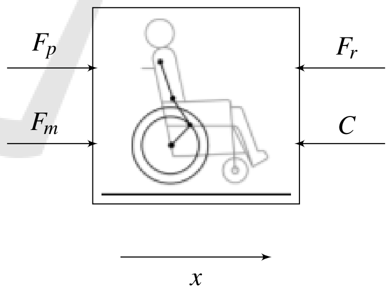

Figure 1: Dynamic Model of the motion wheelchair-user.

2.2 Metabolic Cost Function

The energy consumption of the body can be defined by a metabolic function during the propulsion wheelchair. The metabolic function to minimize, representing as the ratio between the power of push and biomechanical efficiency (Oukacha and Boizot, 2020):

$$J = \frac{P}{\rho} \tag{2}$$

The power associated to the work W is:

$$P = \frac{dW}{dt} = F_p\, \dot{x}$$

The work performed during a muscular activity is not constant, because the pace and the capacity of the person change with time. The wrist motion in handrim wheelchair propulsion can generate in two stages: the concentric and eccentric muscle contractions, which produced by the positive and negative work respectively (Williams, 1985). Therefore, the work of the push can be described as a non-differentiable function that is similar to the absolute work. The absolute work of the propulsion force done by a muscle during a period t_0 to T_f is:

$$W = \int_{t_0}^{T_f} |F_p\, \dot{x}|\ \ dt$$

The cost function which allows to measure the quantity consumed to push the wheelchair from a starting position at time zero to some final position at final time T_f, is given by:

$$J = \int_0^{T_f} \frac{|F_p \dot{x}|}{\rho(\dot{x})} dt \tag{3}$$

According to (Cooper, 1990b), the biomechanical efficiency can be formulated by the second equation, which depends of the linear wheelchair velocity. The curve profile obtained from this equation is illustrated in Figure 2:

$$\rho(\dot{x}) = \frac{1}{100}\Big(-0.55\dot{x}^2 + 7.02\dot{x} + 3.15 \Big)$$

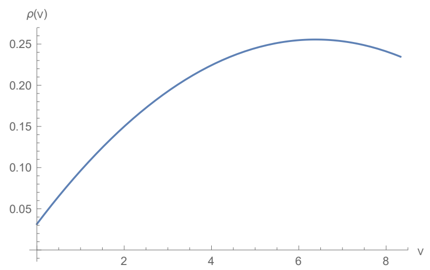

Figure 2: Efficiency function model.

This Efficiency is calculated from the ratio between the power output and the power of the measured oxygen consumption.

2.3 Problem Statement

An optimal control problem is formulated by the dynamic system similar to the one presented in (Cuerva et al., 2016). The cost function is given by the metabolic cost function presented above. Moreover, a control bound and a fixed final time are considered in order to satisfy the upper and lower bounds of the user's force and the duration of the motion. The problem is given by the following equations:

$$\begin{cases} \text{Minimise} \qquad J = \int_0^{T_f} \frac{|F_p v|}{\rho(v)} dt \\ \text{Subject to} \\ \qquad \dot{x} = v; \\ \qquad \dot{v} = \frac{1}{M}\Big(F_p + F_m - F_r\ \text{sign}(v) - C\ v \Big) \\ \qquad \dot{E} = |F_m v| \\ \qquad E \leq W_1 \\ \qquad |\dot{v}| \leq 3 \\ \qquad \begin{pmatrix} x(0) = 0 \\ v(0) = 0 \end{pmatrix}, \quad \begin{pmatrix} x(T_f) = 10 \\ v(T_f) = 0 \end{pmatrix} \\ \qquad Lb \leq F_p \leq Ub \\ \qquad T_f > 0 : fixed \end{cases} \tag{4}$$

Where $W_1 = \frac{3}{4}W$ is the energy consumed by the motor, when this maximum value (W) is calculated by solving the optimisation problem (4), without user interaction ($F_m = 0$). Thus $W = 219.5J$.

A boundary constraint ($|\dot{v} \leq 3|$) is made regarding to the acceleration ant the deceleration to ensure user's comfort (Karmarkar et al., 2008).

Following (Berret et al., 2008), the problem (4) admits at least one solution. According to (Oukacha and Boizot, 2020), the optimal control problem (4) is independent of the position x, the trajectories including an arc with $v < 0$ is not an optimal trajectory. Thus, the speed is positive or null ($v \geq 0$) during wheelchair propulsion, as suggested in (Cuerva et al., 2016).

The optimal control problem (4) could be defined in free final time. In this case, the time is an unknown variable of this problem, we then have an additional constraint.

3 PONTRYAGIN'S MAXIMUM PRINCIPLE

A study is based on the Pontryagin's Maximum Principle (PMP) (Pontryagin et al., 1962), which gives a necessary optimality condition of the optimal control problem. Let us introduce $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ the adjoint vector of the state vector $X = (x, v, E)$ and the Hamiltonian function is defined by:

$$H(X,\lambda,F_p) = -\frac{|F_p|v}{\rho(v)} + \lambda_3 |F_m(F_p)|v + \mu(E - W_1)$$
$$+\lambda_1 v + \lambda_2 \frac{1}{M}\left(F_p + F_m(F_p) - F_r - C\,v\right)$$

Let (X^*, F_p^*) be an optimal solution, then the PMP asserts the existence of an absolutely continuous function λ: $[0,T_f] \to \Re^3$ and μ is the Lagrange multiplier of the constraint. The necessary optimality conditions are as follows:

1. There exists $t \to \mu(t) \geq 0$, such that the adjoint vectors satisfies:

$$\begin{cases} \dot{\lambda}_1 & = -\frac{\partial H(X,\lambda,F_p)}{\partial x} = 0 \\ \dot{\lambda}_2 & = -\frac{\partial H(X,\lambda,F_p)}{\partial v} \\ & = \frac{|F_p|}{\rho(v)} - \frac{|F_p|\rho'(v)v}{\rho(v)^2} - \lambda_1 + \frac{\lambda_2 C}{M} - \lambda_3 |F_m(F_p)| \\ \dot{\lambda}_3 & = -\frac{\partial H(X,\lambda,F_p)}{\partial E} = -\mu \end{cases}$$

   where $\rho'(v) = \frac{\partial \rho(v)}{\partial v}$

2. The mapping $t \to \mu(t)$ is continuous along the boundary arc and verifies:

$$\mu(t)\,(E(t) - W_1) = 0, \qquad \forall t \in [0, T_f]$$

3. $H(X,\lambda,F_p)$ is constant, since the optimal control problem (4) is autonomous. Therefore, the optimal control maximizes almost everywhere the Hamiltonian:

$$H = \max_{F_p} H(X,\lambda,F_p)$$
$$= \lambda_1 v - \frac{\lambda_2}{M}(F_r + Cv) + \mu(E - W_1) + \max_{F_p}\{\phi(t)\}$$

4. If the final time is free, then the Hamiltonian $H(X,\lambda,F_p) = 0$ along the trajectory.

5. The candidate control strategy is given by:

$$F_p = \underset{Ub \leq F_p \leq Lb}{\arg\max}\ \{\phi(t)\}$$
$$= \begin{cases} Ub & \text{if } \phi(t) > 0 \quad \text{(bang)} \\ F_p \in]Lb, Ub[& \text{if } \phi(t) = 0 \quad \text{(singular)} \\ Lb & \text{if } \phi(t) < 0 \quad \text{(bang)} \end{cases}$$

Where $\phi(t) = \frac{-|F_p|v}{\rho(v)} + \lambda_3 |F_m| v + \frac{\lambda_2}{M}\left(F_p + F_m\right)$, is called the switching function.

The optimal control strategy vanishes in terms the $F_m = F_m(F_p)$. In the section that follows, both assistance will be present, where her solution varies between: bang-bang, inactivated and singular arc. An **extremal** is a solution λ of the above equations. A portion of the trajectory is a **bang** type, when the control variable is equal to its maximum, or its minimum. A **trajectory with inactivation** is defined when the control variable is null over a time interval. However, a **singular arc** is a portion of the trajectory along which the control does not achieved its upper (constant or non constant arc), as in Figure 3.

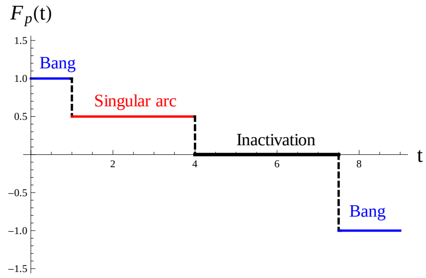

Figure 3: Control strategy example with $|F_p| \leq 1$.

The problem solving is based on the trapezoidal direct collocation method developing in Matlab (Betts, 2010).

4 NUMERICAL RESULTS

This part is devoted to present the results of numerical simulation studies, using the constant and proportional assistance. For all simulations, the data are retrieved from the article (Cuerva et al., 2016), their values are: $M = 110kg$, $F_r = 8.9N$, $C = 4.6N\ s/m$.

4.1 Constant Assistance

The constant assistance is defined by a simple gain (K_1), which represents the assistance force provided by the motor. The value of this gain is connected to the user's force (F_p), with respect to a threshold. An approximation of the push force is given by a numerical approach of the threshold. This push force (F_p) is expressed as a hyperbolic tangent function:

$$F_m(F_p) = K1\ \tanh(F_p) \qquad (5)$$

The energy used by the motor is:

$$E(F_m, v) = \int_0^{T_f} |F_m\ v|\ dt = \int_0^{T_f} K_1 |\tanh(F_p)|\ v\ dt \qquad (6)$$

We have $|\tanh(F_p)| \leq 1$, the equation (6) become:

$E(F_m,v) \leq \int_0^{T_f} K_1\ v\ dt = K_1 \int_0^{T_f} \frac{dx}{dt}\ dt = K_1 \int_0^{T_f} dx$

Therefore, $E(F_m,v) \leq K_1 x(T_f)$ and we have $E(F_m,v) \leq W_1$. In extreme case, the motor consumes $E(F_m,v) = W_1$.

Consequently:

$E(F_m,v) = W_1 \leq K_1 x(T_f) \Longrightarrow K_1 \geq \frac{W_1}{x(T_f)} = 16.46.$

In this case, the switching function $\phi(t)$ is:

$$\phi(t) = \left(\frac{-|F_p|}{\rho(v)} + K_1\lambda_3|\tanh(F_p)| \right) v + \frac{\lambda_2}{M}\left(F_p + K_1 \tanh(F_p) \right)$$

They noted that the switching function has a quite complex expression. Thus, an analytical study the sign of this function is not easy. A numerical method is used to solve the optimal control problem (4). Figures 4 shows some optimal solutions, which varied according to the upper bound controls included.

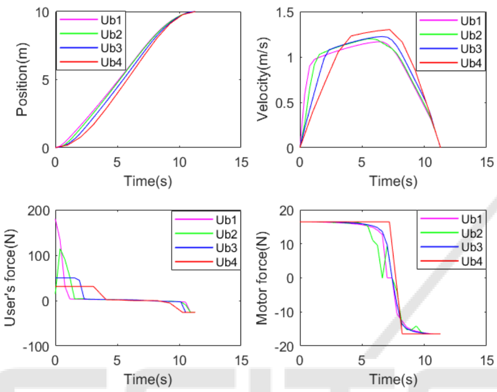

Figure 4: Optimal trajectories with $Lb \geq F_p \leq Ub_i, i = 1,..,4$ and $Ub1 > Ub2 > Ub3 > Ub4$.

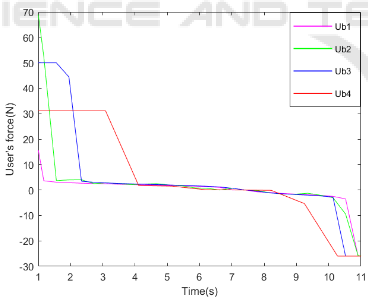

Figure 5: Optimal control strategies with $Lb \geq F_p \leq Ub_i$ $, i = 1,..,4$ and $Ub1 > Ub2 > Ub3 > Ub4$.

In this case, the authors of the work (Cuerva et al., 2016) solved the optimal control problem with $tanh(F_p) \approx sign(F_p)$, since the latter problem does not converge. In this order, a comparison is established with the optimal control problem (4) and with the same condition (cf. Figure 6). Therefore, the motor force and motor energy are becoming:

$$F_m(F_p) = K1 \text{ sign}(F_p) \tag{7}$$

$$E(F_m, v) \approx \int_0^{T_f} K_1 \, v \, dt \tag{8}$$

The energy consumed by the motor depends only on the wheelchair speed. The gain value K_1 is obtained from the equation (8), as follows:

$$E(F_m, v) \approx \int_0^{T_f} K_1 \, v \, dt \leq \frac{3}{4}W \implies K_1 \int_0^{T_f} dx \leq \frac{3}{4}W$$

Thus, $K_1 \leq \frac{3\,W}{4\,x(T_f)} = 16.46$. The switching function $\phi(t)$ become:

$$\phi(t) = \frac{-|F_p|v}{\rho(v)} + \frac{\lambda_2}{M}\left(F_p + K_1 sign(F_p) \right)$$

The last expression of the $\phi(t)$ of the form:

$$\phi(t) = -K|F_p| + MF_p \mp N$$

where $K = \frac{v}{\rho(v)}$, $M = \frac{\lambda_2}{M}$ and $N = \frac{\lambda_2}{M}K_1$.

As the before case, an analytical study of this switching function is not simple, since the value of N varies as a function of time.

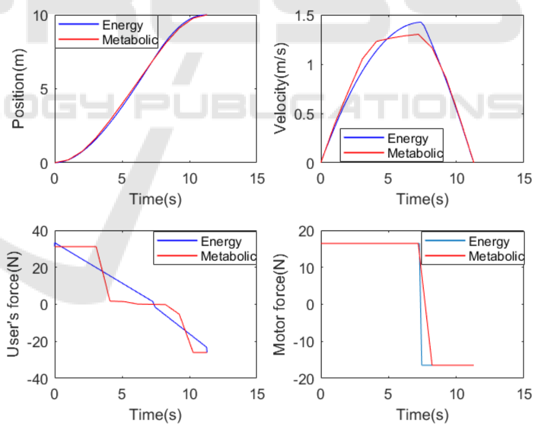

Figure 6: Comparison between the problem (Cuerva et al., 2016) (Ref) and problem (4) (New) with $tanh(F_p) \approx sign(F_p)$ for the constant assistance.

In this case, the goal is to compare the problem posed in (4) without and with $tanh(F_p) \approx sign(F_p)$. For the same initial conditions, the optimal solutions are illustrated by the Figures 6.

4.2 Proportional Assistance

This kind of assistance is inspired from the work of (Cooper et al., 2002), where the motor force (F_m)

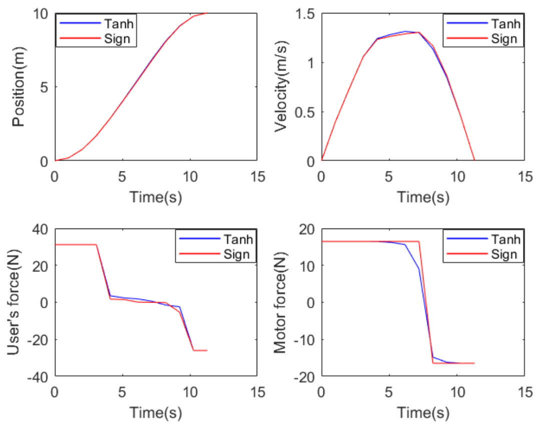

Figure 7: Comparison between the problem (4) without and with $tanh(F_p) \approx sign(F_p)$ for the constant assistance.

is given by the use's force (F_p) multiplied by a gain (K_2) (cf. equation (9)):

$$F_m(F_p) = K_2 \ F_p \tag{9}$$

The energy generated by the motor during the acceleration and deceleration phases is given by:

$$E(F_m, v) = \int_0^{T_f} |F_m \ v| \ dt$$
$$= \int_0^{T_f} \frac{K_2}{K_2+1}\left(M|\dot{v}| + \text{sign}(v)F_r + C \ v\right) v \ dt$$

The gain value K_2 is calculated with the same principle adapted to the previous assistance study, then $K_2 = 1.87$. In the case of the proportional assistance, the switching function $\phi(t)$ is given by:

$$\phi(t) = -\left(\frac{v}{\rho(v)} - K_2\lambda_3 v\right)|F_p| + \frac{\lambda_2}{M}(1+K_2)F_p$$

The last function is of the form:

$$\phi(t) = -G(t)|F_p(t)| + L(t)F_p$$

where $G = \frac{v}{\rho(v)} - K_2\lambda_3 v$ and $L = \frac{\lambda_2}{M}(1+K_2)$.

The maximisation condition of the control (Oukacha and Boizot, 2020):

$$F_p = \underset{Ub \le F_p \le Lb}{\arg\max} \ \{\phi(t)\}$$
$$= \begin{cases} Ub & \text{if } \ L > G & \text{(bang)} \\ F_p \in [0, Ub] & \text{if } \ L = G & \text{(singular)} \\ 0 & \text{if } \ -G < L < G & \text{(inactive)} \\ F_p \in [Lb, 0] & \text{if } \ L = -G & \text{(singular)} \\ Lb & \text{if } \ L < -G & \text{(bang)} \end{cases}$$

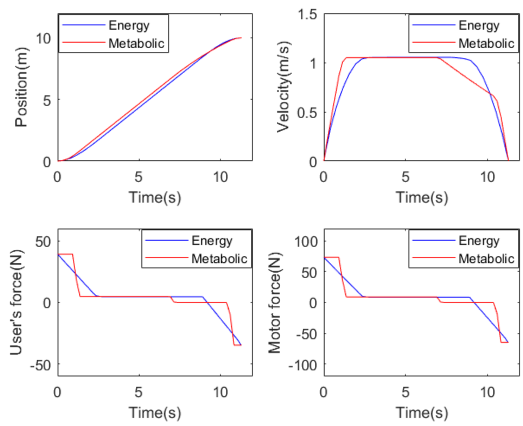

Figure 8: Comparison between the problem (Cuerva et al., 2016) and problem (4) for the proportional assistance.

Cost Function Value

Table 1 summarizes the results of the cost function for the different strategies of the control, with the two assistances.

Table 1: Comparison betwen the cost functions.

Cost	Constant assist.	Proportional assist.
J_{Energy}	4201.06	2115.98
$J_{Metabolic}$	1208.84	—
$J^{Sign}_{Metabolic}$	1137.22	727.48
$J^{UB1}_{Metabolic}$	775.47	—
$J^{UB2}_{Metabolic}$	984.51	—
$J^{UB3}_{Metabolic}$	1019.22	—
$J^{UB4}_{Metabolic}$	1208.84	—

Where J_{Energy} and $J^{Sign}_{Metabolic}$ are the costs corresponding to (Cuerva et al., 2016) and the optimal control problem (4) with $tanh(F_p) \approx sign(F_p)$, for each one of these two assistance (Figures 6 and 8 respectively). $J_{Metabolic}$, $J^{Ub1}_{Metabolic}$, $J^{Ub2}_{Metabolic}$, $J^{Ub3}_{Metabolic}$ and $J^{Ub4}_{Metabolic}$ are the costs to the optimal strategies of the problem (4), for the constant assistance. These costs associated to Figures 7 and 4 (or Figure 5) respectively. One notes that the both cost functions $J_{Metabolic}$ and $J^{Ub4}_{Metabolic}$ represented the same control strategy.

5 DISCUSSION

The Figures 4, 6, 7 and 8 present the optimal trajectories: position, velocities, user's force and the motor force in this order.

The Figure 4 represents the solving of the optimal control problem (4), with the constant assistance. The optimal strategy (Figure 5) varies to the upper bound of the user's force. All control strategies started with the value of the upper bound, then its are gradually decreasing, except for the green curve (c.f. Figure 5 with Ub2) which pushes to the limit and then decreased. The positions and the velocities have a very similar profile. The peaks of user's force occur almost at the same time. When the higher the applied force a small cost function value, as observed in Table 1. For these upper bounds of the control arranged in this sequence: $Ub1 > Ub2 > Ub3 > Ub4$, these costs functions are classified in the inverse order: $J^{Ub1}_{Metabolic} < J^{Ub2}_{Metabolic} < J^{Ub3}_{Metabolic} < J^{Ub4}_{Metabolic}$.

Figures 6 and 8 illustrate a comparison between the problem described in (Cuerva et al., 2016) and the optimal control problem posed in (4). The optimal solution of each problem is indicated by the indices *Energy* and *Metabolic* respectively. For each assistance (constant and proportional), the both Figures 6 and 8, generated a control strategy, which starts and ends at the same values. In other words, we presume that the users have the same maximum force in the handrim wheelchair propulsion. For the constant assistance, the position trajectories are identical, but the velocity profile is just similar at the beginning and end and not achieving the maximum value. The middle part corresponds more or less to singular arc, which is not constant over time (Figure 5). The optimal strategy is composed of bang-bang and singular arc. The results also noted that the two peaks of user's force happening at the same time, as shown in the graph of the motor force. Because, the switching time of the both control are produced simultaneously. In the proportional assistance, the position trajectories are almost identical. In addition, the velocity profiles are identical at the beginning and the end. Thus, a part of the trajectory, which corresponds to the singular arc of the control, is composed of: bang-bang, inactivation, singular arc. In the both assistance, the cost function value given by the optimal control problem (4) is less than that the one achieved by (Cuerva et al., 2016) with three methods (c.f. Table 1). This can be explained by the presence of a period where the user' force is zero (inactivation period in the proportional assistance) or almost zero (singular arc in the constant assistance) on the time interval. Minimising a function of this type implies the presence of inactivation period. This phenomena takes place because the work of both agonistic and antagonistic muscles acting on a joint during rapid motion (Berret et al., 2008), which produced by the hydrolysis of ATP to ADP. During the inactivation period, the user's force applied at each joint is null. Therefore, the cost function is also equal to zero during this inactivation period.

Figure 7 represent a comparison for optimal control problem (4), without and with $tanh(F_p) = sign(F_p)$. As we can see, all the curves overlaid in each case, which is confirmed by the cost functions, since the two strategies expend almost the same amount of energy (Table 1).

6 CONCLUSION

This paper addresses an optimal control problem with minimal user effort during the propulsion wheelchair. The optimal control problem with state constraint is formulated using a power assisted wheelchair propulsion. The assistance represented a human-machine interaction, where a cooperation between the motor force and the user's force is enabled. Both assistance are described: constant and proportional. The optimal control problem with state constraint processed by the Pontryagin's Maximum Principle and then a numerical resolution is achieved when this problem is complex. The cost function is given by the metabolic function, which is a compromise between the absolute work of the user's force and his/her performance in the wheelchair. Finally, a comparison is established between two costs functions using this optimal control problem.

The numerical simulations show that the proposed metabolic cost function reduces the physical effort in comparison with the energy of user's force. All strategies obtained by solving the optimal control problem (4) costs less that the other three approaches presented in (Cuerva et al., 2016), for each assistance. The non-differentiable of this function allows to produce a period when the user's force is null.

Contrary to (Oukacha and Boizot, 2020), this work also includes singular arcs non-constants and the control strategy vanishes in the terms of the motor force model, which is proportional to the user's force applied on the handrim wheelchair propulsion.

The work is realized in QBA framework project (Bentaleb et al., 2019). This paper presented initial simulation results of the biomechanical part. The future work is to develop this model in order to realize the experimental part in the PSCHITT-PMR platform.

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